Question no1:

$2x+5=\sqrt{7x+16}$

$Solution: 2x+5=\sqrt{7x+16}$ ....(1)

Taking square on both side

$(2x+5)^2=(\sqrt{7x+16})^2$

$(2x)^2+(5)^2+2(2x)(5)=7x+16$

$4x^2+25+20x=7x+16$

$4x^2+20x-7x+25-16=0$

$4x^2+13x+9=0$

$4x^2+4x+9x+9=0$

$4x(x+1)+9(x+1)=0$

$(x+1)(4x+9)=0$

$x+1=0 \quad or \quad 4x+9=0$

$x=-1 \quad or \quad 4x=-9$

$x=-1 \quad or \quad x=\frac{-9}{4}$

Checking:

put $x=-1$ *in the equcation*

$2x+5=\sqrt{7x+16}$

$2(-1)+5=\sqrt{7(-1)+16}$

$-2+5=\sqrt{-7+16}$

$3=\sqrt{9}$

$3=3$ *which is true*

$x=\frac{-9}{4}$ *in the equcation*

$2x+5=\sqrt{7x+16}$

$2\left(\frac{-9}{4}\right)+5=\sqrt{7\left(\frac{-9}{4}\right)+16}$

$\frac{-18}{4}+5=\sqrt{\frac{-63}{4}+16}$

$\frac{-18+20}{4}=\sqrt{\frac{-63+64}{4}}$

$\frac{2}{4}=\sqrt{\frac{1}{4}} \quad , \quad \frac{1}{2}=\frac{1}{2}$ *which is true*

Solution set $\left\{-1,\frac{-9}{4}\right\}$

Question no 2:

$\sqrt{x+3}=3x-1$

$Solution: \sqrt{x+3}=3x-1$

Taking square on both side

$(\sqrt{x+3})^2=(3x-1)^2$

$x+3=(3x)^2+(1)^2-2(3x)(1)$

$x+3=9x^2+1-6x=0$

$9x^2-6x-x+1-3=0$

$9x^2-7x-2=0$

$9x^2-9x+2x-2=0$

$9x(x-1)+2(x-1)=0$

$(x-1)(9x+2)$

$x-1=0 \quad or \quad 9x+2=0$

$x=1 \quad or \quad 9x=-2$

$x=1 \quad or \quad x=-\frac{2}{9}$

Checking:

put $x=1$ *in the equation*

$\sqrt{x+3}=3x-1$

$\sqrt{1+3}=3(1)-1$

$\sqrt{4}=2 \Rightarrow 2=2$ *which is true*

put $x=-\frac{2}{9}$ *in the equation*

$\sqrt{x+3}=3x-1$

$\sqrt{-\frac{2}{9}+3}=3\left(-\frac{2}{9}\right)-1$

$\sqrt{\frac{-2+27}{9}}=-\frac{6}{9}-1$

$$\sqrt{\frac{25}{9}} = \frac{-6-9}{9}$$

$$\frac{5}{3} = \frac{-15}{9} \Rightarrow \frac{5}{3} = -\frac{5}{3}$$

$$As -\frac{2}{9}\ is\ an\ extraneous\ root$$

$$So\ the\ solution\ set\ is\ \{1\}$$

Question no 3:

$$4x = \sqrt{13x+14} - 3$$

$$4x = \sqrt{13x+14} - 3 \quad ......(1)$$

$$4x + 3 = \sqrt{13x+14}$$

$$(4x+3)^2 = \left(\sqrt{13x+14}\right)^2$$

$$(4x)^2 + (3)^2 + 2(4x)(3) = 13x + 14$$

$$16x^2 + 9 + 24x = 13x + 14$$

$$16x^2 + 9 + 24x - 13x - 14 = 0$$

$$16x^2 + 11x - 5 = 0$$

$$16x^2 + 16x - 5x - 5 = 0$$

$$16x(x+1) - 5(x+1) = 0$$

$$(x+1)(16x-5) = 0$$

$$x+1 = 0 \quad or \quad 16x - 5 = 0$$

$$x = -1 \quad or \quad 16x = 5$$

$$x = -1 \quad or \quad x = \frac{5}{16}$$

$$x = -1$$

$$4x = \sqrt{13x+14} - 3$$

$$4(-1) = \sqrt{13(-1)+14} - 3$$

$$-4 = \sqrt{-13+14} - 3$$

$$-4 = \sqrt{1} - 3$$

$$-4 = 1 - 3$$

$-4 \neq -2$ which is not true

$$Put\ x = \frac{5}{16}\ in\ the\ equatoin$$

$$4x = \sqrt{13x+14} - 3$$

$$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right) + 14} - 3$$

$$\frac{20}{16} = \sqrt{\frac{65}{16} + 14} - 3$$

$$\frac{5}{4} = \sqrt{\frac{65+224}{16}} - 3$$

$$\frac{5}{4} = \sqrt{\frac{289}{16}} - 3$$

$$\frac{5}{4} = \frac{17}{4} - 3$$

$$\frac{5}{4} = \frac{17-12}{4}$$

$$\frac{5}{4} = \frac{5}{4}\ which\ is\ true$$

So, the solution set is $\left\{\frac{5}{16}\right\}$

Question no 4:

$$\sqrt{3x+100} - x = 4$$

$$\sqrt{3x+100} - x = 4 \quad ......(1)$$

$$\sqrt{3x+100} = 4 + x$$

$$Taking\ square\ on\ both\ side$$

$$\left(\sqrt{3x+100}\right)^2 = (4+x)^2$$

$$3x + 100 = (4)^2 + (x)^2 + 2(4)(x)$$

$$3x + 100 = 16 + x^2 + 8x$$

$$x^2 + 8x + 16 - 3x - 100 = 0$$

$$x^2 + 5x - 84 = 0$$

$$x^2 + 12x - 7x - 84 = 0$$

$$x(x+12) - 7(x+12) = 0$$

$$(x+12)(x-7) = 0$$

$$x + 12 = 0 \quad or \quad x - 7 = 0$$

$x=-12 \quad or \quad x=7$

$x=-12$

$\sqrt{3x+100}-x=4$

$\sqrt{3(-12)+100}-(-12)=4$

$\sqrt{-36+100}+12=4$

$\sqrt{64}+12=4$

$8+12=4$

$20\neq 4$

$So -12\ is\ an\ extraneous\ root$

$put\ x=7\ in\ the\ equation$

$\sqrt{3x+100}-x=4$

$\sqrt{3(7)+100}-(7)=4$

$\sqrt{21+100}-7=4$

$\sqrt{121}-7=4$

$11-7=4$

$4=4\ which\ is\ true$

$So\ the\ solution\ set\ is\ \{7\}$

Question no 5:

$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$ ......(1)

Squaring both sides:

$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$

$\left(\sqrt{x+5}\right)^2+\left(\sqrt{x+21}\right)^2+2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right)=x+60$

$x+5+x+21+2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right)=x+60$

$2x+26+2\sqrt{x^2+21x+5x+105}=x+60$

$2\sqrt{x^2+26x+105}=x+60-2x-26$

$2\sqrt{x^2+26x+105}=-x+34$

$2\sqrt{x^2+26x+105}=34-x$

Again squaring both sides:

$\left(2\sqrt{x^2+26x+105}\right)^2=(34-x)^2$

$4(x^2+26x+105)=(34)^2+x^2-2(34)(x)$

$4x^2+104x+420=1156+x^2-68x$

$4x^2+104x+420-1156-x^2+68x=0$

$3x^2+172x-736=0$

$ax^2+bx+c=0$

$a=3 \quad b=172 \quad c=-736$

By using quadratic formula:

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(172)\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$

$x=\dfrac{-172\pm\sqrt{29584+8832}}{6}$

$x=\dfrac{-172\pm\sqrt{38416}}{6}$

$x=\dfrac{-172\pm 196}{6}$

$x=\dfrac{-172-196}{6} \quad or \quad x=\dfrac{-172+196}{6}$

$x=\dfrac{-368}{6} \quad or \quad x=\dfrac{24}{6}$

$x=\dfrac{-184}{3} \quad or \quad x=4$

Checking:

Putting $x=4$ *in equation* (1)

$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$

$\sqrt{9}+\sqrt{25}=\sqrt{64}$

$3+5=8$

$8=8 \quad which\ is\ true$

Putting $x=\dfrac{-184}{3}$

$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

$\sqrt{\left(\dfrac{-184}{3}\right)+5}+\sqrt{\left(\dfrac{-184}{3}\right)+21}=\sqrt{\left(\dfrac{-184}{3}\right)+60}$

$$\sqrt{\frac{-184+15}{3}}+\sqrt{\frac{-184+63}{3}}=\sqrt{\frac{-184+180}{3}}$$

$$\sqrt{\frac{-169}{3}}+\sqrt{\frac{-121}{3}}=\sqrt{\frac{-4}{3}}$$

$$\sqrt{\frac{-1\times169}{3}}+\sqrt{\frac{-1\times121}{3}}=\sqrt{\frac{-1\times4}{3}}$$

$$\frac{13i}{\sqrt{3}}+\frac{11i}{\sqrt{3}}=\frac{2i}{\sqrt{3}} \qquad \because i=\sqrt{-1}$$

$$\frac{13i+11i}{\sqrt{3}}=\frac{2i}{\sqrt{3}}$$

$$\frac{24i}{\sqrt{3}}=\frac{2i}{\sqrt{3}} \quad \text{which is not true}$$

As $x=\frac{-184}{3}$ is extraneous root

Solution set is $\{4\}$

Question no 6:

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$$

Solution:

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6} \quad ......(1)$$

Squaring both sides:

$$\left(\sqrt{x+1}+\sqrt{x-2}\right)^2=\left(\sqrt{x+6}\right)^2$$

$$(x+1)^2+(x-2)^2+2(x+1)(x-2)=x+6$$

$$x+1+x-2+2\sqrt{(x+1)(x-2)}=x+6$$

$$2x-1+2\sqrt{x^2-2x+1x-2}=x+6$$

$$2\sqrt{x^2-x-2}=x+6-2x+1$$

$$2\sqrt{x^2-x-2}=7-x$$

Again squaring both sides

$$\left(2\sqrt{x^2-x-2}\right)^2=(7-x)^2$$

$$4\left(x^2-x-2\right)=(7)^2+x^2-2(7)(x)$$

$$4x^2-4x-8=49+x^2-14x$$

$$4x^2-x-4x+14x-8-49=0$$

$$3x^2+10x-57=0$$

$$3x^2+19x-9x-57=0$$

$$3(3x+19)-3(3x+19)=0$$

$$(3x+19)(x-3)=0$$

$$3x+19=0 \quad or \quad x-3=0$$

$$3x=-19 \quad or \quad x=3$$

$$x=\frac{-19}{3}$$

Checking:

Putting $x=3$ in the equation (1)

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$$

$$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$$

$$\sqrt{4}+\sqrt{1}=\sqrt{9}$$

$$2+1=3$$

$$3=3 \quad \text{which is true}$$

Putting $x=\frac{-19}{3}$ in the equation (1)

$$\sqrt{\left(\frac{-19}{3}\right)+1}+\sqrt{\left(\frac{-19}{3}\right)-2}=\sqrt{\left(\frac{-19}{3}\right)+6}$$

$$\sqrt{\frac{-19+3}{3}}+\sqrt{\frac{-19-6}{3}}=\sqrt{\frac{-19+18}{3}}$$

$$\sqrt{\frac{-1\times16}{3}}+\sqrt{\frac{-1\times25}{3}}=\sqrt{\frac{-1\times1}{3}}$$

$$\frac{4i}{\sqrt{3}}+\frac{5i}{\sqrt{3}}=\frac{1i}{\sqrt{3}}$$

$$\frac{4i+5i}{\sqrt{3}}=\frac{1i}{\sqrt{3}}$$

$$\frac{9i}{\sqrt{3}}=\frac{1i}{\sqrt{3}} \quad \text{which is not true}$$

$As\ x=\frac{-19}{3} is\ an\ extraneous\ root$

$So\ the\ solution\ set\ is\ \{3\}$

Question no 7:

$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

$Solution:$

$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x} \quad ......(1)$

$Squaring\ both\ sides:$

$\left(\sqrt{11-x}-\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$

$\left(\sqrt{11-x}\right)^2+\left(\sqrt{6-x}\right)^2-2\left(\sqrt{11-x}\right)\left(\sqrt{6-x}\right)=27-x$

$11-x+6-x-2\sqrt{(11-x)(6-x)}=27-x$

$17-2x-2\sqrt{66-11x-6x+x^2}=27-x$

$-2\sqrt{66-17x+x^2}=27-x-17+2x$

$11-x+6-x-2\sqrt{(11-x)(6-x)}=27-x$

$-2\sqrt{66-17x+x^2}=27-x-17+2x$

$-2\sqrt{66-17x+x^2}=10+x$

$Again\ squaring\ both\ sides$

$\left(-2\sqrt{66-17x+x^2}\right)^2=(10+x)^2$

$(-2)^2\left(\sqrt{66-17x+x^2}\right)^2=(10+x)^2$

$4\left(66-17x+x^2\right)=(10)^2+(x)^2+2(10)(x)$

$264-68x+4x^2=100+x^2+20x$

$4x^2-x^2-68x-20x+264-100=0$

$3x^2-88x+164=0$

$By\ applying\ quadratic\ formula$

$a=3 \quad b=-88 \quad c=164$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-88)\pm\sqrt{(-88)^2-4(3)(164)}}{2(3)}$

$x=\frac{88\pm\sqrt{7744-1968}}{6}$

$x=\frac{88\pm\sqrt{5776}}{6}$

$x=\frac{88-76}{6} \quad or \quad x=\frac{88+76}{6}$

$x=\frac{12}{6} \quad or \quad x=\frac{164}{6}$

$x=2 \quad or \quad x=\frac{82}{3}$

$Checking:$

$Putting\ x=\frac{82}{3}\ in\ the\ equation\ (1)$

$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

$\sqrt{11-\frac{82}{3}}-\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$

$\sqrt{\frac{33-82}{3}}-\sqrt{\frac{18-82}{3}}=\sqrt{\frac{81-82}{3}}$

$\sqrt{\frac{1-\times49}{3}}-\sqrt{\frac{-1\times64}{3}}=\sqrt{\frac{-1\times1}{3}}$

$\frac{7i}{\sqrt{3}}-\frac{8i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \qquad \because i=\sqrt{-1}$

$\frac{4i-8i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$

$\frac{-i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \quad which\ is\ not\ true$

$Putting\ x=2\ in\ the\ equation\ (1)$

$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

$\sqrt{11-2}-\sqrt{6-2}=\sqrt{27-2}$

$\sqrt{9}-\sqrt{4}=\sqrt{25}$

$3-2=5$

$1=5$ *which is not true*

Here 2 *and* $\frac{82}{3}$ *are extraneous roots.*

So the solution set is $\{\ \}$

Question no 8:

$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

Solution:

$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

takinig squarinng on both side

$(\sqrt{4a+x}-\sqrt{a-x})^2=(\sqrt{a})^2$

$(\sqrt{4a+x})^2+(\sqrt{a-x})^2-2(\sqrt{4a+x})(\sqrt{a-x})=(\sqrt{a})^2$

$4a+x+a-x-2\sqrt{(4a+x)(a-x)}=a$

$5a-2\sqrt{4a^2-4ax+ax-x^2}=a$

$-2\sqrt{4a^2-3ax-x^2}=a-5a$

$-2\sqrt{4a^2-3ax-x^2}=-4a$

$\sqrt{4a^2-3ax-x^2}=2a$

taking again square on both side

$\left(\sqrt{4a^2-3ax-x^2}\right)^2=(2a)^2$

$4a^2-3ax-x^2=4a^2$

$4a^2+3ax+x^2-4a^2=0$

$3ax+x^2=0$

$x(3a+x)=0$

$x=0 \qquad 3a+x=0$

$x=0 \qquad x=-3a$

Checking:

Put $x=0$ *in the equation*

$\sqrt{4a+0}-\sqrt{a-0}=\sqrt{a}$

$2\sqrt{a}-\sqrt{a}=\sqrt{a}$

$\sqrt{a}=\sqrt{a}$ *which is true*

Put $x=-3a$ *in the equation*

$\sqrt{4a-3a}-\sqrt{a+3a}=\sqrt{a}$

$\sqrt{a}-\sqrt{2a}=\sqrt{a}$

$-\sqrt{a}=\sqrt{a}$ *which is not true*

As $-3a$ *is extraneous root*

so the solution set is $\{0\}$

Question no 9:

$\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$

Solution:

$\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1 \quad ......(1)$

Let $x^2+x=y \quad ......(2)$

Put it in equation (1)

$\sqrt{y+1}-\sqrt{y-1}=1$

$\sqrt{y+1}=1+\sqrt{y-1}$

$y+1=1+y-1+2\sqrt{y-1}$

$y+1=y+2\sqrt{y-1}$

$y+1-y=2\sqrt{y-1}$

$1=2\sqrt{y-1}$

Taking again squaring both sides

$(1)^2=\left(2\sqrt{y-1}\right)^2$

$1=4(y-1)$

$1=4y-4$

$1+4=4y$

$5=4y$

From equation (2) *put* $y=x^2+x$

$5=4(x^2+x)$

$5=4x^2+4x$

$0=4x^2+4x-5$

$4x^2+4x-5=0$

By applying quadratic formula

$a=4 \quad b=4 \quad c=-5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$$

$$x=\frac{-4\pm\sqrt{16+80}}{8}$$

$$x=\frac{-4\pm\sqrt{96}}{8}$$

$$x=\frac{-4\pm\sqrt{16\times6}}{8}$$

$$x=\frac{-4\pm4\sqrt{6}}{8}$$

$$x=\frac{4(-1\pm\sqrt{6})}{8}$$

$$x=\frac{-1\pm\sqrt{6}}{2}$$

$$\text{The solution is}\left\{\frac{-1\pm\sqrt{6}}{2}\right\}$$

Question N0 10:

$$\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$$

Solution:

$$\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3 \quad ......(1)$$

$$\text{Let } x^2+3x=y \quad ......(2)$$

Put it in equation (1)

$$\sqrt{y+8}+\sqrt{y+2}=3$$

$$\sqrt{y+8}=3-\sqrt{y+2}$$

Taking square on both sides

$$\left(\sqrt{y+8}\right)^2=\left(3-\sqrt{y+2}\right)^2$$

$$y+8=(3)^2+\left(\sqrt{y+2}\right)^2-2(3)(\sqrt{y+2})$$

$$y+8=9+y+2-6\sqrt{y+2}$$

$$y+8=11+y-6\sqrt{y+2}$$

$$y-y+8-11=6\sqrt{y+2}$$

$$-3=6\sqrt{y+2}$$

Again squaring both sides

$$(-3)^2=\left(6\sqrt{y+2}\right)^2$$

$$9=36(y+2)$$

$$9=36y+72$$

$$9-72=36y$$

$$-63=36y$$

$$36y=-63$$

$$\text{Put } x^2+3x=y$$

$$36\left(x^2+3x\right)=-63$$

$$36x^2+108x+63=0$$

Solving by quadratic formula

$$a=36 \quad b=108 \quad c=63$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-108\pm\sqrt{(108)^2-4(36)(63)}}{2(36)}$$

$$x=\frac{-108\pm\sqrt{11664-9072}}{72}$$

$$x=\frac{-108\pm\sqrt{2592}}{72}$$

$$x=\frac{-108\pm\sqrt{4\times4\times9\times9\times2}}{72}$$

$$x=\frac{-108\pm\sqrt{4^2\times9^2\times2}}{72}$$

$$x=\frac{-108\pm4\times9\sqrt{2}}{72}$$

$$x=\frac{-108\pm36\sqrt{2}}{72}$$

$$x=\frac{36(-3\pm\sqrt{2})}{72}$$

$$x = \frac{-3 \pm \sqrt{2}}{72}$$

$$So\ the\ solution\ set\ is\ \left(\frac{-3 \pm \sqrt{2}}{72}\right)$$

Question no 11:

$$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$$

Solution:

$$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$$

$$Put\ x^2+3x=y \ ....(1)$$

$$\sqrt{y+9}+\sqrt{y+4}=5$$

$$\sqrt{y+9}=5-\sqrt{y+4}$$

Taking square on both side

$$\left(\sqrt{y+9}\right)^2=\left(5-\sqrt{y+4}\right)^2$$

$$y+9=(5)^2+y+4-2(5)\sqrt{y+4}$$

$$y+9=25+y+4-10\sqrt{y+4}$$

$$y+9-25-y-4=-10\sqrt{y+4}$$

$$-20=-10\sqrt{y+4}$$

$$2=\sqrt{y+4}$$

Taking again square on both side

$$4=y+4 \Rightarrow y=0$$

$$Put\ y=0\ in\ equation\ 1$$

$$x^2+3x=0 \Rightarrow x(x+3)=0$$

$$x=0 \quad or \quad x+3=0$$

$$x=0 \quad or \quad x=-3$$

$$So\ the\ solution\ set\ is\ \{0,-3\}$$