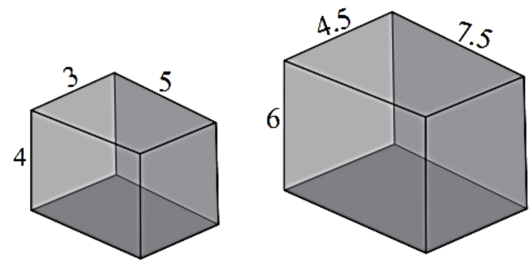
Unit 9 Similar Figures

EXERCISE 9.1

1. Find whether the solids are similar. All lengths are in cm.



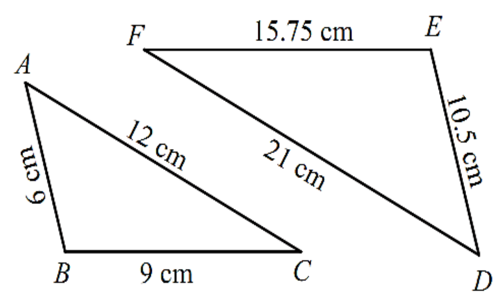
Solution

Ratio of corresponding sides are given as follows;

$\frac{4.5}{3} = \frac{7.5}{5} = \frac{6}{4} = \mathbf{1.5}$

Hence given Figures are similar.

2. In triangle ABC, the sides are given as $m\overline{AB} = 6$ cm, $m\overline{BC} = 9$ cm and $m\overline{CA} = 12$ cm. In triangle DEF, the sides are given as $m\overline{DE} = 10.5$ cm, $m\overline{EF} = 15.75$ cm, and $m\overline{FD} = 21$ cm. Prove that the triangles are similar.



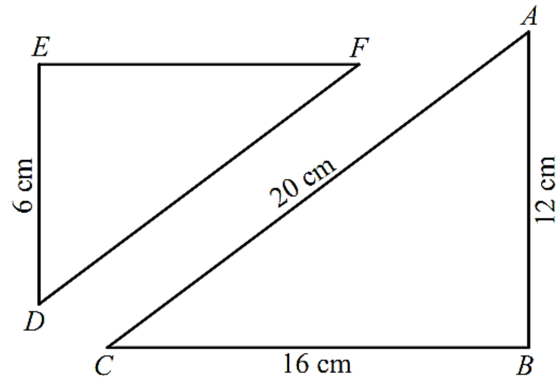
Solution

Ratio of corresponding sides are given as follows;

$\frac{21}{12} = \frac{10.5}{6} = \frac{15.75}{9} = \mathbf{1.75}$

Hence given triangles are similar.

3. In the given figure, $\Delta ABC \sim \Delta DEF$, $m\overline{AB} = 12$ cm, $m\overline{AC} = 20$ cm and $m\overline{BC} = 16$ cm. In ΔDEF, $m\overline{DE} = 6$ cm. Find $m\overline{DF}$ and $m\overline{EF}$.



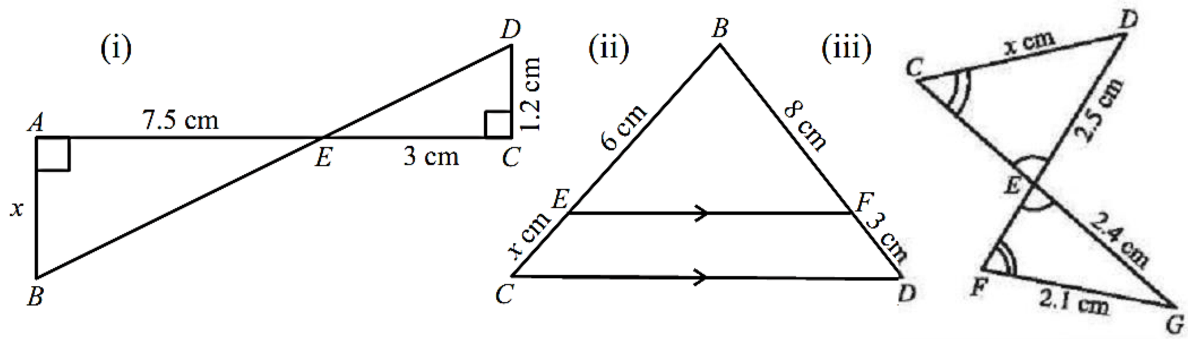
Solution

Since $\Delta ABC \sim \Delta DEF$

Therefore corresponding sides have same ratio.

Using $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{BC}}{m\overline{EF}}$ $\frac{12}{6} = \frac{16}{m\overline{EF}}$ $m\overline{EF} = 8cm$	Similarly $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}}$ $\frac{12}{6} = \frac{20}{m\overline{DF}}$ $m\overline{DF} = 10cm$

4. Find the value of x in each of the following:



Solution

4(i) since figures are similar, therefore

$\frac{7.5}{3} = \frac{x}{1.2} \Rightarrow x = \frac{7.5}{3} \times 1.2 \Rightarrow x = 3cm$

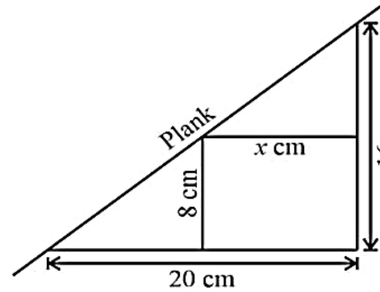
4(ii) since figures are similar, therefore

$\frac{8+3}{8} = \frac{6+x}{6} \Rightarrow 6 + x = \frac{11}{8} \times 6 \Rightarrow x = 2.25cm$

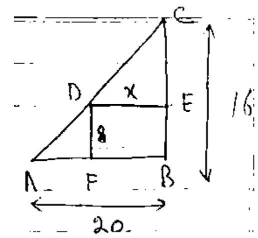
4(iii) since figures are similar, therefore

$\frac{x}{2.1} = \frac{2.5}{2.4} \Rightarrow x = \frac{2.5}{2.4} \times 2.1 \Rightarrow x = 2.19cm$

5. A plank is placed straight upstairs that 20 cm wide and 16 cm deep. A rectangular box of height 8 cm and width x cm is placed on a stair under the plank. Find the value of x.



Solution



In $\Delta ABC \sim \Delta AFD$

$\frac{BC}{DF} = \frac{AB}{AF} \Rightarrow \frac{16}{8} = \frac{20}{AF} \Rightarrow AF = 10\ cm$

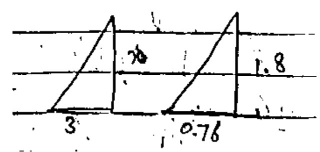
Since $AB = AF + FB$

$\Rightarrow 20 = 10 + FB \Rightarrow FB = 10cm$

$\Rightarrow x = 10cm \qquad \because DE = FB$

6. A man who is 1.8 m tall casts a shadow of a 0.76 m in length. If at the same time a telephone pole costs a 3 m shadow, find the height of the pole.

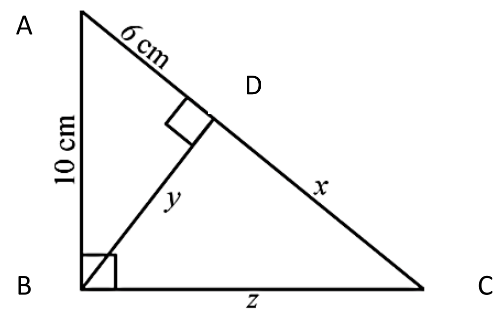
Solution



Since figures are similar, therefore

$\frac{3}{0.76} = \frac{x}{1.8} \Rightarrow x = \frac{3}{0.76} \times 1.8 \Rightarrow x = 7.11m$

7. Find the values of x, y and z in the given figure.



Solution

In $\Delta ABC \sim \Delta ABD$

$\frac{AB}{AD} = \frac{AC}{AB} \Rightarrow \frac{10}{6} = \frac{x+6}{10} \Rightarrow 6x + 36 = 100 \Rightarrow x = 10\frac{2}{3}cm = 10.667cm$

In ΔABD using Pythagoras Theorem

$H^2 = P^2 + B^2 \Rightarrow (10)^2 = (6)^2 + y^2 \Rightarrow 100 = 36 + y^2 \Rightarrow y^2 = 64 \Rightarrow y = 8cm$

In ΔABC we have $Hyp = AC = x + 6 \Rightarrow AC = 10.667 + 6 \Rightarrow AC = 16.667cm$

In ΔABC using Pythagoras Theorem

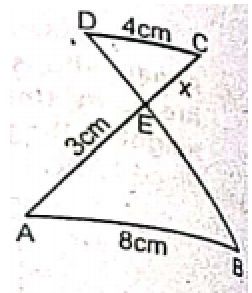
$H^2 = P^2 + B^2 \Rightarrow (16.667)^2 = (10)^2 + z^2 \Rightarrow 277.778 = 100 + z^2$

$\Rightarrow z^2 = 177.778 \Rightarrow z = 13\frac{1}{3}cm = 13.334cm$

8. Draw an isosceles trapezoid $ABCD$ where $\overline{AB} \parallel \overline{CD}$ and $m\overline{AB} > m\overline{CD}$. Draw diagonals $\overline{AC}$ and $\overline{BD}$, intersecting at E. Prove that $\triangle ABE$ is similar to $\triangle CDE$. If $m\overline{AB}$ = 8 cm, $m\overline{CD}$ = 4 cm, and $m\overline{AE}$ = 3 cm, find the length of $\overline{CE}$.

Solution

Given that $\Delta ABE \sim \Delta CDE$



$\frac{AB}{CD} = \frac{AE}{CE}$

$\Rightarrow \frac{8}{4} = \frac{3}{x}$

$\Rightarrow x = 1.5\ cm$

9. A regular dodecagon has its side lengths decreased by a factor of $\frac{1}{\sqrt{2}}$. If the perimeter of the original dodecagon is 72 cm. What is the side length of scaled dodecagon?

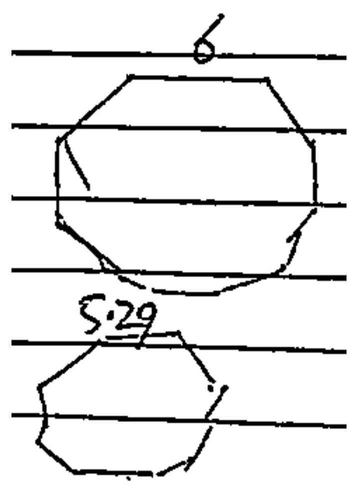
Solution

Dodecagon is 12 sided figure.

Perimeter = 72cm

Since perimeter is sum of all sides, so let x is length of sides, then

x + x + x + … 12 times = 72



12x = 72

x = 6cm

after $\frac{1}{\sqrt{2}}$ decrease it becomes

Scaled side length $= S' = x \times \frac{1}{\sqrt{2}}$

$S' = 6 \times \frac{1}{\sqrt{2}} = \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$ cm

EXERCISE 9.2

1. Find the ratio of the areas of similar figures if the ratio of their corresponding lengths are: (i) 1:3 (ii) 3:4 (iii) 2:7 (iv) 8:9 (v) 6:5

Solution

1(i): $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9} \Rightarrow \boldsymbol{A_1 : A_2 = 1:9}$

1(ii): $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16} \Rightarrow \boldsymbol{A_1 : A_2 = 9:16}$

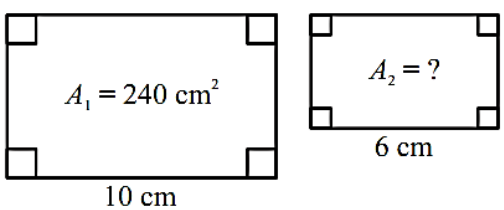
1(iii): $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{2}{7}\right)^2 = \frac{4}{49} \Rightarrow \boldsymbol{A_1 : A_2 = 4:49}$

1(iv): $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{8}{9}\right)^2 = \frac{64}{81} \Rightarrow \boldsymbol{A_1 : A_2 = 64:81}$

1(v): $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{6}{5}\right)^2 = \frac{36}{25} \Rightarrow \boldsymbol{A_1 : A_2 = 36:25}$

2. Find the unknowns in the following figures:

(i)

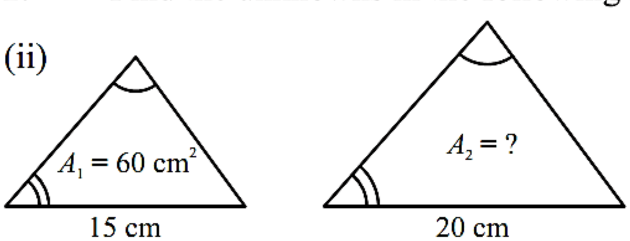


Solution

$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{240}{A_2} = \left(\frac{10}{6}\right)^2 \Rightarrow \frac{240}{A_2} = \frac{100}{36} \Rightarrow A_2 = \frac{240\times 36}{100} \Rightarrow \boldsymbol{A_2 = 86.4\text{cm}^2}$

2. Find the unknowns in the following figures:

(ii)

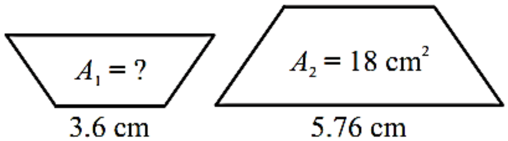


Solution

$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{60}{A_2} = \left(\frac{15}{20}\right)^2 \Rightarrow \frac{60}{A_2} = \frac{225}{400} \Rightarrow A_2 = \frac{60\times 400}{225} \Rightarrow \boldsymbol{A_2 = 106.67\text{cm}^2}$

2. Find the unknowns in the following figures:

(iii)

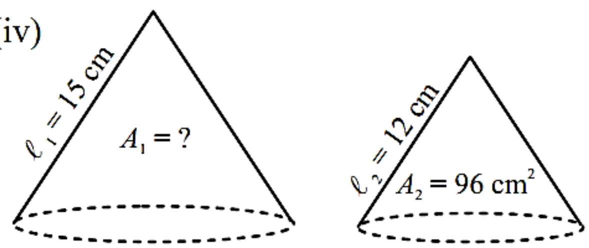


Solution

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{A_1}{18} = \left(\frac{3.6}{5.76}\right)^2 \Rightarrow \frac{A_1}{18} = \frac{12.96}{33.1776} \Rightarrow A_1 = \frac{12.96\times 18}{33.1776} \Rightarrow \boldsymbol{A_1 = 7.03125\mathrm{cm}^2}$$

2. Find the unknowns in the following figures:

(iv)

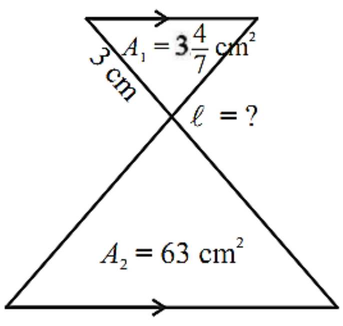


Solution

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{A_1}{96} = \left(\frac{15}{12}\right)^2 \Rightarrow \frac{A_1}{96} = \frac{225}{144} \Rightarrow A_1 = \frac{225\times 96}{144} \Rightarrow \boldsymbol{A_1 = 150\mathrm{cm}^2}$$

2. Find the unknowns in the following figures:

(v)



Solution

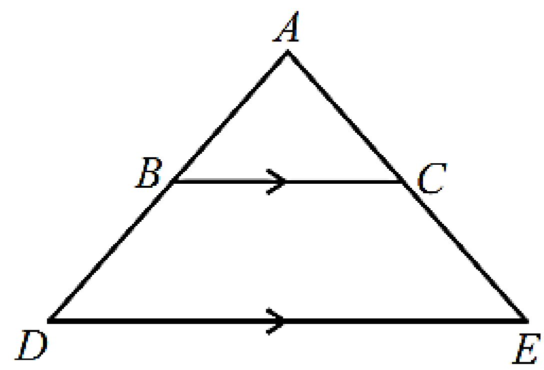
$$A_1 = 3\tfrac{4}{7} = 3.57, A_2 = 63, l_1 = 3, l_2 = ?$$

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{3.57}{63} = \left(\frac{3}{l_2}\right)^2 \Rightarrow 0.0587 = \left(\frac{3}{l_2}\right)^2 \Rightarrow 0.239 = \frac{3}{l_2} \Rightarrow l_2 = \frac{3}{0.239} \Rightarrow \boldsymbol{l_2 = 12.55\mathrm{cm}^2}$$

3. Given that area of $\Delta ABC = 36\ \text{cm}^2$ and $m\overline{AB} = 6\ \text{cm}$, $m\overline{BD} = 4\ \text{cm}$. Find

(a) the area of ΔADE

(b) the area of trapezium $BCED$



Solution

(a) the area of ΔADE

$l_1 = m\overline{AB} = 6\text{cm}, l_2 = m\overline{AD} = m\overline{AB} + m\overline{BD} = 6\text{cm} + 4\text{cm} = 10\text{cm}$

$A_1 = 36\text{cm}^2$, $A_2 = ?$

$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{36}{A_2} = \left(\frac{6}{10}\right)^2 \Rightarrow \frac{36}{A_2} = \frac{36}{100} \Rightarrow A_2 = \frac{36\times100}{36} \Rightarrow \boldsymbol{A_2 = 100\text{cm}^2}$

(b) the area of trapezium $BCED$

Area of trapezium BCED = Area of ΔADE − Area of ΔABC

Area of trapezium BCED = $100 - 36$

Area of trapezium BCED = 64cm^2

4. Given that ΔABC and ΔDEF are similar, with a scale factor of $k = 3$. If the area of ΔABC is 50 cm^2, find the area of triangle ΔDEF?

Solution

Scale Factor = k = 3 ; Area of ΔABC = 50cm^2

Using formula: $\frac{\text{Area of }\Delta\text{DEF}}{\text{Area of }\Delta\text{ABC}} = \text{k}^2$

Area of ΔDEF = (Area of ΔABC)(k^2) = (50)(3^2) = (50)(9)

Area of ΔDEF = 450cm^2

5. Quadrilaterals $ABCD$ and $EFGH$ are similar, with a scale factor of $k = \frac{1}{4}$. If the area of quadrilateral $ABCD$ is 64 cm^2, find the area of quadrilateral $EFGH$.

Solution

Scale Factor = k = $\frac{1}{4}$; Area of Quadrilateral ABCD = 64cm^2

Using formula: $\frac{\text{Area of Quadrilateral EFGH}}{\text{Area of Quadrilateral ABCD}} = \text{k}^2$

$\text{Area of Quadrilateral EFGH} = (\text{Area of Quadrilateral ABCD})(\text{k}^2)$

$\text{Area of Quadrilateral EFGH} = (64)\left(\frac{1}{4}\right)^2 = (64)\left(\frac{1}{16}\right) = 4\text{cm}^2$

6. The areas of two similar triangles are 16 cm^2 and 25 cm^2. What is the ratio of a pair of corresponding sides?

Solution

$$\left(\frac{l_1}{l_2}\right)^2 = \frac{A_1}{A_2} \Rightarrow \left(\frac{l_1}{l_2}\right)^2 = \frac{16}{25} \Rightarrow \frac{l_1}{l_2} = \sqrt{\frac{16}{25}} \Rightarrow \frac{l_1}{l_2} = \frac{4}{5} \Rightarrow \boldsymbol{l_1 : l_2 = 4 : 5}$$

7. The areas of two similar triangles are 144 cm^2 and 81 cm^2. If the base of the large triangle is 30 cm, find the corresponding base of the smaller triangle.

Solution

We have to find here l_2.

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{144}{81} = \left(\frac{30}{l_2}\right)^2 \Rightarrow \frac{12}{9} = \frac{30}{l_2} \Rightarrow l_2 = \frac{9\times 30}{12} \Rightarrow \boldsymbol{l_2 = 22.5}$$

8. A regular heptagon is inscribed in a larger regular heptagon and each side of the larger heptagon is 1.7 times the side of the smaller heptagon. If the area of the smaller heptagon is known to be 100 cm², find the area of the larger heptagon.

Solution

Scale Factor = k = 1.7 ; Area smaller heptagon = 100cm^2

Using formula: $\frac{\text{Area larger heptagon}}{\text{Area smaller heptagon}} = \text{k}^2$

$\text{Area larger heptagon} = (\text{Area smaller heptagon})(\text{k}^2)$

$\text{Area larger heptagon} = (100)(1.7)^2 = (100)(2.89) = 289\text{cm}^2$

EXERCISE 9.3

1. The radii of two spheres are in the ratio 3 : 4. What is the ratio of their volumes?

Solution

$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \frac{V_1}{V_2} = \left(\frac{3}{4}\right)^3 \Rightarrow \frac{V_1}{V_2} = \frac{27}{64} \Rightarrow \boldsymbol{V_1 : V_2 = 27 : 64}$

2. Two regular tetrahedrons have volumes in the ratio 8 : 27. What is the ratio of their sides?

Solution

$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 \Rightarrow \frac{8}{27} = \left(\frac{l_1}{l_2}\right)^3 \Rightarrow \sqrt[3]{\frac{8}{27}} = \sqrt[3]{\left(\frac{l_1}{l_2}\right)^3} \Rightarrow \frac{2}{3} = \frac{l_1}{l_2} \Rightarrow \boldsymbol{l_1 : l_2 = 2 : 3}$

3. Two right cones have volumes in the ratio 64 : 125. What is the ratio of:

(a) their heights (b) their base areas?

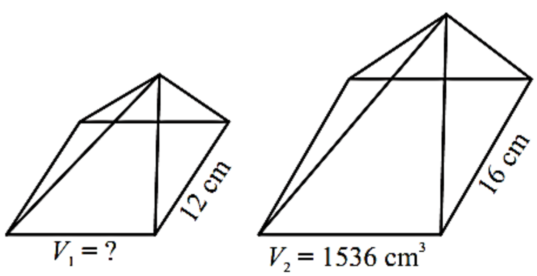
Solution

(a) $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3 \Rightarrow \frac{64}{125} = \left(\frac{h_1}{h_2}\right)^3 \Rightarrow \sqrt[3]{\frac{64}{125}} = \sqrt[3]{\left(\frac{h_1}{h_2}\right)^3} \Rightarrow \frac{4}{5} = \frac{h_1}{h_2} \Rightarrow \boldsymbol{h_1 : h_2 = 4 : 5}$

(b) $\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2 \Rightarrow \frac{A_1}{A_2} = \left(\frac{4}{5}\right)^2 \Rightarrow \frac{A_1}{A_2} = \frac{16}{25} \Rightarrow \boldsymbol{A_1 : A_2 = 16 : 25}$

4. Find the missing value in the following similar solids.

(i)



Solution

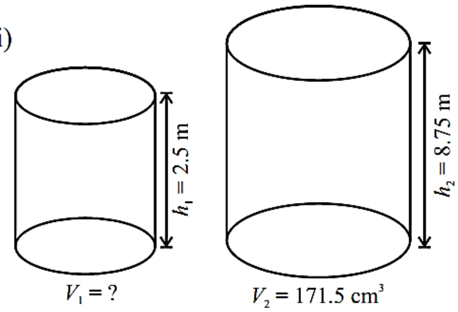
$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 \Rightarrow \frac{V_1}{1536} = \left(\frac{12}{16}\right)^3 \Rightarrow \frac{V_1}{1536} = \left(\frac{3}{4}\right)^3 \Rightarrow \frac{V_1}{1536} = \frac{27}{64}$

$\Rightarrow V_1 = \frac{1536 \times 27}{64}$

$\Rightarrow \boldsymbol{V_1 = 648\text{cm}^3}$

4. Find the missing value in the following similar solids.

(ii)



Solution

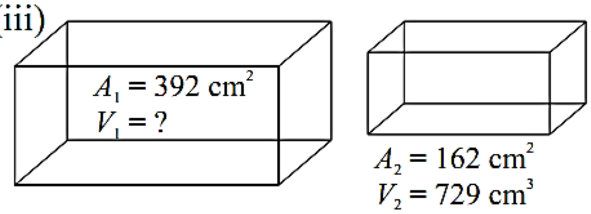
$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3 \Rightarrow \frac{V_1}{171.5} = \left(\frac{2.5}{8.75}\right)^3 \Rightarrow \frac{V_1}{171.5} = \frac{15.625}{669.921}$

$\Rightarrow V_1 = \frac{15.625 \times 171.5}{669.921}$

$\Rightarrow \boldsymbol{V_1} = \mathbf{4cm^3}$

4. Find the missing value in the following similar solids.

(iii)



Solution

Using formula: $\frac{A_1}{A_2} = \mathrm{k}^2$

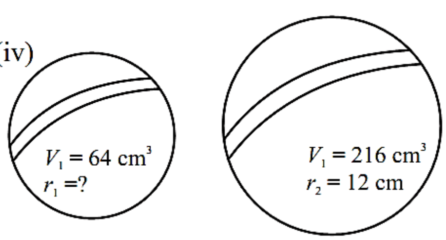
$\Rightarrow \frac{392}{162} = \mathrm{k}^2 \Rightarrow \mathrm{k}^2 = 2.420 \Rightarrow \mathrm{k} = 1.56$

Again using formula: $\frac{V_1}{V_2} = \mathrm{k}^3$

$\Rightarrow \frac{V_1}{729} = (1.56)^3 \Rightarrow V_1 = (729)(3.80) \Rightarrow \boldsymbol{V_1} \approx \mathbf{2744cm^3}$

4. Find the missing value in the following similar solids.

(iv)



Solution

$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \frac{64}{216} = \left(\frac{r_1}{12}\right)^3 \Rightarrow \sqrt[3]{\frac{64}{216}} = \sqrt[3]{\left(\frac{r_1}{12}\right)^3} \Rightarrow \frac{4}{6} = \frac{r_1}{12} \Rightarrow r_1 = \frac{4 \times 12}{6}$

$\Rightarrow \boldsymbol{r_1} = \mathbf{8\ cm}$

5. The ratio of the corresponding lengths of two similar canonical cans is 3 : 2.
   (i) The larger canonical can have surface area of 96 m^2. Find the surface area of the smaller canonical can.
   (ii) The smaller canonical can have a volume of 240 m^3. Find the volume of larger canonical can.

Solution

(a) $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{96}{A_2} = \left(\frac{3}{2}\right)^2 \Rightarrow \frac{96}{A_2} = \frac{9}{4} \Rightarrow A_2 = \frac{96\times4}{9} \Rightarrow \boldsymbol{A_2 = 42.67\mathrm{m}^2}$

(b) $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 \Rightarrow \frac{V_1}{240} = \left(\frac{3}{2}\right)^3 \Rightarrow \frac{V_1}{240} = \frac{27}{8} \Rightarrow V_1 = \frac{240\times27}{8} \Rightarrow \boldsymbol{V_1 = 8107\mathrm{m}^3}$

6. The ratio of the heights of two similar cylindrical water tanks is 5 : 3.
   (i) If the surface area of the larger tank is 250 square metres, find the surface area of the smaller tank.
   (ii) If the volume of the smaller tank is 270 cubic metres, find the volume of the larger tank.

Solution

(a) $\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2 \Rightarrow \frac{250}{A_2} = \left(\frac{5}{3}\right)^2 \Rightarrow \frac{250}{A_2} = \frac{25}{9} \Rightarrow A_2 = \frac{250\times9}{25} \Rightarrow \boldsymbol{A_2 = 90\mathrm{m}^2}$

(b) $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3 \Rightarrow \frac{V_1}{270} = \left(\frac{5}{3}\right)^3 \Rightarrow \frac{V_1}{270} = \frac{125}{27} \Rightarrow V_1 = \frac{270\times125}{27} \Rightarrow \boldsymbol{V_1 = 1250\mathrm{m}^3}$

EXERCISE 9.4

1. (i) What is the sum of the interior angles of a decagon (10-sided polygon)?
(ii) Calculate the measure of each interior angle of a regular hexagon.
(iii) What is each exterior angle of a regular pentagon?
(iv) If the sum of the interior angles of a polygon is 1260°, how many sides does the polygon have?

Solution

(i) Sum of the Interior Angle $= (n-2)\times 180° = (10-2)\times 180° = 8\times 180° = 1440°$

(ii) Measure of the Interior Angle $= \frac{\text{Sum of the Interior Angle}}{\text{n}} = \frac{(\text{n}-2)\times 180°}{\text{n}} = \frac{(6-2)\times 180°}{6} = 120°$

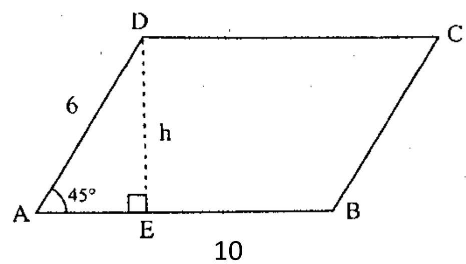
(iii) Measure of each Exerior Angle $= \frac{360°}{\text{n}} = \frac{360°}{5} = 72°$

(iv) Sum of the Interior Angle of Polygon $= (n-2)\times 180°$

$\Rightarrow 1260° = (n-2)\times 180° \Rightarrow \frac{1260°}{180°} = \text{n}-2 \Rightarrow 7 = \text{n}-2 \Rightarrow \text{n} = 9$

2. In a parallelogram $ABCD$, $m\overline{AB} = \underline{10}$ cm, $m\overline{AD} = 6$ cm and $m\angle BAD = 45°$.
Calculate the area of $ABCD$.

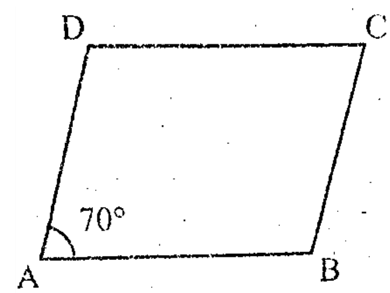
Solution



Given ABCD is a parallelogram.
Also $\text{m}\overline{AB} = 10\text{cm}, \text{m}\overline{AD} = 6\text{cm}, \text{m}\angle\text{BAD} = 45°$
Area of parallelogram ABCD= Base $\times$ Height
Area of parallelogram ABCD= $\overline{AB} \times \overline{ED}$
Area of parallelogram ABCD= $\overline{AB} \times \overline{AD}sin\theta$
Area of parallelogram ABCD= $10 \times 6sin45°$
Area of parallelogram ABCD= 42.43cm^2

3. In a parallelogram $ABCD$ if $m\angle DAB = 70°$, find the measures of all other angles in the parallelogram.

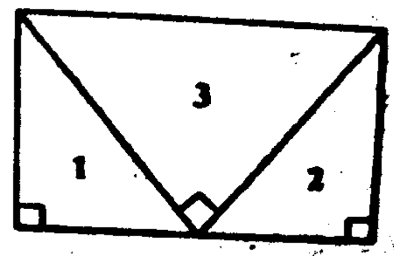
Solution



$\text{m}\angle\text{DAB} = 70°$
$\text{m}\angle\text{DAB} = \text{m}\angle\text{BCD} = 70°$ ∵opposite angles of ||gram
$\text{m}\angle\text{DAB} + \text{m}\angle\text{ABC} = 180°$ $\because AD \parallel BC$
$70° + \text{m}\angle\text{ABC} = 180°$
$\text{m}\angle\text{ABC} = 110°$
also $\text{m}\angle\text{CDA} = 110°$ $\because A\text{m}\angle\text{ABC} = \text{m}\angle\text{CDA}$

4. A shape is created by cutting a square in half diagonally and then attaching a right-angled triangle to the hypotenuse of each half. Explain why this shape can tessellate and calculate the interior angle of the new shape.

Solution



The new shape can tessellate because it is composed of triangles which are inherently able to tessellate when arranged approximately that they fit together perfectly without gaps.
Also, the sum of interior angles is 360°.

Explanation

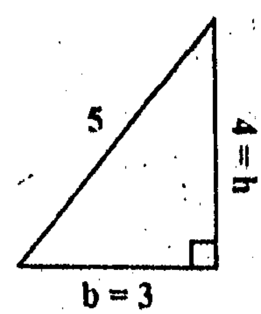
The new shape can tessellate because it is composed of triangles which are inherently able to tessellate when arranged approximately.
The original square when certain half diagonally, forms two congruent right angled triangles.
Attaching right angled triangles to the hypotenuse of these two halves does not change their overall symmetry. The new shape remains geometrically compatible with tessellation because the triangles angles ensure that they fit together perfectly without gaps.

Interior angles of the new shape

The sum of interior angles of a hexagon is (6 - 2) × 180° = 720°.
Since the shape is composed of two identical parts, the sum of interior angles of one part is 720° ÷ 2 = 360°.
Since the shape has 6 sides, the interior angle of the new shape is 360° ÷ 6 = 60° (for the equilateral triangle) and 120° (for the isosceles triangle).

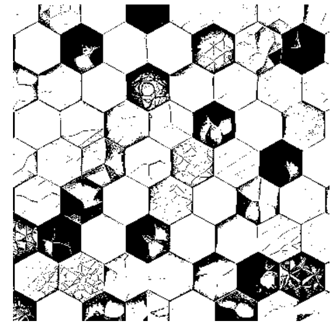
5. A tessellation is created by repeatedly reflecting a basic shape. The basic shape is a right-angled triangle with sides of length 3, 4, and 5 units. Find: The minimum number of reflections needed to create a tessellation that covers a square with an area of 3600 square units.

Solution



Area of triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$
Area of triangle $= \frac{1}{2} \times 3 \times 4 = 6$ units
Number of triangles $= \frac{\text{area of square}}{\text{area of triangles}} = \frac{3600}{6} = 600$
So 600 reflections needed to cover the square.

6. A tessellation is created using regular hexagons. Each hexagon has a side length of 5 cm. Find the total area of the tessellation if it consists of 25 hexagons and total perimeter of the outer edge of the tessellation, assuming it's a perfect hexagon.



Solution

Area of hexagon $= \frac{3\sqrt{3}}{2}a^2 = \frac{3\sqrt{3}}{2} \times 5^2 = \frac{3\sqrt{3}}{2} \times 25 = 64.95\text{cm}^2$

Area of 25 hexagon $= 64.95 \times 25 = 1623.75\text{cm}^2$

Perimeter $= 6 \times (5 \times 5) = 6 \times 25 = 150\text{cm}$

Note:

length of 1 side is 5×5 since each side consists of 5 hexagons

7. A rectangular floor is 12 m by 15 m. How many square tiles, each 1 m by 1 m, are needed to cover the floor?

Solution

Number of tiles $= \frac{\text{area of floor}}{\text{area of one tile}} = \frac{12\times15}{1\times1} = \frac{180\text{m}^2}{1\text{m}^2} = 180$ tiles

8. A rectangular wall is 10 m tall and 12 m wide. How many gallons of paint are needed to cover the wall, if one gallon covers 35 m^2?

Solution

Number of gallons $= \frac{\text{area of rectangular wall}}{\text{coverage per gallon}} = \frac{10\times12}{350} = \frac{120\text{m}^2}{350\text{m}^2} = 0.343$ gallons

9. A rectangular wall has a length of 10 m and a width of 4 meters. If 1 litre of paint covers 7 m^2, how many liters of paint are needed to cover the wall?

Solution

Paint needed $= \frac{\text{area of the wall}}{\text{paint per liter}} = \frac{10\times4}{7} = \frac{40\text{m}^2}{7\text{m}^2} = 5.71 \approx 6$ liters

10. A window has a trapezoidal shape with parallel sides of 3 m and 1.5 m and a height of 2 m. Find the area of the window.

Solution

Area of trapezoidal $= \frac{\text{sum of parallel sides}}{2} \times \text{height} = \frac{3+1.5}{2} \times 2 = 4.5\text{m}^2$

REVIEW EXERCISE 9

1. Four options are given against each statement. Encircle the correct one.

(i) If two polygons are similar, then:
   (a) ✓ their corresponding angles are equal.
   (b) their areas are equal.
   (c) their volumes are equal.
   (d) their corresponding sides are equal.

(ii) The ratio of the areas of two similar polygons is:
   (a) equal to the ratio of their perimeters.
   (b) ✓ equal to the square of the ratio of their corresponding sides.
   (c) equal to the cube of the ratio of their corresponding sides.
   (d) equal to the sum of their corresponding sides.

(iii) If the volume of two similar solids is 125 cm³ and 27 cm³, the ratio of their corresponding heights is -----------.
   (a) 3:5 (b) ✓ 5:3 (c) 25:9 (d) 9:25

(iv) The exterior angle of regular pentagon is:
   (a) 40° (b) 45° (c) 60° (d) ✓ 72°

(v) A parallelogram has an area of 64 cm² and a similar parallelogram has an area of 144 cm². If a side of the smaller parallelogram is 8 cm, the corresponding side of the larger parallelogram is:
   (a) 10 cm (b) 12 cm (c) ✓ 18 cm (d) 16 cm

(vi) The total number of diagonals in a polygon with 9 sides is:
   (a) 18 (b) 21 (c) 25 (d) ✓ 27

(vii) Two spheres are similar, and their radii are in the ratio 4:5. If the surface area of the larger sphere is 500π cm², what is the surface area of the smaller sphere?
   (a) 256π cm² (b) 320π cm² (c) ✓ 400π cm² (d) 405π cm²

(viii) A regular polygon has an exterior angle of 30°. How many diagonals does the Polygon have?
   (a) ✓ 54 (b) 90 (c) 72 (d) 108

(ix) In a regular hexagon, the ratio of the length of a diagonal to the side length is:

(a) $\sqrt{3}:1$ (b) ✓ $2:1$ (c) $3:2$ (d) $2:3$

(x) A regular polygon has an interior angle of 165°. How many sides does it have?

(a) 15 (b) 16 (c) 20 (d) ✓ 24

2. If the sum of the interior angles of a polygon is 1080°, how many sides does the polygon has?

Solution

Sum of the Interior Angle of Polygon $= (n-2) \times 180°$

$\Rightarrow 1080° = (n-2) \times 180° \Rightarrow \frac{1080°}{180°} = \mathrm{n} - 2 \Rightarrow 6 = \mathrm{n} - 2 \Rightarrow \mathrm{n} = 8$

3. Two similar bottles are such that one is twice as high as the other. What is the ratio of their surface areas and their capacities?

Solution

(a) $\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2 \Rightarrow \frac{A_1}{A_2} = \left(\frac{2x}{1x}\right)^2 = \left(\frac{2}{1}\right)^2 = \frac{4}{1} \Rightarrow \boldsymbol{A_1 : A_2 = 4 : 1}$

(b) $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3 \Rightarrow \frac{V_1}{V_2} = \left(\frac{2}{1}\right)^3 = \frac{8}{1} \Rightarrow \boldsymbol{V_1 : V_2 = 8 : 1}$

4. Each dimension of a model car is $\frac{1}{10}$ of the corresponding car dimension. Find the ratio of:

(a) the areas of their windscreens (b) the capacities of their boots

(c) the widths of the cars (d) the number of wheels they have.

Solution

(a) $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{A_1}{A_2} = \left(\frac{1}{10}\right)^2 = \frac{1}{100} \Rightarrow \boldsymbol{A_1 : A_2 = 1 : 100}$

(b) $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 \Rightarrow \frac{V_1}{V_2} = \left(\frac{1}{10}\right)^3 = \frac{1}{1000} \Rightarrow \boldsymbol{V_1 : V_2 = 1 : 1000}$

(c) $\frac{l_1}{l_2} = \frac{1}{10} \Rightarrow \boldsymbol{l_1 : l_2 = 1 : 10}$

(d) ratio of wheels of car $= \frac{4}{4} = \frac{1}{1}$

$\Rightarrow$ **ratio of number of wheels of car = 1 : 1**

5. Three similar jugs have heights 8 cm, 12 cm and 16 cm. If the smallest jug holds $\frac{1}{2}$ litre, find the capacities of the other two.

Solution

(a) $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3 \Rightarrow \frac{\frac{1}{2}}{V_2} = \left(\frac{8}{12}\right)^3 = \left(\frac{2}{3}\right)^3 \Rightarrow \frac{\frac{1}{2}}{V_2} = \frac{8}{27} \Rightarrow V_2 = \frac{1\times 27}{2\times 8} \Rightarrow \boldsymbol{V_2 = 1.69}$ **liter**

(b) $\frac{V_1}{V_3} = \left(\frac{h_1}{h_3}\right)^3 \Rightarrow \frac{\frac{1}{2}}{V_3} = \left(\frac{8}{16}\right)^3 = \left(\frac{1}{2}\right)^3 \Rightarrow \frac{\frac{1}{2}}{V_3} = \frac{1}{8} \Rightarrow V_3 = \frac{1\times 8}{2\times 1} \Rightarrow \boldsymbol{V_3 = 4}$ **liter**

6. Three similar drinking glasses have heights 7.5 cm, 9 cm and 10.5 cm. If the tallest glass holds 343 millilitres, find the capacities of the other two.

Solution

(a) $\frac{V_1}{V_3} = \left(\frac{h_1}{h_3}\right)^3 \Rightarrow \frac{V_1}{0.343} = \left(\frac{7.5}{10.5}\right)^3 = \left(\frac{1.5}{2.1}\right)^3 \Rightarrow \frac{V_1}{0.343} = \frac{3.375}{9.261} \Rightarrow V_1 = \frac{3.375\times 0.343}{9.261}$
$\Rightarrow \boldsymbol{V_1 = 0.125}$ **liter** $= \mathbf{125}$ **mL**

(b) $\frac{V_2}{V_3} = \left(\frac{h_2}{h_3}\right)^3 \Rightarrow \frac{V_2}{0.343} = \left(\frac{9}{10.5}\right)^3 \Rightarrow \frac{V_2}{0.343} = \frac{729}{1157.625} \Rightarrow V_2 = \frac{729\times 0.343}{1157.625}$
$\Rightarrow \boldsymbol{V_2 = 0.216}$ **liter** $= \mathbf{216}$ **mL**

7. A toy manufacturer produces model cars which are similar in every way to the actual cars. If the ratio of the door area of the model to the door area of the car is 1 cm to 2500 cm, find:
   (a) the ratio of their lengths
   (b) the ratio of the capacities of their petrol tanks
   (c) the width of the model, if the actual car is 150 cm wide
   (d) the area of the rear window of the actual car if the area of the rear window of the model is 3 cm^2.

Solution

(a) $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{1}{2500} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \sqrt{\frac{1}{2500}} = \sqrt{\left(\frac{l_1}{l_2}\right)^2} \Rightarrow \frac{1}{50} = \frac{l_1}{l_2} \Rightarrow \boldsymbol{l_1 : l_2 = 1 : 50}$

(b) $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 \Rightarrow \frac{V_1}{V_2} = \left(\frac{1}{50}\right)^3 = \frac{1}{125000} \Rightarrow \boldsymbol{V_1 : V_2 = 1 : 125000}$

(c) $\frac{w_1}{w_2} = \frac{l_1}{l_2} \Rightarrow \frac{w_1}{150} = \frac{1}{50} \Rightarrow w_1 = \frac{150}{50} \Rightarrow \boldsymbol{w_1 = 3}$**cm**

(d) $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{3}{A_2} = \left(\frac{1}{50}\right)^2 \Rightarrow \frac{3}{A_2} = \frac{1}{2500} \Rightarrow A_2 = 3 \times 2500 \Rightarrow \boldsymbol{A_2 = 7500}$**cm**$^{\mathbf{2}}$

8. The ratio of the areas of two similar labels on two similar jars of coffee is 144 : 169. Find the ratio of

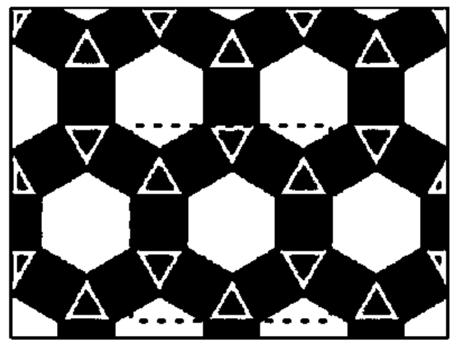
   (a) the heights of the two jars (b) their capacities.

Solution

(a) $\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2 \Rightarrow \frac{144}{169} = \left(\frac{h_1}{h_2}\right)^2 \Rightarrow \sqrt{\frac{144}{169}} = \sqrt{\left(\frac{h_1}{h_2}\right)^2} \Rightarrow \frac{12}{13} = \frac{h_1}{h_2} \Rightarrow \boldsymbol{h_1 : h_2 = 12 : 13}$

(b) $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3 \Rightarrow \frac{V_1}{V_2} = \left(\frac{12}{13}\right)^3 = \frac{1728}{2197} \Rightarrow \boldsymbol{V_1 : V_2 = 1728 : 2197}$

9. A tessellation of tiles on a floor has been made using a repeating pattern of a regular hexagon, six squares and six equilateral triangles. Find the total area of a single pattern with side length $\frac{1}{2}$ metre of each polygon.



Solution

The pattern will be 12 sided regular polygon (Dodecagon)

Area of Dodecagon $= 3(2+\sqrt{3})a^2$

Area of Dodecagon $= 3(2+\sqrt{3}) \times \left(\frac{1}{2}\right)^2$

Area of Dodecagon $= 3(2+\sqrt{3}) \times \frac{1}{4}$

Area of Dodecagon $= 2.8\text{m}^2$