MathCity.org *Merging man and maths*	**Exercise 2.8 (Solutions)** **Mathematics (Science Group): 10th** Written by Amir Shehzad, Version: 1.0

Q.1 The product of two positive consecutive numbers is 182. Find the numbers.

Solution:

Suppose first positive number=x

Second positive number=x+1

According to given condition:

$$x(x+1)=182$$
$$x^2+x-182=0$$
$$x^2+14x-13x-182=0$$
$$x(x+14)-13(x+14)=0$$
$$(x+14)(x-13)=0$$
$$x+14=0 \quad \text{or} \quad x-13=0$$
$$x=-14 \quad \text{or} \quad x=13$$

As x is positive number therefore we neglect the negative value, So x=13

Then first positive number=x=13

Second positive number=x+1

=13+1=14

So, 13 and 14 are two required consecutive positive numbers.

Q.2 the sum of the square of three positive consecutive numbersis77.Find them.

Solution:

Let x, (x+1) and (x+2) be the three consecutive positive number

According to given condition:

$$x^2+(x+1)^2+(x+2)^2=77$$
$$x^2+\left[x^2+(1)^2+2(1)(x)\right]+\left[(x)^2+(2)^2+2(x)(2)\right]=77$$
$$x^2+x^2+1+2x+x^2+4+4x=77$$
$$3x^2+6x+5-77=0$$
$$3x^2+6x-72=0$$
$$3\left[x^2+2x-24\right]=0$$
$$x^2+2x-24=0 \quad \because 3\neq 0$$
$$x^2+6x-4x-24=0$$
$$x(x+6)-4(x+6)=0$$
$$(x+6)(x-4)=0$$
$$x+6=0 \quad \text{or} \quad x-4=0$$
$$x=-6 \quad \text{or} \quad x=4$$

As x is a positive number therefore we neglect the negative value and we take positive value of x like x=4

First positive number=x=4

Second positive number=x+1=4+1=5

Third positive number=x+2=4+2=6

So, 4, 5 and 6 are three required positive numbers.

Q.3 The sum of five times a number and the square of the number is 204. Find the number.

Solution: Let required number=x

Five times the number=5x

Square of number= x^2

According to given condition:

$$x^2+5x=204 \Rightarrow x^2+5x-204=0$$

$$x^2+17x-12x-204=0$$
$$x(x+17)-12(x+17)=0$$
$$(x+17)(x-12)=0$$
$$x+17=0 \quad \text{or} \quad x-12=0$$
$$\boxed{x=-17} \quad \text{or} \quad \boxed{x=12}$$

So, required number is -17 and 12.

Q.4 The product of five less than three times a certain number and one less than four times the number is 7. Find the number.

Solution: Let required number=x

Five less than three times the number=3x-5

One less than four times the number=4x-1

According to given condition:

$$(3x-5)(4x-1)=7$$
$$12x^2-3x-20x+5-7=0$$
$$12x^2-23x-2=0$$
$$12x^2-24x+x-2=0$$
$$12x(x-2)+1(x-2)=0$$
$$(x-2)(12x+1)=0$$
$$x-2=0 \quad \text{or} \quad 12x+1=0$$
$$\boxed{x=2} \quad \text{or} \quad 12x=-1$$
$$\boxed{x=\frac{-1}{12}}$$

So, required number is 2 or $\frac{-1}{12}$

Q.5 The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number.

Solution: Let required number=x

Reciprocal of the number= $\frac{1}{x}$

According to given condition:

$$x-\frac{1}{x}=\frac{15}{4}$$
$$\frac{x^2-1}{x}=\frac{15}{4}$$
$$4(x^2-1)=15x$$
$$4x^2-4-15x=0$$
$$4x^2-15x-4=0$$
$$4x^2-16x+1x-4=0$$
$$4x(x-4)+1(x-4)=0$$
$$(x-4)(4x+1)=0$$
$$x-4=0 \quad \text{or} \quad 4x+1=0$$
$$x=4 \quad \text{or} \quad 4x=-1$$
$$\boxed{x=4} \quad \text{or} \quad \boxed{x=\frac{-1}{4}}$$

So, required number is 4 or $\frac{-1}{4}$

Q.6 The sum of squares of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number.

Solution: Let

Digits at unit's place of a number=x

Digits at ten's place of a number=y

Required number=10y+x

According to first condition:

$$x^2+y^2=65 \quad \text{.....(i)}$$

According to second condition:

$$10y+x=9(x+y)$$
$$10y+x=9x+9y$$
$$10y-9y=9x-x$$
$$y=8x \quad \text{.....(ii)}$$

Put value of y in eq. (i)

$$x^2+(8x)^2=65$$
$$x^2+64x^2=65$$
$$65x^2=65$$
$$x^2=1$$
$$\sqrt{x^2}=\pm\sqrt{1}$$
$$x=\pm1$$
$$x=1 \quad \text{or} \quad x=-1$$

As x is a digit at unit's place which is always positive therefore we neglect the negative value and take the positive value like x=1

Put x=1 in eq. (i)

$$y=8(1)$$
$$y=8$$

So, required number $=10y+x$

$$=10(8)+1$$
$$=80+1=\boxed{80}$$

Q .7 The sum of the co-ordinates of a point is 9 and sum of their squares is 45. Find the co-ordinates of the point.

Solution: Let (x, y) are co-ordinates of required point.

According to given condition:

$$x+y=9 \qquad \text{.....(i)}$$
$$x^2+y^2=45 \qquad \text{.....(ii)}$$

From equation (i)

$$x+y=9$$
$$x=9-y \qquad \text{.....(iii)}$$

Putting this in eq. (ii), we get

$$(9-y)^2+y^2=45$$
$$(9)^2-2(9)(y)+(y)^2+(y)^2=45$$
$$81-18y+y^2+y^2=45$$
$$2y^2-18y+81-45=0$$
$$2y^2-18y+36=0$$
$$2(y^2-9y+8)=0$$
$$\therefore y^2-9y^2+18=0 \qquad \because 2\neq 0$$
$$y^2-6y-3y+18=0$$
$$y(y-6)-3(y-6)=0$$
$$(y-6)(y-3)=0$$
$$y-6=0 \quad \text{or} \quad y-3=0$$
$$y=6 \quad \text{or} \quad y=3$$

Putting values of y in eq. (iii)

$$y=6 \quad \text{or} \quad y=3$$
$$x=9-6 \quad \text{or} \quad x=9-6$$
$$\boxed{x=3} \quad \text{or} \quad \boxed{x=6}$$

The co-ordinates of the point are either (3,6) or (6,3).

Q.8 Find two integers whose sum is 9 and the difference of their squares is also 9.

Solution: Suppose x and y are two integers

According to given condition:

$$x+y=9 \qquad \text{.....(i)}$$
$$x^2-y^2=9 \qquad \text{.....(ii)}$$

From eq. (i)

$$x+y=9$$
$$x=9-y \qquad \text{.....(iii)}$$

Putting value of x in eq. (ii)

$$(9-y)^2-y^2=9$$
$$(9)^2+(y)^2-2(9)(y)-y^2=0$$
$$81+y^2-18y-y^2-9=0$$
$$72-18y=0$$
$$-18y=-72$$

$$y=\frac{-72}{-18} \Rightarrow \boxed{y=4}$$

Putting the value of y in eq. (iii)

$$x=9-y$$
$$x=9-4 \Rightarrow \boxed{x=5}$$

So, 4 and 5 are required integers.

Q.9 Find two integers whose difference is 4 and whose squares differ by 72.

Solution: Let x and y are two integers

According to given condition:

$$x-y=4 \quad .....(i)$$
$$x^2-y^2=72 \quad .....(ii)$$

From eq. (i)

$$x=4+y \quad .....(iii)$$

Putting the value of x in eq. (ii)

$$(4+y)^2-y^2=72$$
$$\left[(4)^2+(y)^2+2(4)(y)\right]-y^2=72$$
$$16+y^2+8y-y^2=72$$
$$16+8y=72$$
$$8y=72-16$$
$$8y=56$$
$$y=\frac{56}{8} \Rightarrow \boxed{y=7}$$

Putting the value of y in eq. (iii)

$$x=4+y$$
$$x=4+7 \Rightarrow \boxed{x=11}$$

So, required integers are 7 and 11.

Q.10 Find the dimensions of a rectangle, whose perimeter is 80cm and its area is 375 cm^2

Solution:

Let width of a rectangle=x cm

Length of rectangle =y cm

Perimeter of rectangle=80cm

Area of rectangle=375 cm^2

$\because$ 2(L+W) =P

2(x+ y) =80

$$x+y=\frac{80}{2}$$

x+ y=40 .....(i)

Area=Length $\times$ Width

375= $x \times y$

xy =375 .....(ii)

From eq. (i)

$$x+y=40$$
$$x=40-x$$

Put it in eq. (ii)

$$x(40-x)=375$$
$$40x-x^2=375$$
$$0=x^2-40x+375$$
$$x^2-40x+375=0$$
$$x^2-25x-15x+375=0$$
$$x(x-25)-15(x-25)=0$$
$$(x-15)(x-25)=0$$
$$x-15=0 \quad \text{or} \quad x-25=0$$
$$x=15 \quad \text{or} \quad x=25$$

Putting the value of x in eq. (i)

$$x=15 \quad \text{or} \quad x=25$$
$$15+y=40 \quad \text{or} \quad 25+y=40$$
$$y=40-15 \quad \text{or} \quad y=40-25$$
$$\boxed{y=25} \quad \text{or} \quad \boxed{y=15}$$

If x=15 then y=25 and x=25 then y=15

So, dimensions of rectangle are either 25cm by 15cm or 15cm by 25cm.