Unit 13 Probability

EXERCISE 13.1

1. Arshad rolls a dice, with sides labelled L, M, N, O, P, U. What is the probability that the dice lands on consonant?

Solution

$S = \{L, M, N, O, P, U\} \quad ; n(S) = 6$

$A = \{L, M, N, P\} \quad ; n(A) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

2. Shazia throws a pair of fair dice. What will be the probability of getting:
   (i) sum of dots is at least 4.
   (ii) product of both dots is between 5 to 10.
   (iii) the difference between both the dots is equal to 4.
   (iv) number at least 5 on the first dice and the number at least 4 on the second dice.

Solution

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),\\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6),\\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$n(S) = 36$

(i) Sum of dots is at least 4
When a pair of fair dice is rolled, the sample space is as follows;
Let A be the even, when Sum of dots is at least 4

A = {(1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 3),
(2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3),
(3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3),
(4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3),
(5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3),
(6, 4), (6, 5), (6, 6)}.

$n(A) = 33$
$P(A) = \frac{n(A)}{n(S)} = \frac{33}{36} = \frac{11}{12}$

(ii) Product of both dots is between 5 to 10
$B = \{(1,5), (1,6), (2,3), (2,4), (2,5), (3,2), (3,3), (4,2), (5,1), (5,2), (6,1)\}$
$n(B) = 11$
$P(B) = \frac{n(B)}{n(S)} = \frac{11}{36}$

(iii) The difference between both the dots is equal to 4
$C = \{(1,5), (2,6), (5,1), (6,2)\}$
$n(C) = 4$
$P(C) = \frac{n(C)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

(iv) Number at least 5 on the first dice and the number at least 4 on the second dice
$D = \{(5,4), (5,5), (5,6), (6,4), (6,5), (6,6)\}$
$n(D) = 4$
$P(D) = \frac{n(D)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

3. One alphabet is selected at random from the word “MATHEMATICS". Find the probability of getting:

(i)	vowel	(ii)	consonant	(iii)	an E
(iv)	an A	(v)	not M	(vi)	not T

Solution
Given word **MATHEMATICS**
Total words are 11
$n(S) = 11$

(i) Vowel $A = \{A, E, A, I\}$; $n(A) = 4$ $\text{P(A)} = \frac{n(A)}{n(S)} = \frac{4}{11}$	**(ii) Consonant** $B = \{M, T, H, M, T, C, S\}$; $n(B) = 7$ $\text{P(B)} = \frac{n(B)}{n(S)} = \frac{7}{11}$
(iii) an E $C = \{E\}$; $n(C) = 1$ $\text{P(C)} = \frac{n(C)}{n(S)} = \frac{1}{11}$	**(iv) an A** $D = \{A, A\}$; $n(D) = 2$ $\text{P(D)} = \frac{n(D)}{n(S)} = \frac{2}{11}$
(v) not M $E = \{A, T, H, E, T, I, C, S\}$; $n(E) = 9$ $\text{P(E)} = \frac{n(E)}{n(S)} = \frac{9}{11}$	**(vi) not T** $F = \{M, A, H, E, M, A, I, C, S\}$; $n(F) = 9$ $\text{P(F)} = \frac{n(F)}{n(S)} = \frac{9}{11}$

4 Aslam rolled a dice. What is the probability of getting the numbers 3 or 4? Also find the probability of not getting the numbers 3 or 4.

Solution

$S = \{1,2,3,4,5,6\} \quad ; n(S) = 6$

$A = \{3,4\} \quad ; n(A) = 2$

$\text{P(A)} = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

Probability of not gettig 3 or 4 $= \text{P(A}') = 1 - \text{P(A)} = 1 - \frac{1}{3} = \frac{2}{3}$

5 Abdul Hadi labelled cards from 1 to 30 and put them in a box. He selects a card at random. What is the probability that selected card containing:

(i) the number 25
(ii) number between 17 to 22
(iii) number at least 20
(iv) number not 27 and 29
(v) number not between 12 – 15

Solution

Since the cards are labeled from 1 to 30, therefore $n(S) = 30$

(i) $A = \{25\} \quad ; n(A) = 1 \Rightarrow \text{P(A)} = \frac{n(A)}{n(S)} = \frac{1}{30}$

(ii) $A = \{17,18,19,20,21,22\} \quad ; n(A) = 6 \Rightarrow \text{P(A)} = \frac{n(A)}{n(S)} = \frac{6}{30} = \frac{1}{5}$

(iii) $A = \{20,21,22,23,24,25,26,27,28,29,30\} \quad ; n(A) = 11 \Rightarrow \text{P(A)} = \frac{n(A)}{n(S)} = \frac{11}{30}$

(iv) $A = \{27,29\} \quad ; n(A) = 2 \Rightarrow \text{P(A)} = \frac{n(A)}{n(S)} = \frac{2}{30} = \frac{1}{15}$

Probability of not gettig 27 and 29 $= \text{P(A}') = 1 - \text{P(A)} = 1 - \frac{1}{15} = \frac{14}{15}$

(v) $A = \{12,13,14,15\} \quad ; n(A) = 4 \Rightarrow \text{P(A)} = \frac{n(A)}{n(S)} = \frac{4}{30} = \frac{2}{15}$

Probability of not gettig 12 to 15 $= \text{P(A}') = 1 - \text{P(A)} = 1 - \frac{2}{15} = \frac{13}{15}$

6. The probability that Ayesha will pass the examination is 0.85. What will be the probability that Ayesha will not pass the examination?

Solution

Ayesha will pass the examinition $= \text{P(A)} = 0.85$

Ayesha will not pass the examinition $= \text{P(A}') = 1 - \text{P(A)} = 1 - 0.85 = 0.15$

7 Taabish tossed a fair coin and rolled a fair dice once. Find the probability of the following events:

(i) tail on coin and at least 4 on dice.

(ii) head on coin and the number 2,3 on dice.

(iii) head and tail on coin and the number 6 on dice.

(iv) not tail on coin and the number 5 on dice.

(v) not head on coin and the number 5 and 2 on dice.

Solution

When a fair coin is tossed and fair dice is rolled, the sample space is as follows;

Die / Coin	1	2	3	4	5	6
H	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
T	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

Sample space = {(H,1), (H,2), (H,3), (H,4), (H,5), (H,6)
(T,1), (T,2), (T,3), (T,4), (T,5), (T,6)}

Therefore $n(S) = 12$

(i) $A = \{(T,4),(T,5),(T,6)\}\ ;n(A) = 3 \Rightarrow \text{P(A)} = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$

(ii) $A = \{(H,2),(H,3)\}\ ;n(A) = 2 \Rightarrow \text{P(A)} = \frac{n(A)}{n(S)} = \frac{2}{12} = \frac{1}{6}$

(iii) $A = \{(H,5),(T,5)\}\ ;n(A) = 2 \Rightarrow \text{P(A)} = \frac{n(A)}{n(S)} = \frac{2}{12} = \frac{1}{6}$

(iv) $A = \{(T,5)\}\quad ;n(A) = 1 \Rightarrow \text{P(A)} = \frac{n(A)}{n(S)} = \frac{1}{12}$

$\text{P(A}') = 1 - \text{P(A)} = 1 - \frac{1}{12} = \frac{11}{12}$

(v) $A = \{1(H,5),(H,2)\}\quad ;n(A) = 2 \Rightarrow \text{P(A)} = \frac{n(A)}{n(S)} = \frac{2}{12} = \frac{1}{6}$

$\text{P(A}') = 1 - \text{P(A)} = 1 - \frac{1}{6} = \frac{5}{6}$

8. A card is selected at random from a well shuffled pack of 52 plying cards. What will be the probability of selecting:

(i) a queen (ii) neither a queen nor a jack

Solution

Since there are 52 playing cards, so $n(S) = 52$

(i) $A = 4$ queen ; $n(A) = 4 \Rightarrow \mathrm{P(A)} = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

(ii) $A = 4$ queen and 4 Jack ; $n(A) = 8 \Rightarrow \mathrm{P(A)} = \frac{n(A)}{n(S)} = \frac{8}{52} = \frac{2}{13}$

Probability of selecting neither a queen nor a jack $= \mathrm{P(A')} = 1 - \mathrm{P(A)}$

Probability of selecting neither a queen nor a jack $= \mathrm{P(A')} = 1 - \frac{2}{13} = \frac{11}{13}$

Probability of selecting neither a queen nor a jack $= \mathrm{P(A')} = \frac{11}{13}$

9. A card is chosen at random from a pack of 52 playing cards. Find the probability of getting:

(i) a jack (ii) no diamond

Solution

Since there are 52 playing cards, so $n(S) = 52$

(i) $A = 4$ jack ; $n(A) = 4 \Rightarrow \mathrm{P(A)} = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

(ii) $A = 13$ diamond ; $n(A) = 13 \Rightarrow \mathrm{P(A)} = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

Probability of selecting no diamond $= \mathrm{P(A')} = 1 - \mathrm{P(A)}$

Probability of selecting no diamond $= \mathrm{P(A')} = 1 - \frac{1}{4}$

Probability of selecting no diamond $= \mathrm{P(A')} = \frac{3}{4}$

EXERCISE 13.2

1. A researcher collected data on number of deaths from Horse-Ricks in Russian Army crops over to years. The table is as follows:

No. of death	0	1	2	3	4	5	6
Frequency	60	50	87	40	32	15	10

Find the relative frequency of the given data.

Solution: using the formula $r.f = \frac{f}{\Sigma f}$

No. of death	f	$r.f.$
0	60	$\frac{60}{294} = \frac{10}{49}$
1	50	$\frac{25}{147}$
2	87	$\frac{29}{98}$
3	40	$\frac{20}{147}$
4	32	$\frac{16}{147}$
5	15	$\frac{5}{98}$
6	10	$\frac{5}{147}$
Total	$\Sigma f = 294$	

2. The frequency of defective products in 750 samples are shown in the following table. Find the relative frequency for the given table.

No. of defectives per sample	0	1	2	3	4	5	6	7	8
No. of sample	120	140	94	85	105	50	40	66	50

Solution: using the formula $\boldsymbol{r.f} = \frac{f}{\sum f}$

No. of defective per sample	f	$r.f.$
0	120	$\frac{4}{25}$
1	140	$\frac{14}{75}$
2	94	$\frac{47}{375}$
3	85	$\frac{17}{150}$
4	105	$\frac{21}{150}$
5	50	$\frac{1}{15}$
6	40	$\frac{4}{75}$
7	66	$\frac{66}{750} = \frac{33}{375}$
8	50	$\frac{1}{15}$
Total	$\sum f = 750$	

3. A quiz competition on general knowledge is conducted. The number of corrected answers out of 5 questions for 100 sets of questions is given below.

X	0	1	2	3	4	5
f	10	23	15	25	18	9

Find the relative frequencies for the given data.

Solution: using the formula $\boldsymbol{r.f} = \frac{f}{\Sigma f}$

X	*f*	*r.f.*
0	10	$\frac{1}{10}$
1	23	$\frac{23}{100}$
2	15	$\frac{3}{20}$
3	25	$\frac{1}{4}$
4	18	$\frac{9}{50}$
5	09	$\frac{9}{100}$
Total	$\Sigma f = 100$	

4. A survey was conducted from the 50 students of a class and asked about their favourite food. The responses are as under:

Name of food item	Biryani	Fresh Juice	Chicken	Bar. B.Q	Sweets
No. of students	40	07	21	15	25

(i) how many percentages of students like biryani?

(ii) how many percentages of students like chicken?

(iii) which food is the least like by the students?

(iv) which food is the most prefer by the students?

Solution

Total number of students = 50, , so $n(S) = 50$

(i) Relative frequency of students who like biryani $= \frac{40}{50} = 0.8 = 80\%$

(ii) Relative frequency of students who like chicken $= \frac{21}{50} = 0.42 = 42\%$

(iii) Fresh Juice is the least like by the students. i.e. 7 students out of 50.

(iv) Biryani is the most prefer by the students. i.e. 40 students out of 50.

5. In 500 trials of a thrown of two dice, what is expected frequency that the sum will be greater than 8?

Solution

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$$
$$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$$
$$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$$
$$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$$
$$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$$
$$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$n(S) = 36$

Let A be the event that sum will be greater than 8;

$A = \{(3,6),(4,5),(4,6),(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)\}$

$n(A) = 10$

$\text{P(A)} = \frac{n(A)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

$\text{Expected Frequency} = \text{E(A)} = \text{N} \times \text{P(A)} = 500 \times \frac{5}{18} = 138.89 \approx 139$

6. What is the expectation of a person who is to get Rs. 120 if he obtains at least 2 heads in single toss of three coins?

Solution

$\text{S} = \{\text{HHH, HTH, HHT, THH, TTH, THT, HTT, TTT}\}$; $n(S) = 8$

$\text{A} = \{\text{HHH, HTH, HHT, THH}\}$; $n(S) = 4$

$\text{P(A)} = \frac{n(A)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

$\text{Expected Frequency} = \text{E(A)} = \text{N} \times \text{P(A)} = 120 \times \frac{1}{2} = 60$

7. Find the expected frequencies of the given data if the experiment is repeated 200 times.

x	0	1	2	3	4	5	6
$P(x)$	0.11	0.21	0.17	0.18	0.09	0.17	0.07

Solution: using the formula $\mathbf{EF = P(X) \times 200}$

X	0	1	2	3	4	5	6
$P(X)$	0.11	0.21	0.17	0.18	0.09	0.17	0.07
Expected Frequency	22	42	34	36	18	34	14

8. The probability of getting 5 sixes while tossing six dice is $\frac{2}{5}$. How many times would you expect it to show 5 sixes?

Solution

$\text{N} = 200$; $\text{P(A)} = \frac{2}{5}$

$\text{Expected Frequency} = \text{E(A)} = \text{N} \times \text{P(A)} = 200 \times \frac{2}{5} = 80 \text{ times}$

REVIEW EXERCISE 13

1. Four options are given against each statement. Encircle the correct option.

(i) Each element of the sample space is called:

(a) event (b) experiment
(c) ✓ sample point (d) outcomes

(ii) An outcome which represents how many times we expect the things to be happened is called:

(a) outcomes (b) ✓ favourable outcome
(c) sample space (d) sample point

(iii) Which one tells us how often a specific event occurs relative to the total number of frequency event or trials?

(a) expected frequency (b) sum of relative frequency
(c) ✓ relative frequency (d) frequency

(iv) Estimated probability of an event occurring is also known as:

(a) ✓ relative frequency (b) expected frequency
(c) class boundaries (d) sum of expected frequency

(v) The sum of all expected frequencies is equal to the fixed number of:

(a) ✓ trials (b) relative frequencies
(c) outcomes (d) events

(vi) The chance of occurrence of a particular event is called:

(a) sample space (b) estimated probability
(c) ✓ probability (d) expected frequency

(vii) An event which will probably occur. It has greater chance to occur is called:

(a) equally likely event (b) ✓ likely event
(c) unlikely event (d) certain event

(viii) Find out the total number of possible sample space when 4 dice are rolled.:

(a) 6^2 (b) 6^3 (c) ✓ 6^4 (d) 6^6

(ix) While rolling a pair of dice, what will be the probability of double 2?

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{5}{6}$ (d)✓ $\frac{1}{36}$

(x) A card is chosen from a pack of 52 playing cards, find the probability of getting no jack and king:

(a) $\frac{2}{13}$ (b)✓ $\frac{11}{13}$ (c) $\frac{2}{52}$ (d) $\frac{11}{52}$

2. Define the following:

(i) relative frequency (ii) expected frequency

Solution

Relative Frequency: Relative Frequency is an estimated probability of an event occurring when an experiment is repeated a fixed number of times.

Expected Frequency: Expected Frequency is a measure that estimates how often an event should be occur depended on probability. Expected frequency is found by using the following method;

Expected Frequency = Total number of trials × Probability of an event

Expected Frequency = E(A) = N × P(A)

3. An urn contains 10 red balls, 5 green balls and 8 blue balls. Find the probability of selecting at random.

(i) a green ball (ii) a red ball (iii) a blue ball

(iv) not a red ball (v) not a green ball

Solution

Since total balls are 23, therefore $n(S) = 10 + 5 + 8 = 23$

(i) $A =$ Green balls $;n(A) = 5 \Rightarrow \mathrm{P(A)} = \frac{n(A)}{n(S)} = \frac{5}{23}$

(ii) $A =$ Red balls $;n(A) = 10 \Rightarrow \mathrm{P(A)} = \frac{n(A)}{n(S)} = \frac{10}{23}$

(iii) $A =$ Blue balls $;n(A) = 8 \Rightarrow \mathrm{P(A)} = \frac{n(A)}{n(S)} = \frac{8}{23}$

(iv) $A =$ Red balls $;n(A) = 10 \Rightarrow \mathrm{P(A)} = \frac{n(A)}{n(S)} = \frac{10}{23}$

Probability of not gettig a red ball $= \mathrm{P(A')} = 1 - \mathrm{P(A)} = 1 - \frac{10}{23} = \frac{13}{23}$

(v) $A =$ Green balls $;n(A) = 5 \Rightarrow \mathrm{P(A)} = \frac{n(A)}{n(S)} = \frac{5}{23}$

Probability of not gettig a green ball $= \mathrm{P(A')} = 1 - \mathrm{P(A)} = 1 - \frac{5}{23} = \frac{18}{23}$

4. Three coins are tossed together. what is the probability of getting:

(i) exactly three heads

(ii) at least two tails

(iii) not at least two heads

(iv) not exactly two heads

Solution

$S = \{HHH, HTH, HHT, THH, TTH, THT, HTT, TTT\}$; $n(S) = 8$

(i) $A = \{HHH\}$; $n(A) = 1 \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$

(ii) $A = \{TTH, THT, HTT, TTT\}$; $n(A) = 4 \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

(iii) $A = \{HHH, HTH, HHT, THH\}$; $n(A) = 4 \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

Probability of not gettig at least two Heads $= P(A') = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

(iv) $A = \{HTH, HHT, THH\}$; $n(A) = 3 \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$

Probability of not gettig exactly two Heads $= P(A') = 1 - P(A) = 1 - \frac{3}{8} = \frac{5}{8}$

5. A card is drawn from a well shuffled pack of 52 playing cards. What will be the probability of getting:

(i) king or jack of red colour

(ii) not "2" of club and spade

Solution

Since there are 52 playing cards, so $n(S) = 52$

(i) $A =$ king or jack of red colour ; $n(A) = 2 + 2 = 4 \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

(ii) $A = 2$ of club and spade ; $n(A) = 2 \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

Probability of selecting not 2 of club and spade $= P(A') = 1 - P(A)$

Probability of selecting not 2 of club and spade $= P(A') = 1 - \frac{1}{26} = \frac{25}{26}$

Probability of selecting not 2 of club and spade $= P(A') = \frac{25}{26}$

6. Six coins are tossed 600 times. The number of occurrence of tails are recorded and shown in the table given below:

No. of tails	0	1	2	3	4	5	6
Frequency	110	90	105	80	76	123	16

Find the relative frequency of given table.

Solution: using the formula $\boldsymbol{r.f} = \frac{\boldsymbol{f}}{\sum \boldsymbol{f}}$

No. of tails	0	1	2	3	4	5	6	Total
f	110	90	105	80	76	123	16	$\Sigma f = 600$
Relative Frequency	$\frac{11}{60}$	$\frac{3}{20}$	$\frac{7}{40}$	$\frac{2}{15}$	$\frac{19}{150}$	$\frac{41}{200}$	$\frac{2}{75}$	

7. From a lot containing 25 items, 8 items are defective. Find the relative frequency of non-defective items, also find the expected frequency of non-defective items.

Solution

Total Items = 25

Defective Items = 8

Non – Defective Items = 25 – 8 = 17

So expected frequency of Non – Defective Items is 17

$$\text{Relative Frequency} = \frac{\text{frequency of Non – Defective Items}}{\text{Total Items}}$$

$$\text{Relative Frequency} = \frac{17}{25} = 0.68$$