

Unit 10

Graphs of Functions

EXERCISE 10.1

1. Sketch the graph of the following linear functions:

(i) $y = 3x - 5$

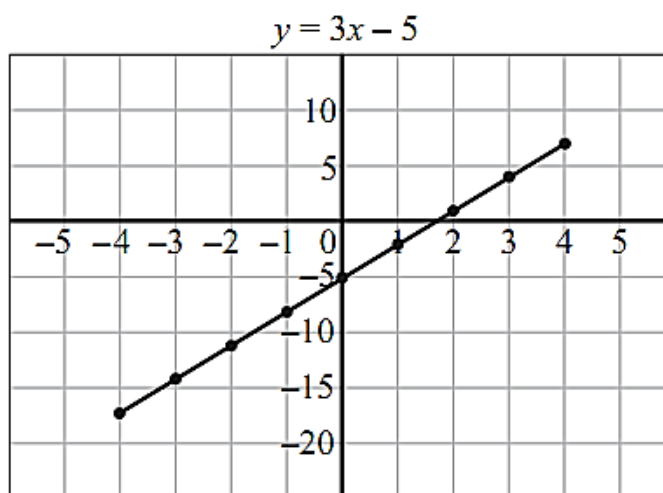
(ii) $y = -2x + 8$

(iii) $y = 0.5x - 1$

Solution

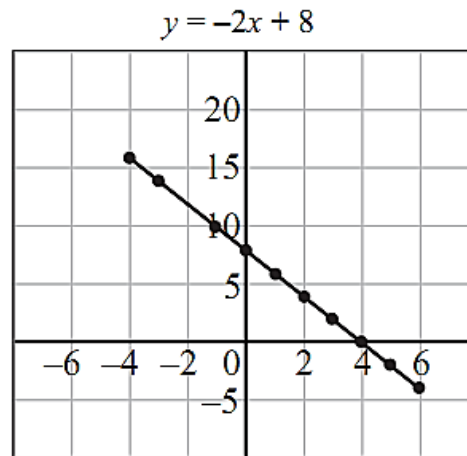
(i)

x	-2	-1	0	1	2	3
$y = 3x - 5$	-11	-8	-5	-2	1	4



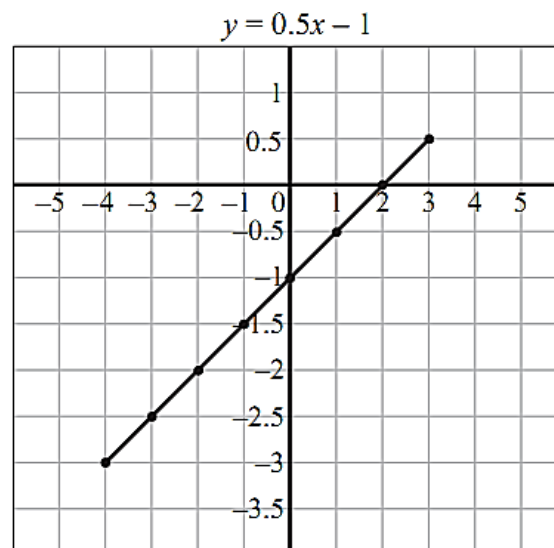
(ii)

x	-2	-1	0	1	2	3
$y = -2x + 8$	12	10	8	6	4	2



(iii)

x	-2	-1	0	1	2	3
$y = 0.5x - 1$	-2	-1.5	-1	-0.5	0	0.5



2. Plot the graph of the following quadratic and cubic functions:

(i) $y = x^3 + 2x^2 - 5x - 6; -3.5 \leq x \leq 2.5$ (ii) $y = x^2 + x - 2$

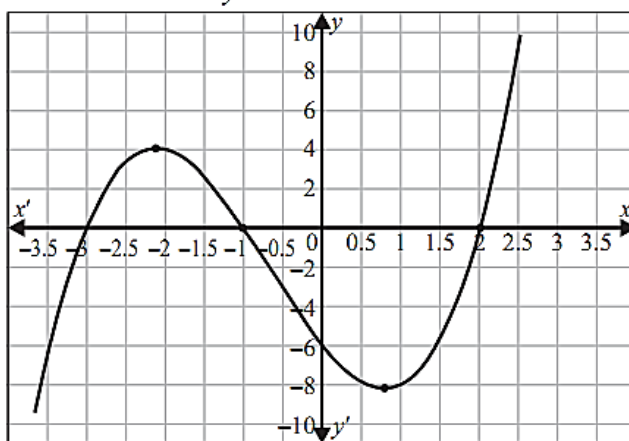
(iii) $y = x^3 + 3x^2 + 2x; -2.5 \leq x \leq 0.5$ (iv) $y = 5x^2 - 2x - 3$

Solution

(i)

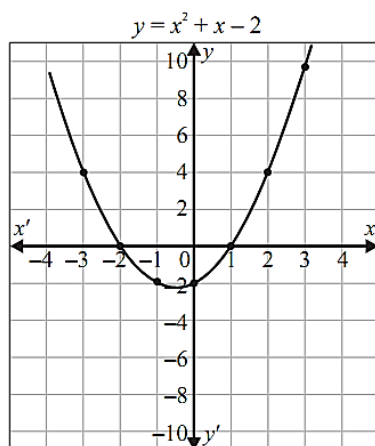
x	-3.5	-3	-2	-1	0	1	2	2.5
y	-6.88	0	4	0	-6	-8	0	9.63

(i) $y = x^3 + 2x^2 - 5x - 6$



(ii)

x	-4	-3	-2	-1	0	1	2	3	4
y	10	4	0	-2	-2	0	4	10	18

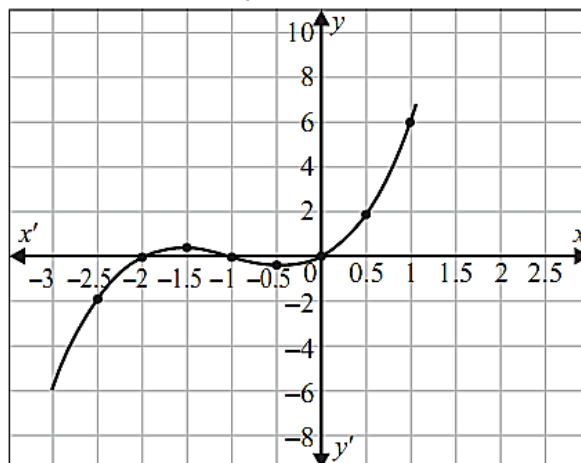


(iii)

x	-2.5	-2	-1.5	-1	-0.5	0	0.5	1
y	-1.875	0	0.375	0	-0.375	0	1.875	6

(iii)

$$y = x^3 + 3x^2 + 2x$$

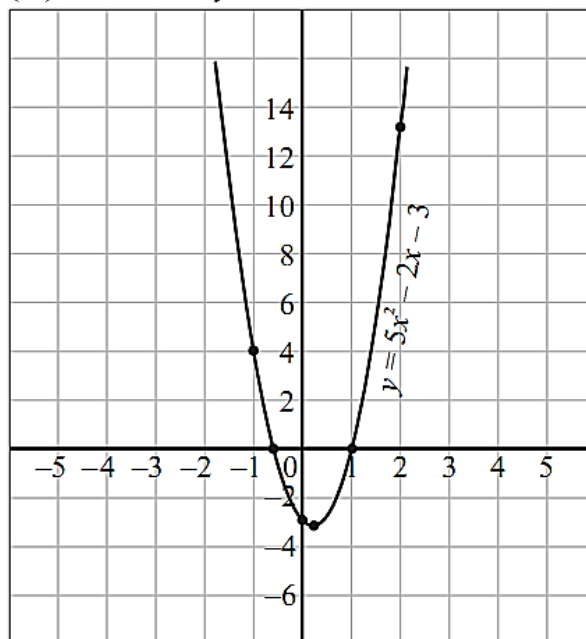


(iv)

x	-2	-1	0	1	2
y	21	4	-3	0	13

(iv)

$$y = 5x^2 - 2x - 3$$



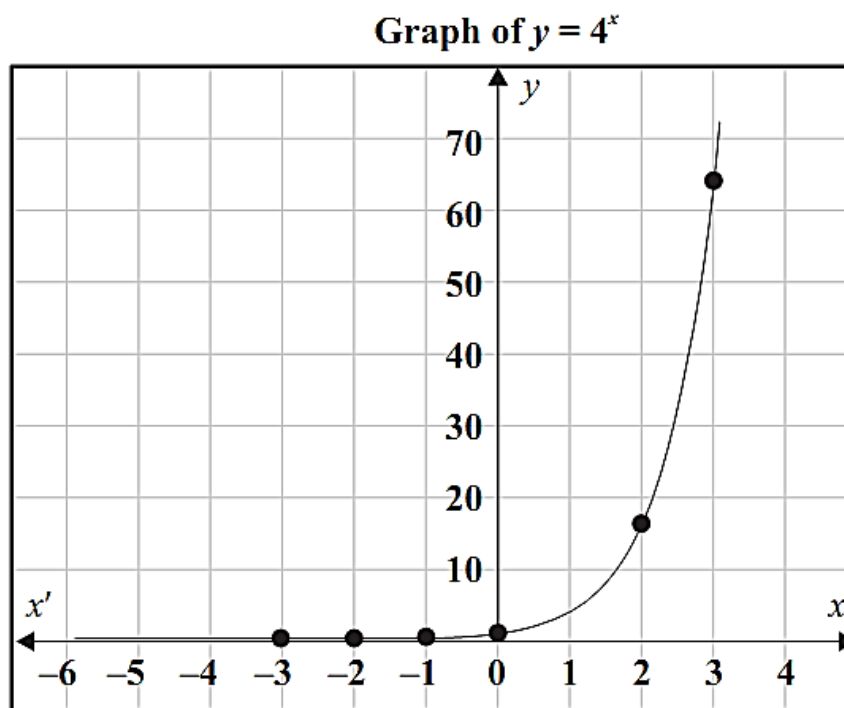
3. Plot the graph of the following functions:

- (i) $y = 4^x$ (ii) $y = 5^{-x}$ (iii) $y = \frac{1}{x-3} \quad x \neq 3$
- (iv) $y = \frac{2}{x} + 3, x \neq 0$ (v) $y = x^{\frac{1}{2}}$ (vi) $y = 3x^{\frac{1}{3}}$
- (vii) $y = 2x^{-2}$

Solution

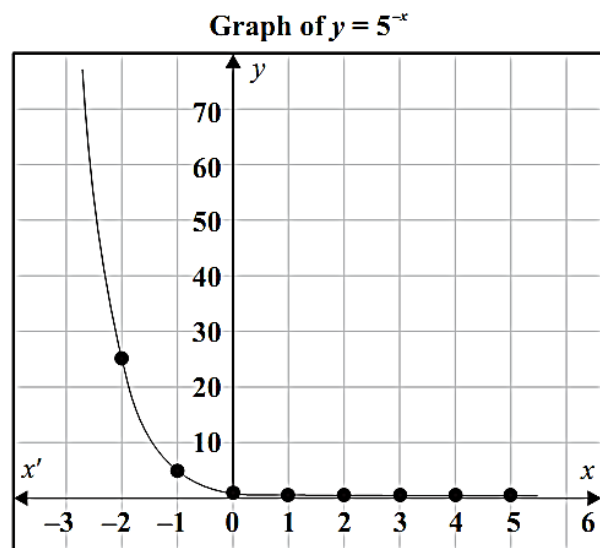
(i)

x	-3	-2	-1	0	1	2	3
y	0.02	0.06	0.25	1	4	16	64



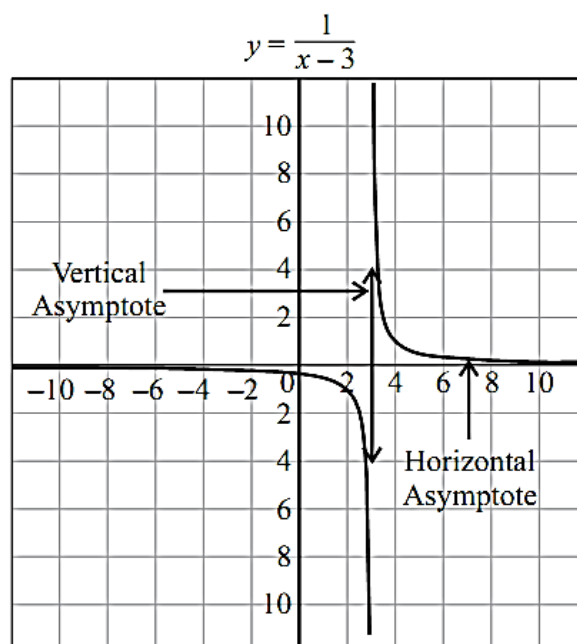
(ii)

x	-3	-2	-1	0	1	2	3
y	125	25	5	1	0.2	0.04	0.008



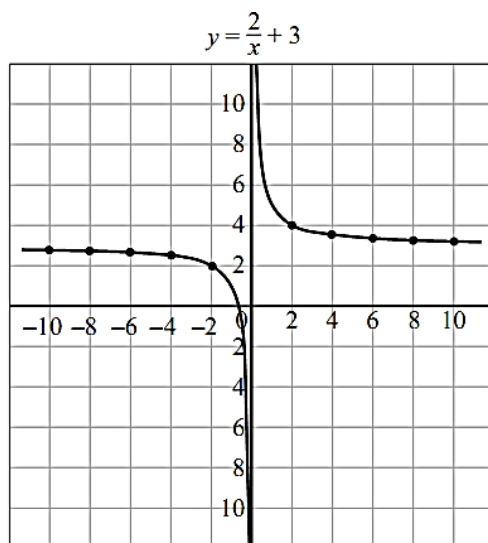
(iii)

x	-2	-1	0	1	2	4	5
y	-0.2	-0.25	-0.3	-0.5	-1	1	0.5



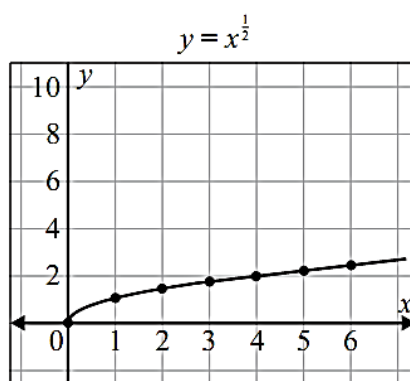
(iv)

x	-3	-2	-1	1	2	3
y	2.3	2	1	5	4	3.7



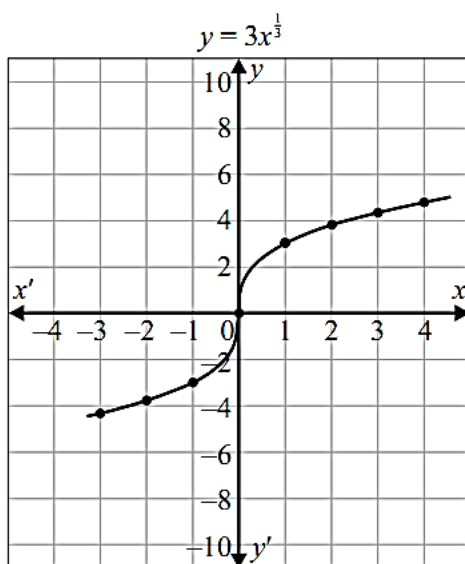
(v)

x	0	1	2	3	4	5	6
y	0	1	1.4	1.7	2	2.2	2.4



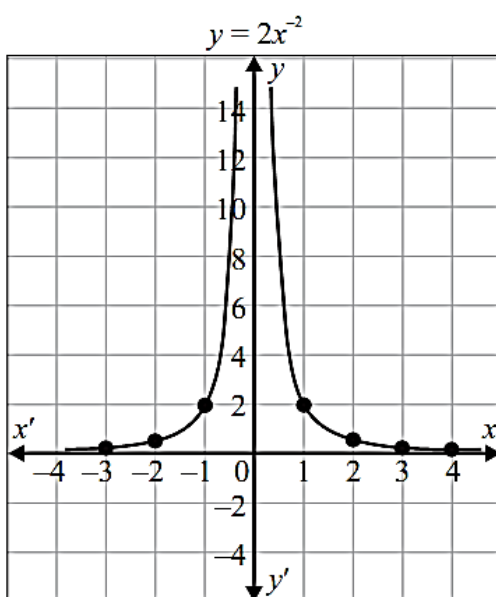
(vi)

x	-1	0	1	2	3	4
y	-3	0	3	3.6	4.2	4.5



(vii)

x	-2	-1.5	-1	-0.5	0.5	1	1.5	2
y	0.5	0.9	2	8	8	2	0.9	0.5



EXERCISE 10.2

1. Plot the graph of $y = 2x^2 - 4x + 3$ for x from -1 to 3 . Draw tangent at $(2, 3)$ and find the gradient.

Solution

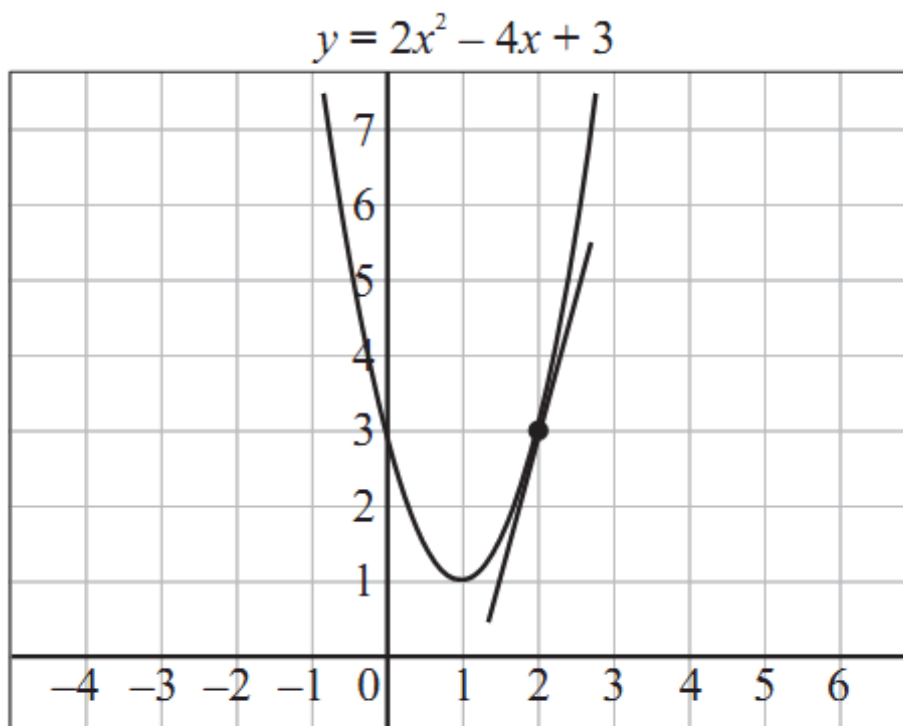
x	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	9	5.5	3	1.5	1	1.5	3	5.5	9

Consider $(2.5, 5.5)$ & $(1.5, 1.5)$

$$\text{Gradient} = \frac{1.5 - 5.5}{1.5 - 2.5} = \frac{-4}{-1}$$

$$\text{Gradient} = 4$$

Graph



$$\text{Gradient} = 4$$

2. Plot the graph of $y = 3x^2 + x + 1$ and draw tangent at $(1, 5)$. Also find gradient of the tangent line at this point.

Solution

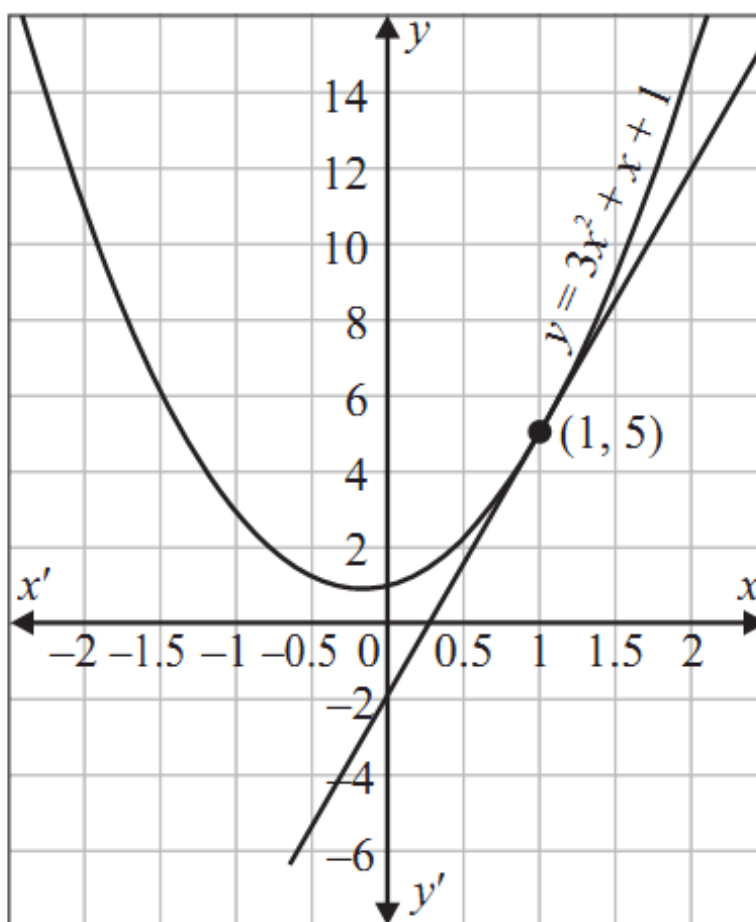
x	-1	-0.5	0	0.5	1	1.5
y	3	1.25	1	2.25	5	9.25

Consider $(1.5, 8.5)$ & $(0.5, 1.5)$

$$\text{Gradient} = \frac{1.5 - 8.5}{0.5 - 1.5} = \frac{-7}{-1}$$

$$\text{Gradient} = 7$$

Graph

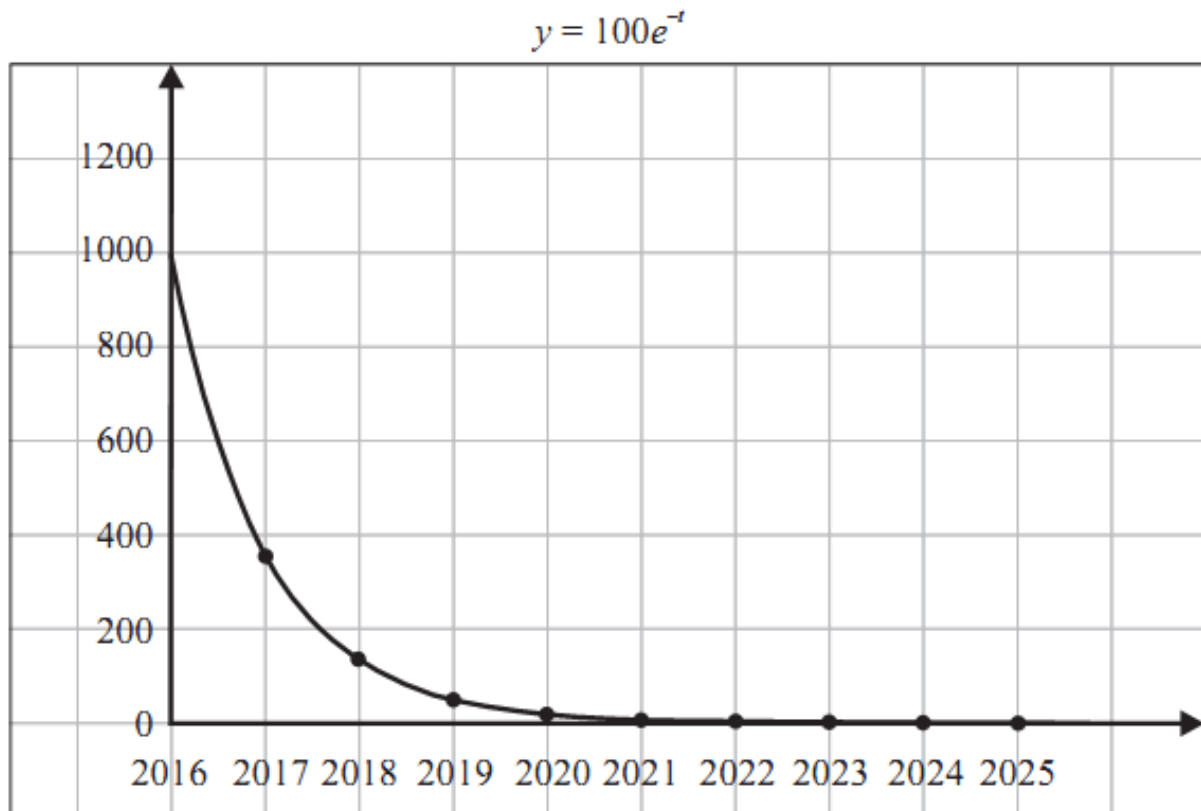


$$\text{Gradient} = 7$$

3. The strength of students in a school was 1000 in 2016. If the strength decay according to the equation $S = 1000 e^{-t}$, where S is the number of students at time t .
- (a) Graph the given equation for $t = 0$ (in 2016) to $t = 9$ (in 2025).
- (b) From the graph, estimate the student's strength in 2019 and in 2023.

Solution

x	0	1	2	3	4	5	6	7	8	9
y	1000	368	135	50	18	7	2	0.9	0.3	0.1



- (b) From the graph, students' strength in 2019 is approximately 50, and in 2023 approximately 1.

4. The demand and supply functions for a product are given by the equations $P_d = 400 - 5Q$, $P_s = 3Q + 24$:

Plot the graph of each function over the interval $Q = 0$ to $Q = 300$.

Solution

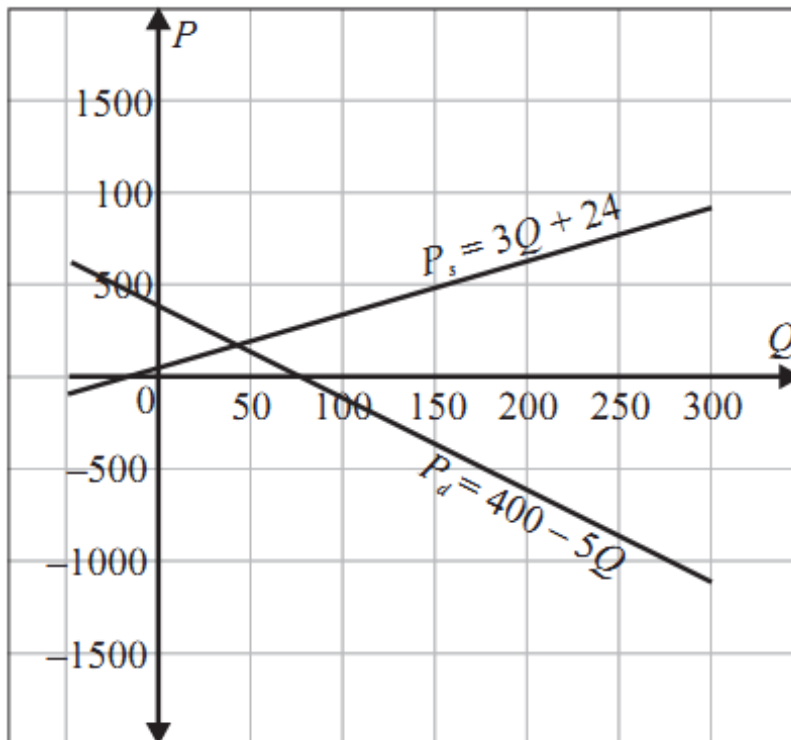
$$P_d = 400 - 5Q$$

Q	0	50	100	150	200	250	300
P_d	400	150	-100	-350	-600	-850	-1100

$$P_s = 3Q + 24$$

Q	0	50	100	150	200	250	300
P_s	24	174	324	474	624	774	924

Graph



5. Shahid's salary $S(x)$ in rupees is based on the following formula:

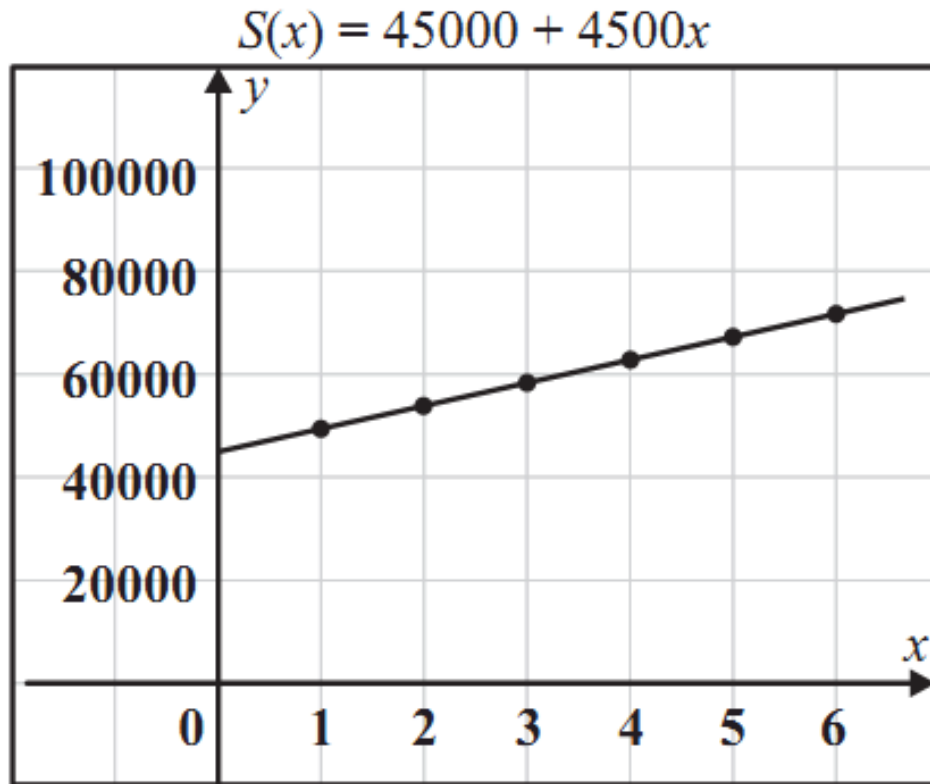
$$S(x) = 45000 + 4500x,$$

where x is the number of years he has been with the company. Sketch and interpret the graph of salary function for $0 \leq x \leq 5$.

Solution

S	0	1	2	3	4	5
$S(x)$	45000	49500	54000	58500	63000	67500

Graph



Shahid's salary increases linearly with years of service and rises by Rs. 4500 for every year.

6. A company manufactures school bags. The cost function of producing x bags is $C(x) = 1200 + 20x$ and the revenue from selling x bags is $R(x) = 50x$.
- Find the break-even point.
 - Determine the profit or loss when 250 bags are sold.
 - Plot the graphs of both the functions and identify the break-even point.

Solution

(a) The break – even point

The break – even point occur when $R(x) = C(x)$

$$50x = 1200 + 20x$$

$$50x - 20x = 1200$$

$$30x = 1200$$

$$x = 40 \text{ bags}$$

(b) Profit or Loss after Sale

$$P(x) = R(x) - C(x)$$

$$P(x) = 50x - 1200 - 20x$$

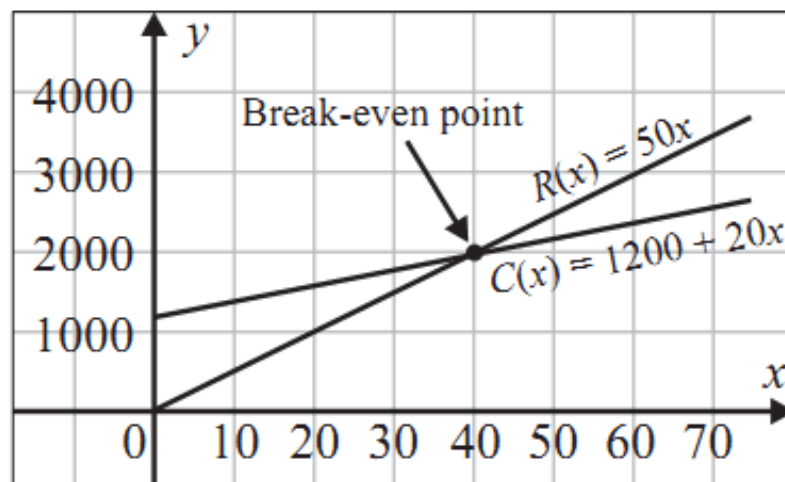
$$P(x) = 30x - 1200$$

$$P(250) = 30(250) - 1200$$

$$\text{Profit} = \text{Rs. } 6300$$

(c) Graph

x	0	30	60	90	120	150	180	210
C(x)	1200	1800	2400	3000	3600	4200	4800	5400
R(x)	0	1500	3000	4500	6000	7500	9000	10500



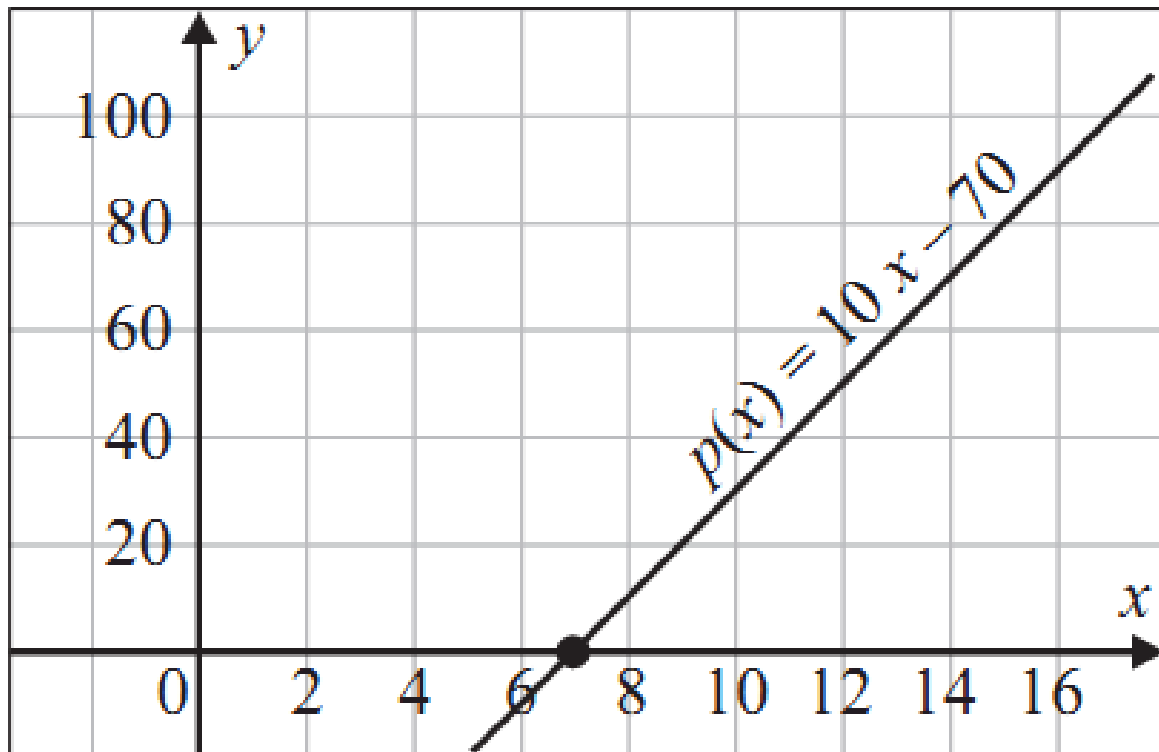
7. A newspaper agency fixed cost of Rs. 70 per edition and marginal printing and distribution costs of Rs. 40 per copy. Profit function is $p(x) = 10x - 70$, where x is the number of newspapers. Plot the graph and find profit for 500 newspapers.

Solution

x	6	7	8	10
P(x)	-10	0	10	30

$$P(500) = 10(500) - 70 = 5000 - 70 = 4930$$

Graph



Profit for 500 newspapers = Rs. 4930

8. Ali manufactures expensive shirts for sale to a school. Its cost (in rupees) for x shirts is $C(x) = 1500 + 10x + 0.2x^2$, $0 \leq x \leq 150$. Plot the graph and find the cost of 200 shirts.

Solution

x	0	50	100	150	200
P(x)	1500	2500	4500	7500	11500

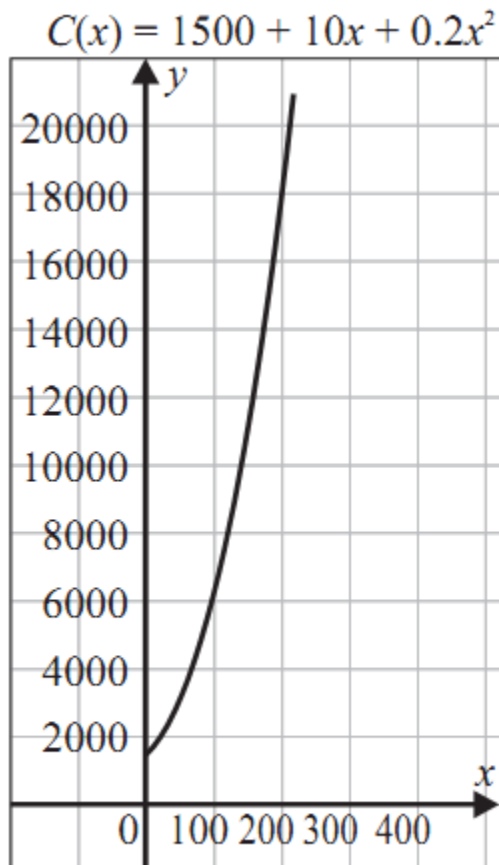
$$C(x) = 1500 + 10x + 0.2x^2$$

$$C(200) = 1500 + 10(200) + 0.2(200)^2$$

$$C(200) = 1500 + 2000 + 8000$$

$$C(200) = 11500$$

Graph



Cost of 200 shirts = Rs. 11500

REVIEW EXERCISE 10

1. Four options are given against each statement. Encircle the correct option.

(i) $x = 5$ represents:

(a) x -axis

(b) y -axis

(c) line $\parallel$ to x -axis

☒ (d) line $\parallel$ to y -axis

(ii) Slope of the line $y = 5x + 3$ is:

(a) 3

(b) -3

☒ (c) 5

(d) -5

(iii) The y - intercepts of $y = -2x - 1$ is:

(a) -2

(b) 2

☒ (c) -1

(d) 1

(iv) The graph of $y = x^3$, cuts the x -axis at:

☒ (a) $x = 0$

(b) $x = 1$

(c) $x = -1$

(d) $x = 2$

(v) The graph of 3^x represents:

☒ (a) growth

(b) decay

(c) both(a)and(b) (d) a line

(vi) The graph of $y = -x^2 + 5$ opens:

(a) upward

☒ (b) downward

(c) left side

(d) right side

(vii) The graph of $y = x^2 - 9$ opens:

☒ (a) upward

(b) downward

(c) left side

(d) right side

(viii) $y = 5^x$ is _____ function.

(a) linear

(b) quadratic

(c) cubic

☒ (d) exponential

(ix) Reciprocal function is:

(a) $y = 7^x$

☒ (b) $y = \frac{2}{x}$

(c) $y = 2x^2$

(d) $y = 5x^3$

(x) $y = -3x^3 + 7$ is _____ function.

(a) exponential

☒ (b) cubic

(c) linear

(d) reciprocal

2. Plot the graph of the following functions:

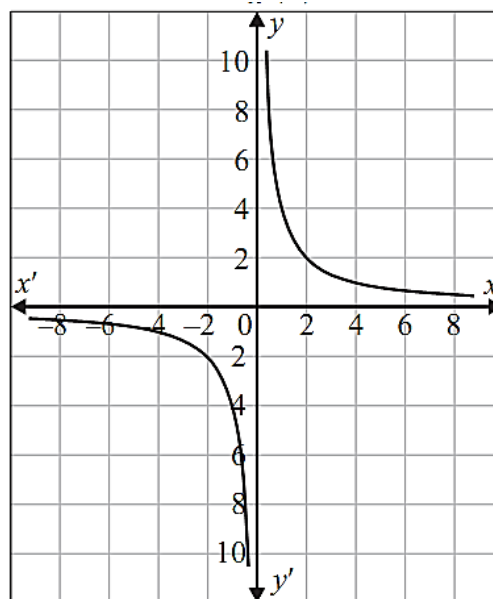
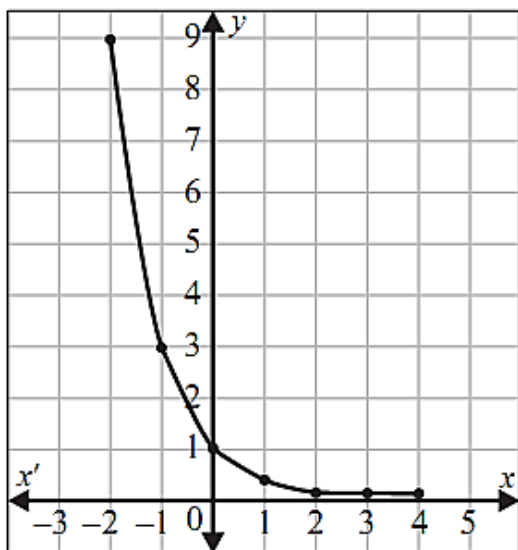
(i) $y = 3^{-x}$ for x from -2 to 4

(ii) $y = \frac{2}{x}, x \neq 0$

Solution

x	$y = 3^{-x}$
-2	9
-1	3
0	1
1	0.33
2	0.11
3	0.04
4	0.01

x	$y = 2/x$
-3	-2/3
-2	-1
-1	-2
-0.5	-4
-0.2	-10
0.2	10
0.5	4
1	2
2	1
3	2/3

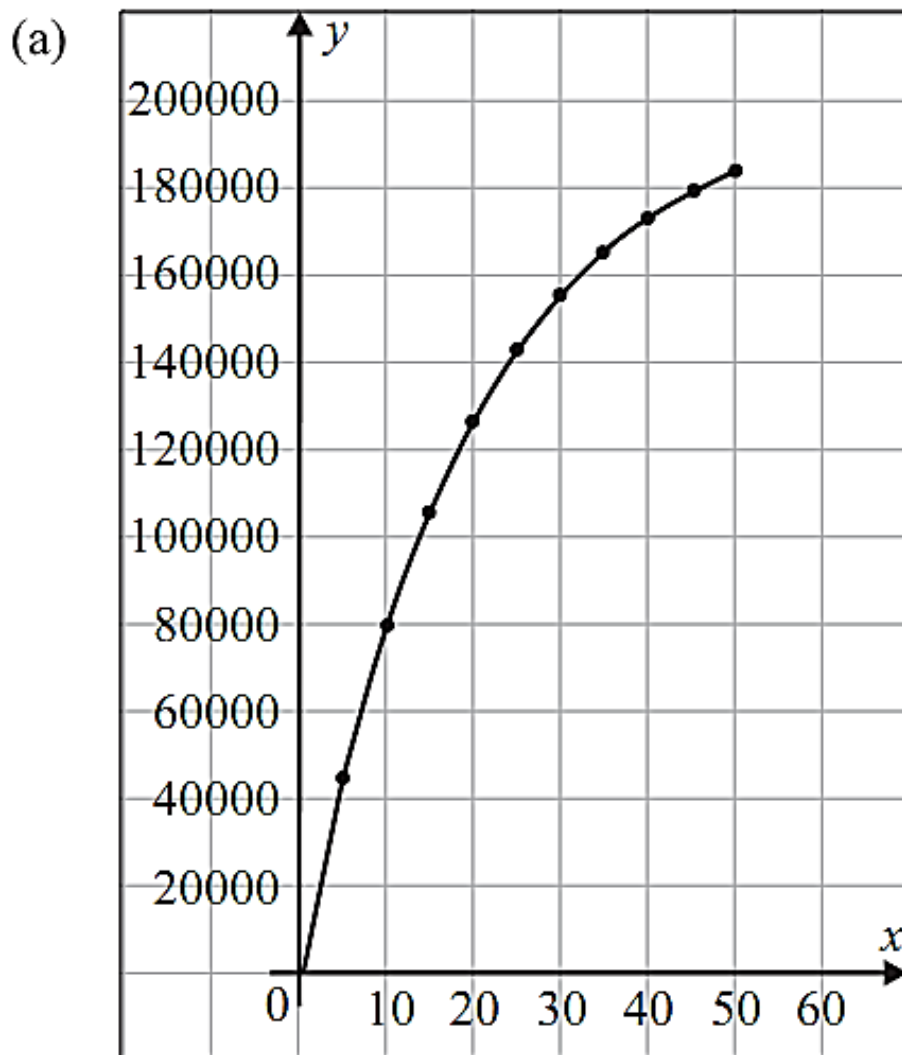


3. Sales for a new magazine are expected to grow according to the equation:
 $S = 200000 (1 - e^{-0.05t})$, where t is given in weeks.

- (a) Plot graph of sales for the first 50 weeks.
 (b) Calculate the number of magazines sold, when $t = 5$ and $t = 35$.

Solution

t	0	10	20	30	40	50
S(t)	0	78694	126424	155374	172933	183583



- (b) For $t = 5$, $S = 44239.84$ and for $t = 35$,
 $S = 165245.2$

4. Plot the graph of following for x from -5 to 5 :

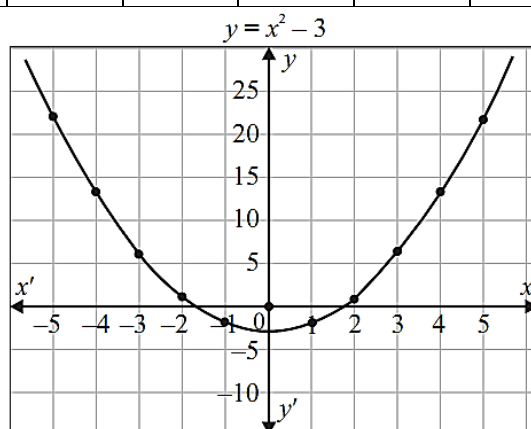
(i) $y = x^2 - 3$

(ii) $y = 15 - x^2$

Solution

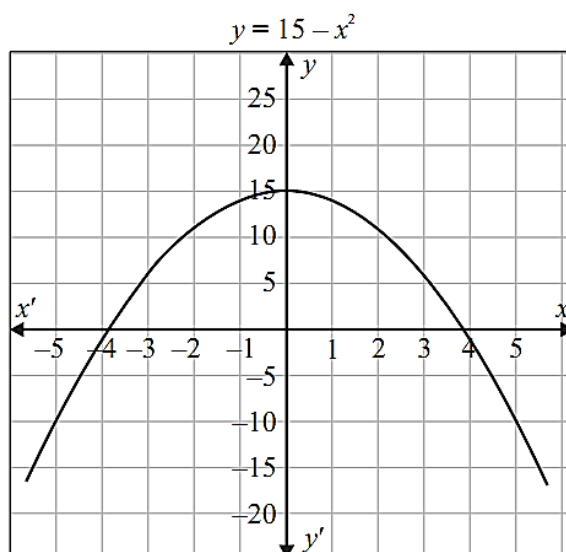
4(i)

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	22	13	6	1	-2	-3	-2	1	6	13	22



4(ii)

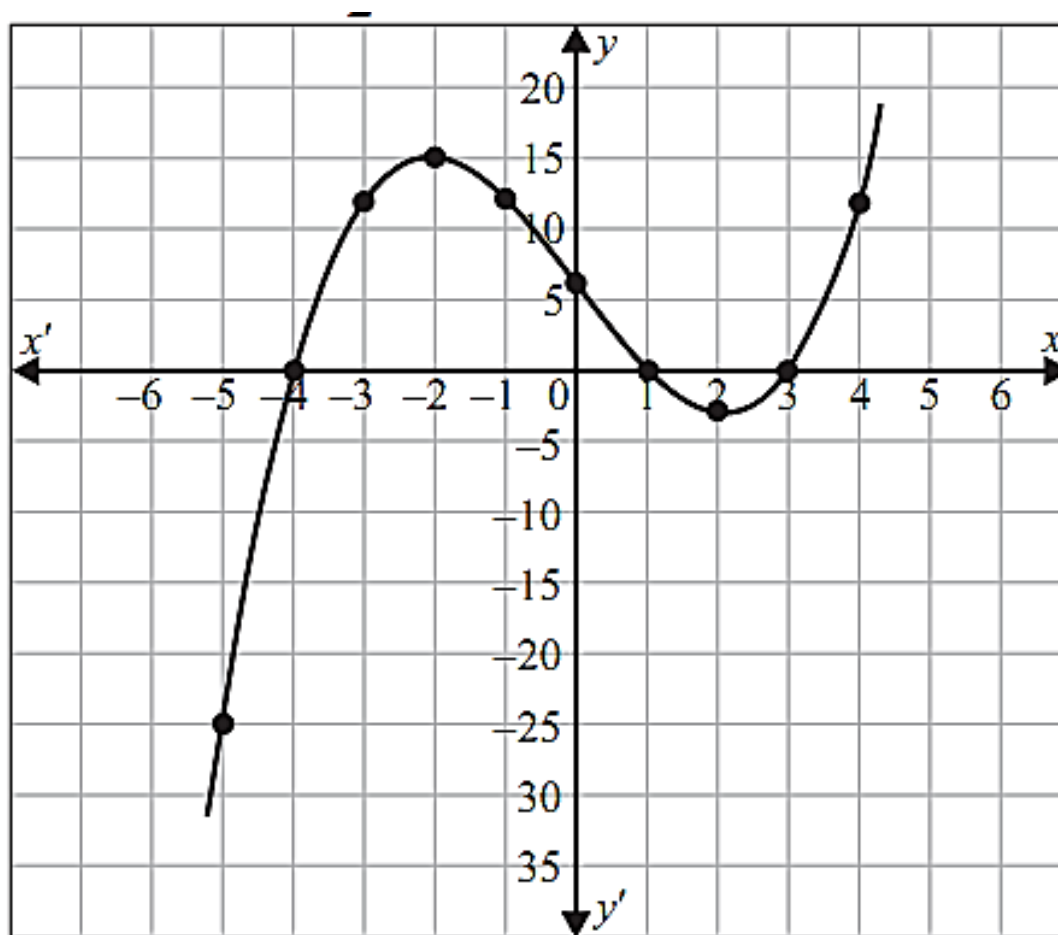
x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	-10	-1	6	11	14	15	14	11	6	-1	-10



5. Plot the graph of $y = \frac{1}{2} (x + 4)(x - 1)(x - 3)$ for x from -5 to 4

Solution

x	-5	-4	-3	-2	-1	0	1	2	3	4
y	-24	0	12	15	12	6	0	-3	0	12



6. The supply and demand functions for a particular market are given by the equations:

$P_s = Q^2 + 5$ and $P_d = Q^2 - 10Q$, where P represents price and Q represents quantity,

Sketch the graph of each function over the interval $Q = -20$ to $Q = 20$.

Solution

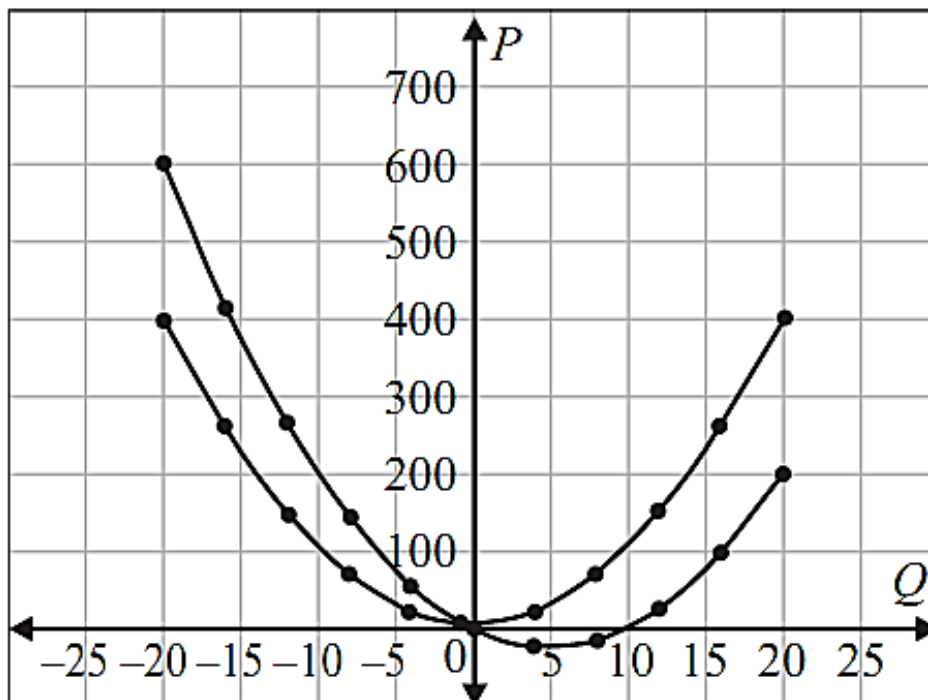
$$P_s = Q^2 + 5$$

Q	-20	-15	-10	-5	0	5	10	15	20
P_s	405	230	105	30	5	30	105	230	405

$$P_d = Q^2 - 10Q$$

Q	-20	-15	-10	-5	0	5	10	15	20
P_d	600	375	200	75	0	-25	0	75	200

Graph



7. A television manufacturer company make 40 inches LEDs. The cost of manufacturing x LEDs is $C(x) = 60,000 + 250x$ and the revenue from selling x LEDs is $R(x) = 1200x$. Find the break-even point and find the profit or loss when 100 LEDs are sold. Identify the break-even point graphically.

Solution

(a) The break – even point (no profit or loss)

The break – even point occur when $R(x) = C(x)$

$$1200x = 60000 + 250x$$

$$x = 63.16 \text{ LED's}$$

(b) Profit or Loss after Sale

$$P(x) = R(x) - C(x)$$

$$P(x) = 1200x - 60000 - 250x$$

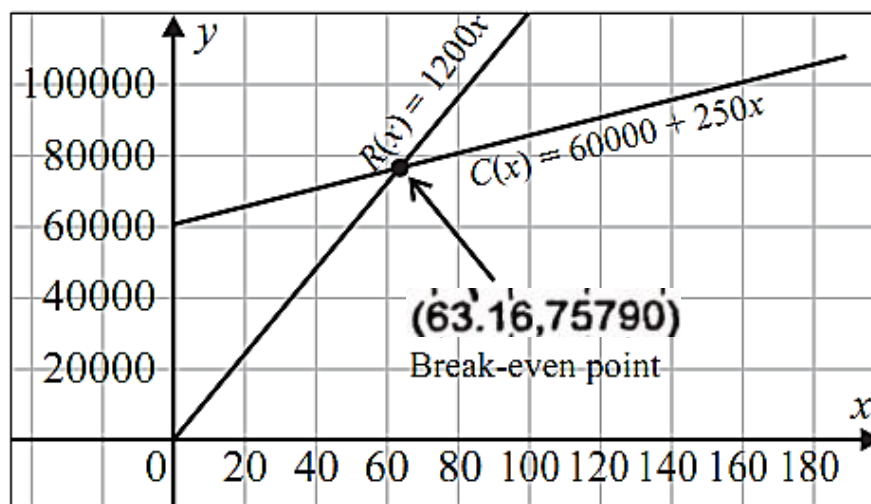
$$P(x) = 950x - 60000$$

$$P(100) = 950(100) - 60000$$

$$\text{Profit} = \text{Rs. } 35000$$

(c) Graph

x	20	40	60	80	100	120	140
C(x)	65000	70000	75000	80000	85000	90000	95000
R(x)	24000	48000	72000	96000	120000	144000	168000



$$\text{Profit} = \text{Rs. } 35000$$