Unit 11 Loci and Construction

EXERCISE 11.1

1. Construct ΔABC with the given measurements and verify that the perpendicular bisectors of the triangle are concurrent.

(i) $m\overline{AB} = 5$ cm, $m\overline{BC} = 6$ cm and $m\overline{AC} = 7$ cm

Solution

(i) Draw a line segment AB of length 5cm long.

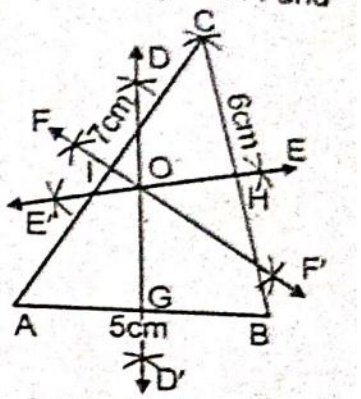
(ii) Using a pair of compasses, draw two arcs with centres at point A and B of radii 7cm and 6cm respectively.

(iii) These two arcs intersect each other at point C.

(iv) Join C with A and B.
Hence, ΔABC is the required triangle.

(v) Draw two arcs above and below AB with more Than half radius of mAB with centre at A.

(vi) Draw two arcs above and below AB with radius more than half of mAB with centre at B.



(vii) Draw a line through the points of intersection of the arcs in step (v) and (vi), we get the perpendicular bisector DD′ of the side AB at G.

(viii) Draw two more perpendicular bisectors EE′ and FF′ of the sides BC and AC at H and I respectively.

Hence, we see that the perpendicular bisectors $\overline{DD'}$, $\overline{EE'}$ and $\overline{FF'}$ are concurrent at point O.

1. Construct ΔABC with the given measurements and verify that the perpendicular bisectors of the triangle are concurrent.

(ii) $m\overline{AB} = 7.1$ cm, $m\angle B = 135°$ and $m\overline{BC} = 6.5$ cm

Solution

(i) Draw a line segment AB of length 7.1cm.

(ii) Draw an angle 135° at point B using a pair of compasses and draw a ray $\overrightarrow{BD}$ through this angle.

(iii) Draw an arc of radius 6.5cm with centre at point B intersecting $\overrightarrow{BD}$ at point C.

(iv) Join C and A.

(v) Hence, ABC is the required triangle.

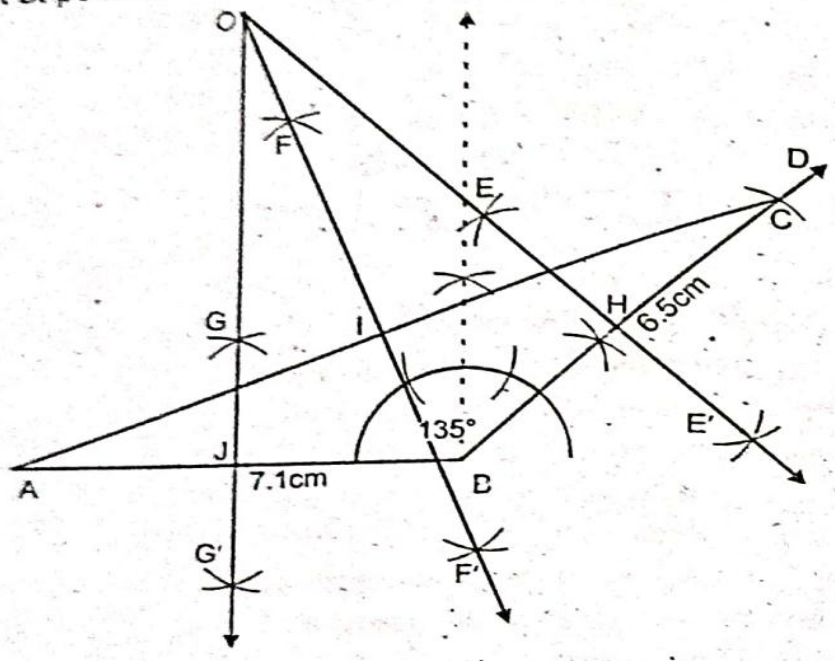
(vi) Draw two arcs above and below $\overline{AB}$ with more than half of $\overline{AB}$ with centre at A.

(vii) Draw two arcs above and below $\overline{AB}$ with radius more than half of $m\overline{AB}$ with centre at B.

(viii) Draw a line through the points of intersection of the arcs in step (vi) and (vii), we get the perpendicular bisector EGG' of the side $\overline{AB}$ at J.

(ix) Draw two more perpendicular bisectors EE' and FF' of the sides $\overline{BC}$ and $\overline{AC}$ at H and I respectively.

Hence, we see that the perpendicular bisector EE', FF' and GG' are concurrent at point O.



2. Construct ΔLMN of the following measurements and verify that the medians of the triangle are concurrent.

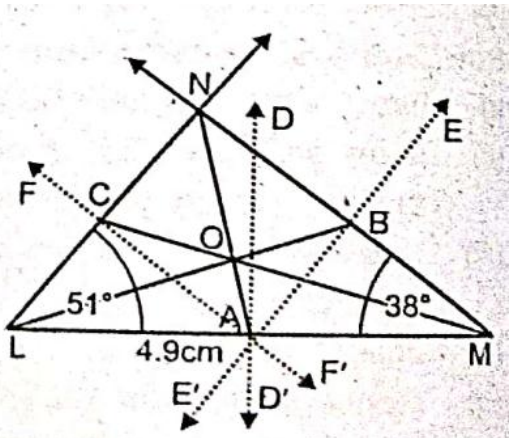
(i) $m\overline{LM} = 4.9$ cm, $m\angle L = 51°$ and $m\angle M = 38°$

Solution

(i) Construct ΔLMN using the given measurements.

(ii) Draw two arcs above and below $\overline{LM}$ with more than half of $\overline{LM}$ with centre at L.

(iii) Draw two arcs above and below $\overline{LM}$ with radius more than half of $m\overline{LM}$ with centre at M.



(iv) Draw a line through the points of intersection of the arcs in step (ii) and (iii), we get the perpendicular bisector DD′ of the side $\overline{LM}$ at A.

(v) Draw two more perpendicular bisectors EE′ and FF′ of the sides $\overline{MN}$ and $\overline{LN}$ at B and C respectively.

(vi) Join point L with opposite midpoint B, so LB is a median.

(vii) Join the point M with opposite midpoint C, we get the median MC and join N with opposite mid point A, we get median $\overline{NA}$.

Hence, we see that the medians $\overline{LB}$, $\overline{MC}$ and $\overline{NA}$ are concurrent at O.

2. Construct ΔLMN of the following measurements and verify that the medians of the triangle are concurrent.

(ii) $m\overline{MN} = 4.8$ cm, $m\angle N = 30°$ and $m\overline{LM} = 8.1$ cm

Solution

(i) Construct ΔLMN using the given measurements.

(ii) Draw two arcs above and below $\overline{MN}$ with more than half of $\overline{MN}$ with centre at M.

(iii) Draw two arcs above and below $\overline{MN}$ with radius more than half of $m\overline{MN}$ with centre at N.

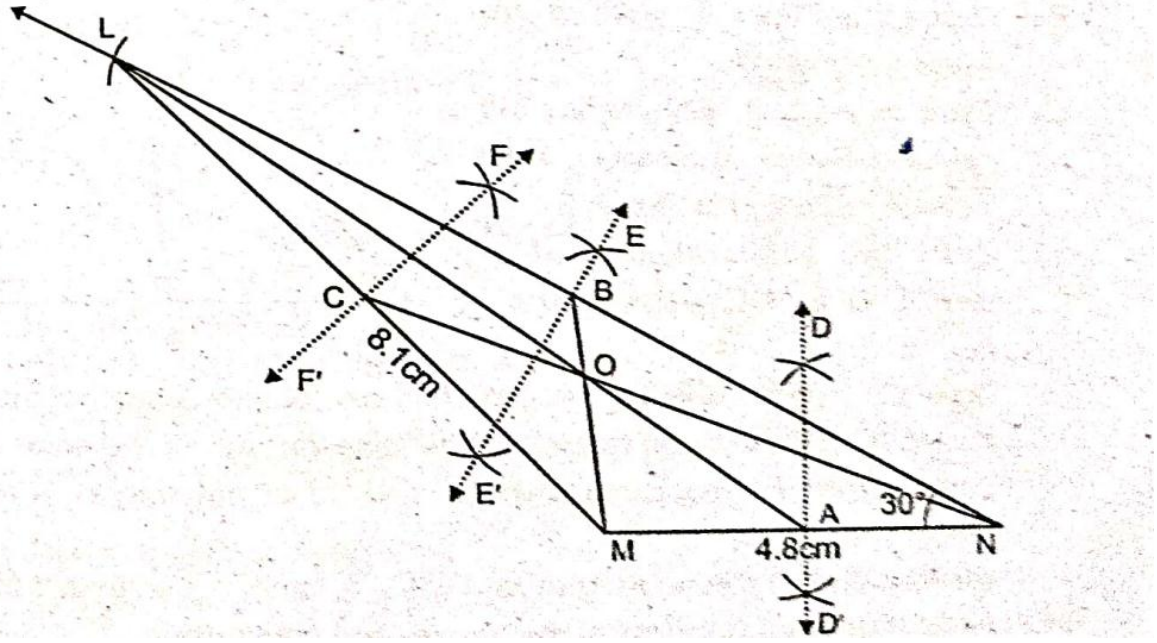
(iv) Draw a line through the points of intersection of the arcs in step (ii) and (iii), we get the perpendicular bisector DD′ of the side $\overline{MN}$ at A.

(v) Draw two more perpendicular bisectors EE′ and FF′ of the sides $\overline{LN}$ and $\overline{LM}$ at B and C respectively.

(vi) Join point N with opposite midpoint C, so $\overline{NC}$ is a median.

(vii) Join the point M with opposite midpoint B, we get the median $\overline{MB}$ and join L with opposite mid point A, we get median $\overline{LA}$.

Hence, we see that the medians $\overline{NC}$, $\overline{MB}$ and $\overline{LA}$ are concurrent at O.



3. Verify that the angle bisectors of ΔABC are concurrent with the following measurement:

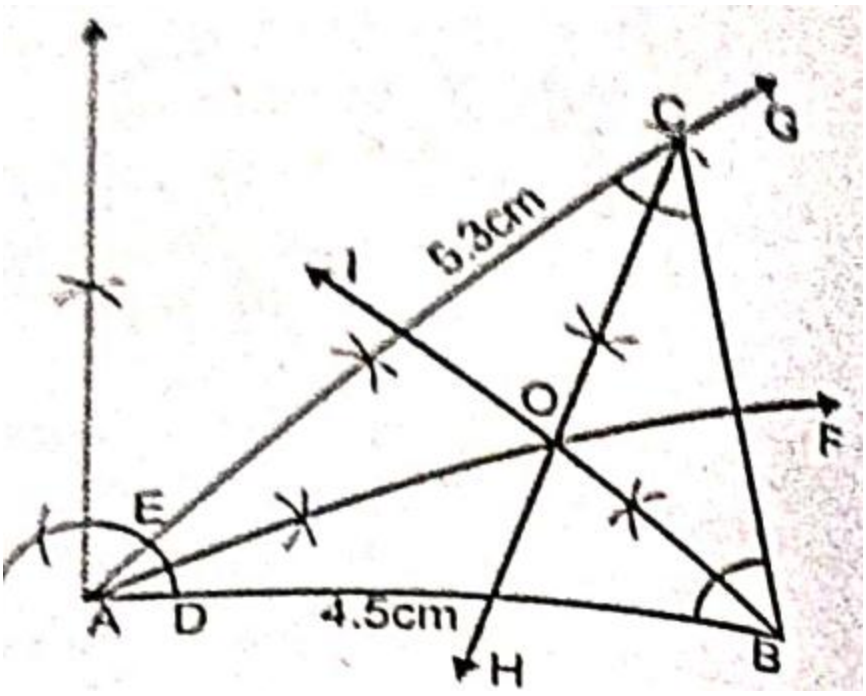
(i) $m\overline{AB} = 4.5$ cm, $m\angle A = 45°$ and $m\overline{AC} = 5.3$ cm

Solution

(i) Construct ΔABC with given lengths and angle.

(ii) Draw an arc of suitable radius with centre at point A intersecting sides $\overline{AB}$ and $\overline{AC}$ at points D and E.

(iii) Draw two arcs with centres at points D and E with suitable radius.



(iv) Draw a ray from A passing through the point of intersection of the arcs in step (iii). Which is the required angle bisector $\overline{AF}$ of the angle A.

(v) Draw two more angle bisectors BI and CH of the angles B and C respectively.

We see that all the angle bisectors $\overline{AF}$, $\overline{BI}$ and $\overline{CH}$ intersect at one point O. i.e, the angle bisectors of the triangle are concurrent.

3. Verify that the angle bisectors of ΔABC are concurrent with the following measurement:

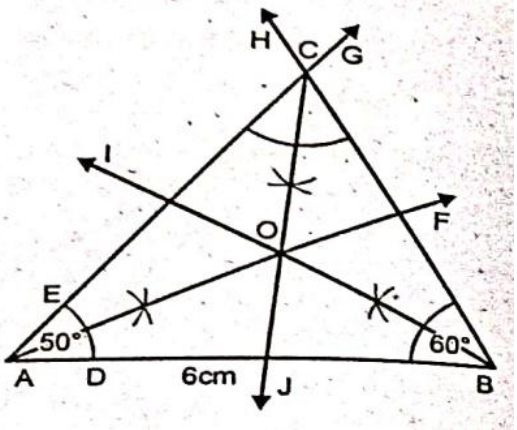
(ii) $m\overline{AB} = 6$ cm, $m\angle A = 150°$ and $m\angle B = 60°$

Solution

(i) Construct ΔABC with given lengths and angle.

(ii) Draw an arc of suitable radius with centre at point A intersecting sides $\overline{AB}$ and $\overline{AC}$ at points D and E.

(iii) Draw two arcs with centres at points D and E with suitable radius.



(iv) Draw a ray from A passing through the point of intersection of the arcs in step (iii). Which is the required angle bisector $\overrightarrow{AF}$ of the angle A.

(v) Draw two more angle bisectors BI and CJ of the angles B and C respectively.

We see that all the angle bisectors $\overrightarrow{AF}$, $\overrightarrow{BI}$ and $\overrightarrow{CJ}$ intersect at one point O. i.e, the angle bisectors of the triangle are concurrent.

4. Given the measurements of ΔDEF : $m\overline{DE} = 4.8$ cm, $m\overline{EF} = 4$ cm and $m\angle E = 45°$, draw altitudes of ΔDEF and find orthocentre.

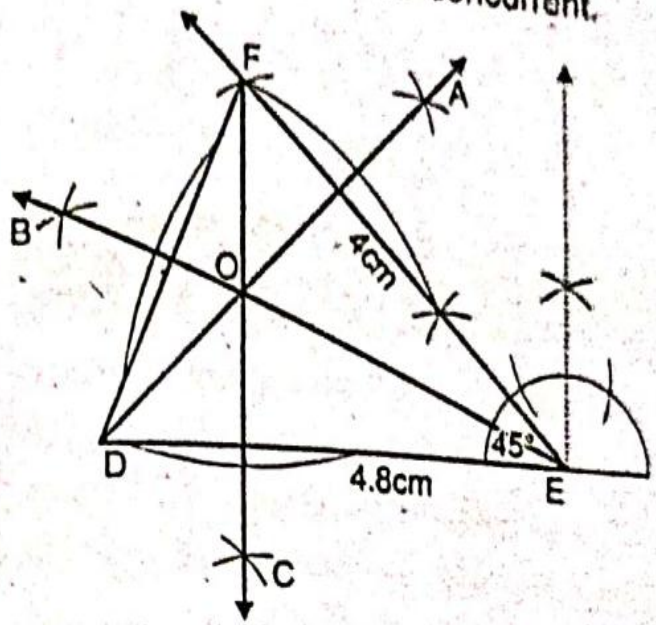
Solution

(i) Construct ΔDEF using the given measurements.

(ii) Draw perpendicular $\overline{DA}$ from D to the opposite side $\overline{EF}$.

(iii) Draw two more perpendiculars $\overline{EB}$ and $\overline{FC}$. The first is from point E to the opposite side $\overline{FD}$ and the other is from point F to the opposite side $\overline{DE}$.

So, $\overrightarrow{DA}$, $\overrightarrow{EB}$ and $\overrightarrow{FC}$ are the altitudes of ΔDEF and they intersect at one point O. i.e., the altitudes of ΔDEF are concurrent.

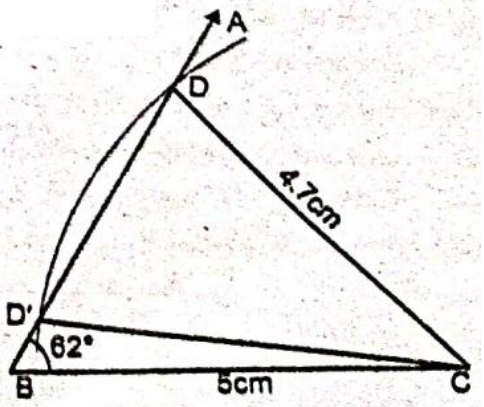


5. Construct the following triangles and find whether there exists any ambiguous case.

(i) ΔBCD ; $m\overline{BC} = 5$ cm, $m\angle B = 62°$ and $m\overline{CD} = 4.7$ cm

Solution

(i) Draw $m\overline{BC}$=5cm.
(ii) Construct an angle 62° at point B using a protractor and draw $\overrightarrow{BA}$ through this angle.
(iii) Draw an arc of radius 4.7 cm with centre at point C.
(iv) This arc intersects $\overrightarrow{BA}$ at two points D and D′.
(v) Join D and D′ with C.
We get two triangles BCD and DCD′.
This is known as ambiguous case.

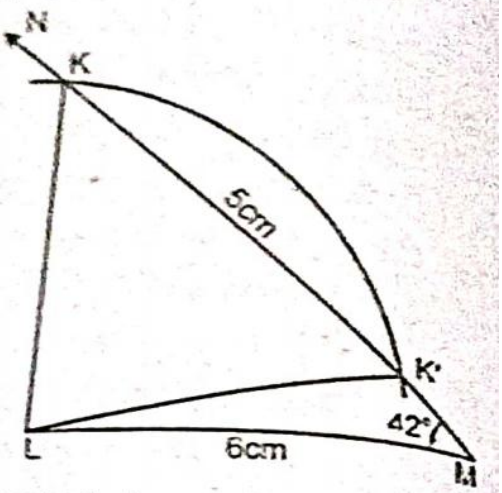


5. Construct the following triangles and find whether there exists any ambiguous case.

(ii) ΔKLM ; $m\overline{LM} = 6$ cm, $m\angle M = 42°$ and $m\overline{LN} = 5$ cm

Solution

(i) Draw $m\overline{LM}$=6cm.
(ii) Construct an angle 42° at point M using a protractor and draw $\overrightarrow{MN}$ through this angle.
(iii) Draw an arc of radius 5cm with centre at point L.
(iv) This arc intersects $\overrightarrow{MN}$ at two points K and K'.
(v) Join K and K' with N.
We get two triangles KLM and K'LM. This is known as ambiguous case.



EXERCISE 11.2

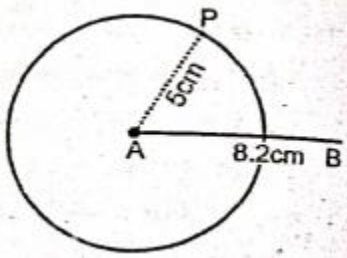
1. Two points A and B are 8.2 cm apart. Construct the locus of points 5 cm from point A.

Solution

(i) Draw a line segment $m\overline{AB}=8.2cm$.

(ii) Draw a circle of radius 5cm with centre at point A.

The locus is 5cm from point A. Any point on this circle is exactly 5cm away from point A.



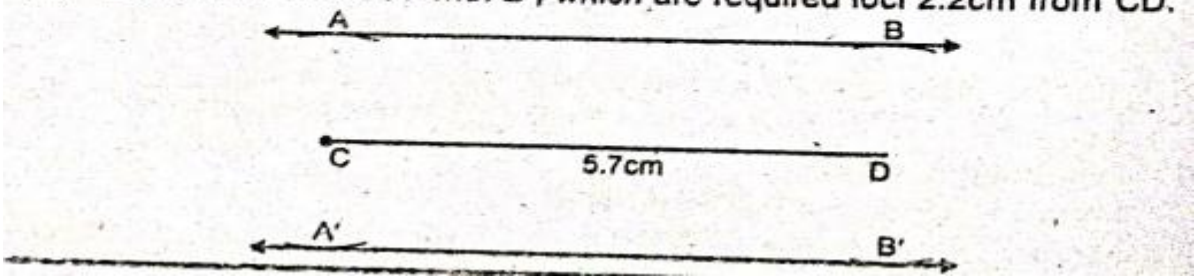
2. Construct a locus of point 2.2 cm from line segment CD of measure 5.7cm.

Solution

(i) Draw a line segment $m\overline{CD}$ = 5.7cm.

(ii) Draw two arcs A,A′ and B,B′ above and below $\overline{CD}$ of radius 2.2cm.

(iii) Join A with B and A′ with B′, which are required loci 2.2cm from $\overline{CD}$.



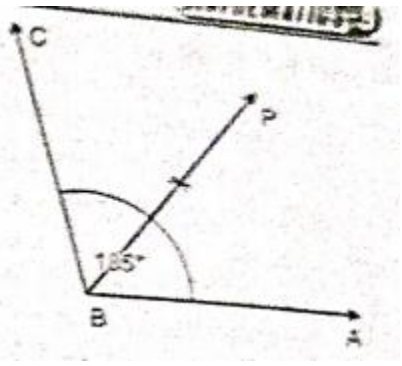
3. Construct an angle $ABC = 105°$. Construct a locus of a point P which moves such that it is equidistant from $\overline{BA}$ and $\overline{BC}$.

Solution

(i) Draw $m\angle\overline{AB}=105°$.

(ii) Draw an angle bisector $\overline{BP}$ of $m\angle ABC$.

All the points on $\overline{BP}$ are equidistant from $\overline{BA}$ and $\overline{BC}$.



4. Two points E and F are 5.4 cm apart. Construct a locus of a point P which moves such that it is equidistant from E and F.

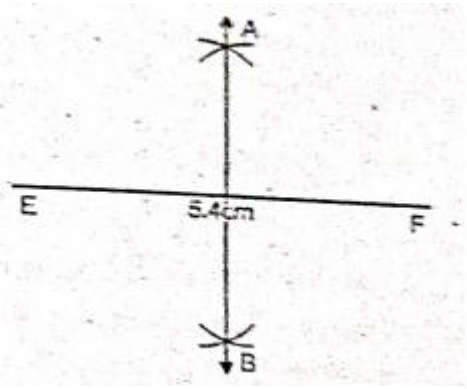
Solution

it is equidistant from E and F.

(i) Draw $m\overline{EF}$ = 5.4cm.

(ii) Draw a right bisector $\overline{AB}$ of line segment $\overline{EF}$.

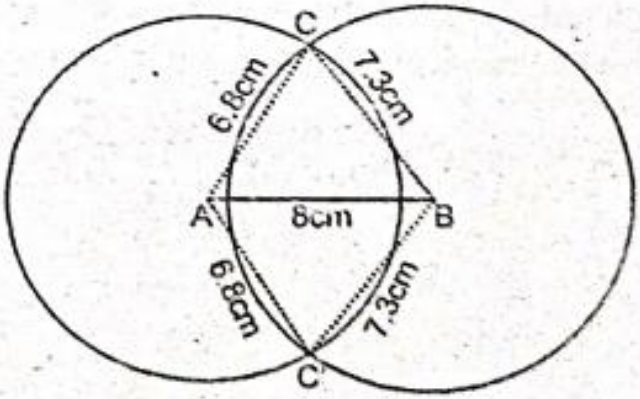
(iii) All the points on $\overline{AB}$ are equidistant from E and F.



5. The island has two main cities A and B 8 km apart. Kashif lives on the island exactly 6.8 km from city A and exactly 7.3 km from city B. Mark with a cross the points on the island where Kashif could live.

Solution

tion: (Here 1km=1cm)



(i) Draw mAB=8cm

(ii) With centre at point A, draw a circle of radius 6.8cm.

(iii) With centre at point B, draw a circle of radius 7.3cm.

(iv) Both the circles intersect each other at point C and C'.

So, C and C are the points on the island where Kashif could live.

6. Construct a triangle CDE with $m\overline{CD} = 7.6$ cm, $m\angle D = 45°$ and $mDE = 5.9$ cm. Draw the locus of all points which are:

(a) equidistant from C and D (b) equidistant from $\overline{CD}$ and $\overline{CE}$

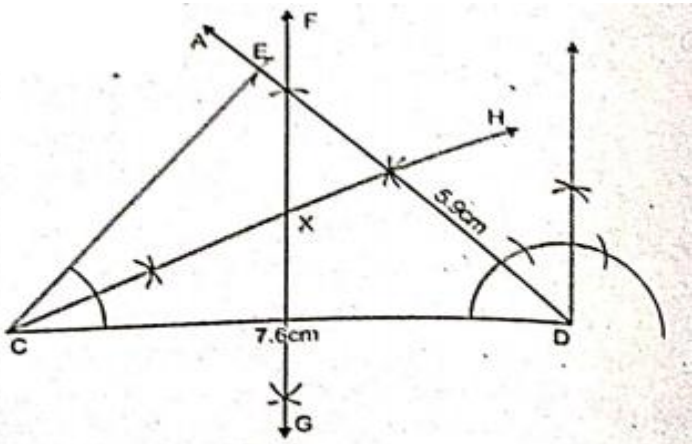
Mark the point X where the two loci intersect.

Solution

(i) Construct a $\triangle CDE$ with the given measurements.

(ii) Draw perpendicular bisector of $\overline{CD}$. All the points located on the line FG are equidistant from C and D.

Draw the angle bisector of $\angle DCE$. All the points located on $\overline{CH}$ are equidistant from $\overline{CD}$ and $\overline{CE}$. Both loci intersect each other at point X.



7. Construct a triangle LMN with $m\overline{LM} = 7$ cm, $m\angle L = 70°$ and $m\angle M = 45°$. Find a point within the triangle LMN which is equidistant from L and M and 3 cm from L.

Solution

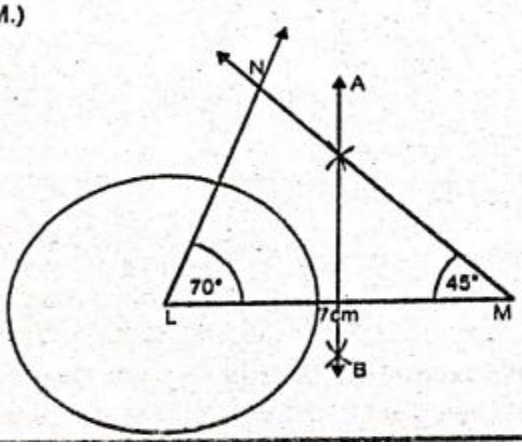
(i) Draw a triangle LMN from the given measurements.

(ii) Draw a right bisector $\overline{AB}$ of line segment LM.

(iii) All the points on $\overline{AB}$ are equidistant from L and M.

(iv) Draw a circle of radius 3cm with centre at point L which is 3cm from L.

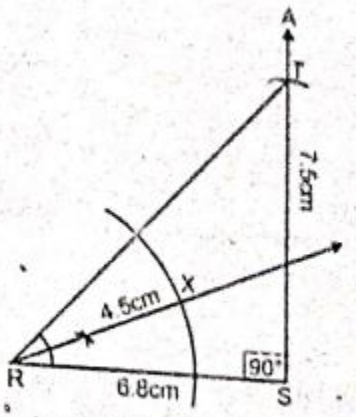
(Note: From the given measurements, it is not possible to draw a point 3cm from L and equistant from L and M.)



8. Construct a right angled triangle RST with $m\overline{RS}$ = 6.8 cm, $m\angle S = 90°$ and $m\overline{ST}$ = 7.5 cm. Find a point within the triangle RST which is equidistant from $\overline{RS}$ and $\overline{RT}$ and 4.5 cm from R.

Solution

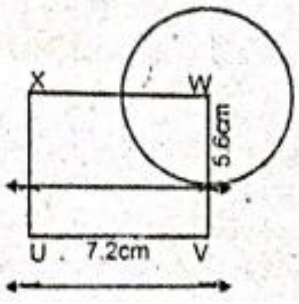
(i) Draw $m\overline{RS}$=6.8cm.
(ii) Construct $m\angle S=90°$ and draw $\overline{SA}$.
(iii) Draw an arc of radius 7.5cm with centre at point S intersecting $\overline{SA}$ at point T.
(iv) Joint T and R.
Hence ΔRST is the triangle.
(v) Draw the angle bisector of $\angle SRT$.
(vi) Draw an arc of radius 4.5cm with centre at R inside the triangle.
(vii) The point of intersection X of angle bisector and the arc inside the circle is the required point which is equidistant from $\overline{RS}$ and $\overline{RT}$ and 4.5cm from R.



9. Construct a rectangle $UVWX$ with $m\overline{UV}$ = 7.2 cm and $m\overline{VW}$ = 5.6 cm. Draw the locus of points at a distance of 2 cm from $\overline{UV}$ and 3.5 cm from W.

Solution

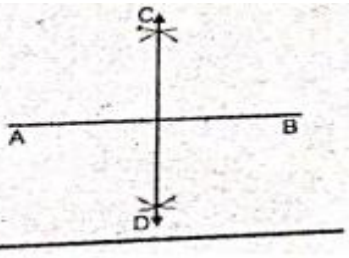
(i) Construct a rectangle UVWX with given lengths.
(ii) Draw a line parallel to $\overline{UV}$ at 2cm. It is a required locus at a distance of 2cm from $\overline{UV}$.
(iii) With centre at W draw a circle of radius 3.5cm. It is required locus which is at the distance of 3.5cm from W.



10. Imagine two cell towers located at points A and B on a coordinate plane. The GPS-enabled device, positioned somewhere on the plane, receives signals from both towers. To ensure accurate navigation, the device is placed equidistant from both towers to estimate its position. Draw this locus of navigation.

Solution

The locus of points equidistant from two points A and B is the perpendicular bisector of the line segment joining A and B. This perpendicular bisector will extend infinitely in both directions and serve as the locus of all points equidistant from two towers.

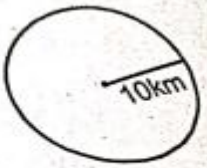


11. Epidemiologists use loci to determine infection zones, especially for contagious diseases, to predict the spread and take containment measures. In the case of a disease outbreak, authorities might determine a quarantine zone within 10 km of the infection source. Draw the locus of all points 10 km from the source defining the quarantine area to monitor and control the disease's spread.

Solution

The locus of all points 10km from the infection source would be a circle with a radius of 10km, centered at the infection source. This circle would define the quarantine zone.

In the below figure, O is the infection source and any point within the circle will be quarantine zone.



12. There is a treasure buried somewhere on the island. The treasure is 24 kilometres from A and equidistant from B and C. Using a scale of 1cm to represent 10 km, find where the treasure could be buried.

Solution

could be buried.

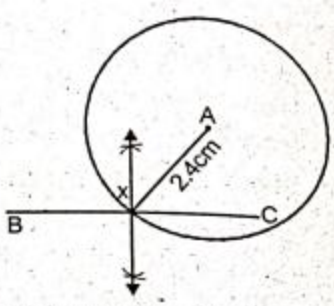
(i) Plot points A,B and C, using appropriate scale (i.e., 1cm=10km).

(ii) With centre at point A, draw a circle of radius 2.4cm.

(iii) Connect points B and C with a straight line.

(iv) Draw perpendicular bisector of $\overline{BC}$.

(v) The treasure lies at point X where the circle centred at A intersects the perpendicular bisector of BC.



13. There is an apple tree at a distance of 90 metres from banana tree in the garden of Sara's house. Sara wants to plant a mango tree M which is 64 metres from apple tree and between 54 and 82 metres from the banana tree. Using a scale of 1cm to represent 10m, Find the points where the mango tree should be planted.

Solution

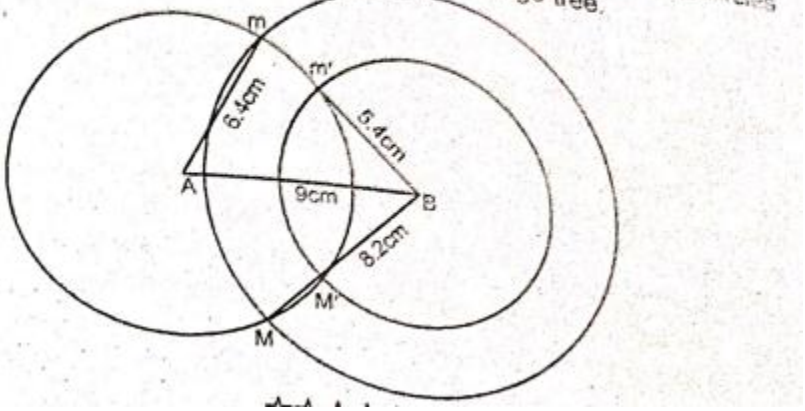
(i) Draw a line segment AB=9cm which is a distance between apple and banana tree.

(ii) With centre at A draw a circle of radius 6.4cm. This circle represents all points that are 64m away from the apple tree.

(iii) With centre at B draw a circle of radius 5.4cm.

(iv) With centre at B draw another arc of radius 8.2cm. These circles represent the range of distances (between 54m and 82m) from the banana tree.

(v) The points where the circle around A intersects with the two circles around B are the possible locations for the mango tree.



REVIEW EXERCISE 11

1. Four options are given against each statement. Encircle the correct option.

(i) A triangle can be constructed if the sum of the measure of any two sides is _____ the measure of the third side.

(a) less than (b) ✓ greater than
(c) equal to (d) greater than and equal to

(ii) An equilateral triangle _____.

(a) ✓ can be isosceles (b) can be right angled
(c) can be obtuse angled (d) has each angle equal to 50^0.

(iii) If the sum of the measures of two angles is less than 90^0, then the triangle is _____.

(a) equilateral (b) acute angled
(c) ✓ obtuse angled (d) right angled

(iv) The line segment joining the midpoint of a side to its opposite vertex in a triangle is called _____.

(a) ✓ median (b) perpendicular bisector
(c) angle bisector (d) circle

(v) The angle bisectors of a triangle intersect at _____.

(a) ✓ one point (b) two points
(c) three points (d) four points

(vi) Locus of all points equidistant from a fixed point is _____.

(a) ✓ circle (b) perpendicular bisector
(c) angle bisector (d) parallel lines

(vii) Locus of points equidistant from two fixed points is ----------

(a) circle (b) ✓ perpendicular bisector
(c) angle bisector (d) parallel lines

(viii) Locus of points equidistant from a fixed line is/are ----------

(a) circle (b) perpendicular bisector
(c) angle bisector (d) ✓ parallel lines

(ix) Locus of points equidistant from two intersecting lines is _____.

(a) circle (b) perpendicular bisector

(c) ✓ angle bisector (d) parallel lines

(x) The set of all points which is farther than 2 km from a fixed point B is a region outside a circle of radius _____ and centre at B.

(a) 1 km (b) 1.9 km

(c) ✓ 2 km (d) 2.1 km

2. Construct a right angled triangle with measures of sides 6 cm, 8 cm and 10 cm.

Solution

8cm and 10cm.

(i) Draw $m\overline{AB}$=6cm.

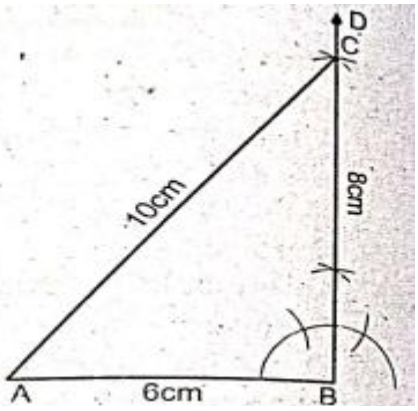
(ii) Construct mB=90° using compass and draw $\overline{BD}$ through this angle.

(iii) Draw an arc of radius 8cm with centre at point B intersecting $\overline{BD}$ at point C.

(iv) With centre at A, draw an arc of radius 10cm intersecting $\overline{BD}$ at point C.

(v) join C with A.

Hence ΔABC is the required right angled triangle.



3. Construct a triangle ABC with $m\overline{AB}$ = 5.3 cm, $m\angle A$ = 30° and $m\angle B$ = 120°. Draw the locus of all points which are equidistant from A and B.

Solution

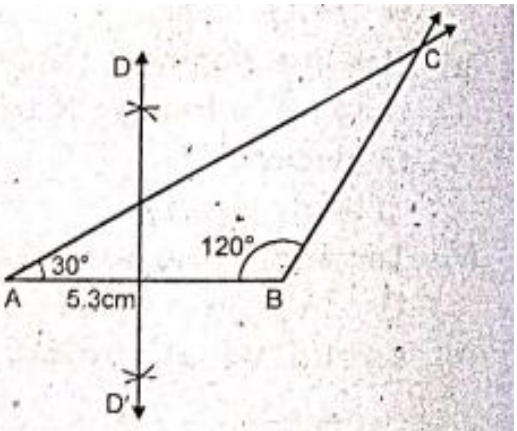
(i) Draw $m\overline{AB}$=5.3 cm.

(ii) Draw angles 30° and 120° at points A and B respectively and draw two rays through these angles from A and B.

(iii) These two rays intersect each other at point C.

Hence, ΔABC is the required triangle.

(iv) Draw a perpendicular bisector $\overline{DD'}$ of $\overline{AB}$. The locus of all the points on $\overline{DD'}$ are equidistant from A and B.



4. Construct a triangle with $m\overline{DE} = 7.3$ cm, $m\angle D = 42°$ and $m\overline{EF} = 5.4$ cm.

Solution

Steps of Construction:

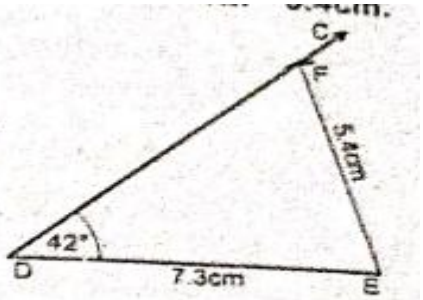
(i) Draw a line segment $\overline{DE}$ of length 7.3cm.

(ii) Draw an angle 42° at point D using a [illegible] and draw a ray $\overline{DC}$ through this angle.

(iii) Draw an arc of radius 5.4cm with centre at point E intersecting $\overline{DC}$ at point F.

(iv) Join E and F.

Hence, ΔDEF is the required triangle.



5. Construct a triangle XYZ with $m\overline{YX} = 8$ cm, $m\overline{YZ} = 7$ cm and $m\overline{XZ} = 6.5$ cm. Draw the locus of all points which are equidistant from $\overline{XY}$ and $\overline{XZ}$.

Solution

Steps [illegible]

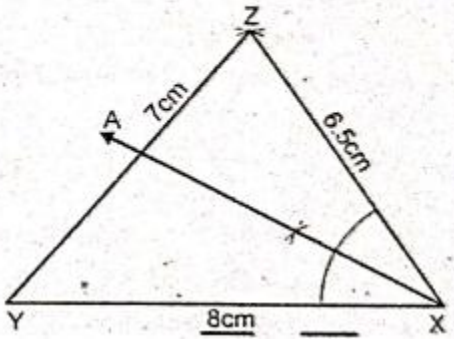
(i) Draw a line segment YX of length 8cm long.

(ii) Using a pair of compasses, draw two arcs with centres at point Y and X of radii 7cm and 6.5cm respectively.

(iii) These two arcs intersect each other at point Z.

(iv) Join Z with X and Y.
Hence, ΔXYZ is the required triangle.

(v) Draw angle bisector of angle XYZ. All the points on $\overline{XA}$ are equidistant from $\overline{XY}$ and $\overline{XZ}$.



6. Construct a triangle FGH such that $m\overline{FG} = m\overline{GH} = 6.4$ cm, $m\angle G = 122°$. Draw the locus of all points which are:

(a) equidistant from F and G,

(b) equidistant from $\overline{FG}$ and $\overline{GH}$.

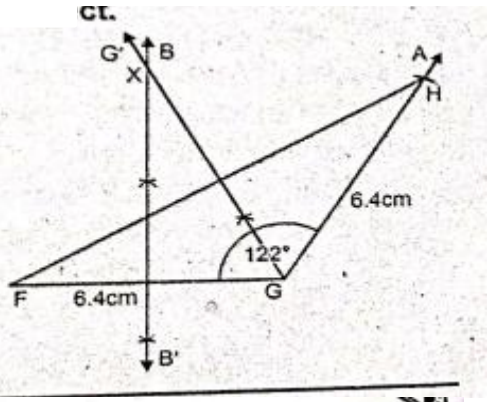
(c) Mark the point where the two loci intersect.

Solution

(i) Draw a line segment FG of length 6.4cm.

(ii) Draw an angle 122° at point G using a protractor and draw a ray $\overrightarrow{GA}$ through this angle.

(iii) Draw an arc of radius 6.4cm with centre at point G intersecting $\overrightarrow{GA}$ at point H.



(iv) Join H and F.
Hence, FGH is the required triangle.

(v) Draw the bisector of $\overline{GF}$. All the points on $\overline{AB}$ are equidistant from F and G.

(vi) Draw angle bisector of m∠FGH.
All the points of $\overrightarrow{GG'}$ are equidstant from $\overline{FG}$ and $\overline{GH}$.

(vii) Two loci intersect each other at point X.

7. Two houses Q and R are 73 metres apart. Using a scale of 1 cm to represent 10 m, construct the locus of a point P which moves such that it is:

(i) at a distance of 32 metres from Q

(ii) at a distance of 48 metres from the line joining Q and R.

Solution

To construct the locus of a point P from Q, using a scale of 1cm to represent 10m.

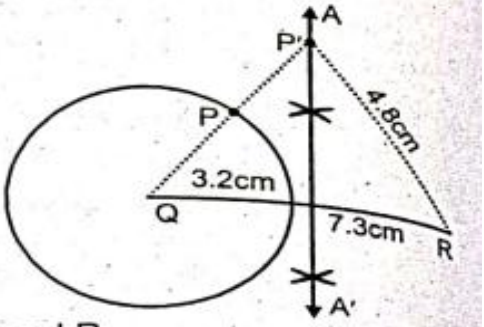
(i) Draw QR=7.3cm.

(ii) The locus of P is a circle centered at Q with a radius of 3.2cm.

(iii) All the points on circle centered at Q are equidistant from Q.

(iv) Draw the perpendicular bisector of $\overline{QR}$.

(v) Mark a point P' on $\overrightarrow{AA'}$ such that point P' is 4.8cm from the line joining Q and R.



8. The field is in the form of a rectangle $ABCD$ with $m\overline{AB}$ = 70m and $m\overline{BC}$ = 60 m. Construct the rectangle $ABCD$ using a scale of 1cm to represent 10 m. Show the region inside the field which is less than 30 m from C and farther than 25 m from $\overline{AB}$.

Solution

To construct a locus use the scale of 1cm equals to 10m.

(i) Construct a rectangle ABCD from the given measurements.

(ii) With centre at point C draw a circle of radius 3cm. The region inside the circle represents the point less than 3cm from C.

(iii) Draw a line parallel to AB which is 2.5cm from $\overline{AB}$. The region above this line represents the points farther than 2.5cm from $\overline{AB}$.