Unit 7 Coordinate Geometry

EXERCISE 7.1

1. Describe the location in the plane of the point $P(x, y)$, for which

(i) $x > 0$ (ii) $x > 0$ and $y > 0$ (iii) $x = 0$ (iv) $y = 0$
(v) $x > 0$ and $y \leq 0$ (vi) $y = 0, x = 0$ (vii) $x = y$
(viii) $x \geq 3$ (ix) $y > 0$ (x) x and y have opposite signs.

Solution

(i) Right half plane (ii) The 1st quadrant (iii) y-axis (iv) x-axis (v) 4th quadrant and negative y-axis (vi) Origin (vii) It is a line bisecting 1st and 3rd quadrant.
(vii) The set of points lying on and right side of the line $x = 3$.
(ix) The set of points lying above x-axis. (x) The set of points in 2nd and 4th quadrants.

2. Find the distance between the points:

(i) $A(6, 7), B(0, -2)$ (ii) $C(-5, -2), D(3, 2)$
(iii) $L(0, 3), M(-2, -4)$ (iv) $P(-8, -7), Q(0, 0)$

Solution

2 (i) $d = |\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0-6)^2 + (-2-7)^2}$
$= \sqrt{(-6)^2 + (-9)^2} = \sqrt{36 + 81} = \sqrt{117} = 3\sqrt{13}$

2 (ii) $d = |\overline{CD}| = \sqrt{\left(3-(-5)\right)^2 + \left(2-(-2)\right)^2}$
$= \sqrt{(8)^2 + (4)^2} = \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5}$

2 (iii) $d = |\overline{LM}| = \sqrt{(-2-0)^2 + \left(3-(-4)\right)^2}$
$= \sqrt{(-2)^2 + (7)^2} = \sqrt{4 + 49} = \sqrt{53}$

2 (iv) $d = |\overline{PQ}| = \sqrt{\left(0-(-8)\right)^2 + \left(0-(-7)\right)^2}$
$= \sqrt{(8)^2 + (7)^2} = \sqrt{64 + 49} = \sqrt{113}$

3. Find in each of the following:

(i) The distance between the two given points

(ii) Midpoint of the line segment joining the two points:

(a) $A(3, 1)$, $B(-2, -4)$ (b) $A(-8, 3)$, $B(2, -1)$

(c) $A\left(-\sqrt{5}, -\frac{1}{3}\right)$, $B\left(-3\sqrt{5}, 5)\right)$

Solution

3(i) $\boldsymbol{A(3,1)}$, $\boldsymbol{B(-2,-4)}$

$$|AB| = \sqrt{(-2-3)^2 + (-4-1)^2} = \sqrt{(-5)^2 + (-5)^2}$$
$$= \sqrt{25+25} = \sqrt{50} = \sqrt{25\times 2} = 5\sqrt{2}$$

$$\text{Midpoint of } AB = \left(\frac{3-2}{2}, \frac{1-4}{2}\right) = \left(\frac{1}{2}, \frac{-3}{2}\right)$$

3(ii) $\boldsymbol{A(-8,3)}$, $\boldsymbol{B(2,-1)}$

$$|\overline{AB}| = \sqrt{\left(2-(-8)\right)^2 + (-1-3)^2}$$
$$= \sqrt{(10)^2 + (-4)^2} = \sqrt{100+16} = \sqrt{116} = 2\sqrt{29}$$

$$\text{mid point of AB} = \left(\frac{-8+2}{2}, \frac{3+(-1)}{2}\right) = \left(\frac{-6}{2}, \frac{2}{2}\right) = (-3,1)$$

3(iii) $\boldsymbol{A(-\sqrt{5}, -\frac{1}{3})}$, $\boldsymbol{B(-3\sqrt{5}, 5)}$

$$|AB| = \sqrt{\left(-3\sqrt{5}+\sqrt{5}\right)^2 + \left(5+\frac{1}{3}\right)^2} = \sqrt{\left(2\sqrt{5}\right)^2 + \left(\frac{16}{3}\right)^2}$$
$$= \sqrt{20+\frac{256}{9}} = \sqrt{\frac{436}{9}} = \sqrt{\frac{4\times 109}{9}} = \frac{2\sqrt{109}}{3}$$

$$\text{Midpoint of } AB = \left(\frac{-\sqrt{5}-3\sqrt{5}}{2}, \frac{-\frac{1}{3}+5}{2}\right) = \left(\frac{-4\sqrt{5}}{2}, \frac{\frac{14}{3}}{2}\right) = \left(-2\sqrt{5}, \frac{7}{3}\right)$$

4. Which of the following points are at a distance of 15 units from the origin?

(i) $(\sqrt{176}, 7)$ (ii) $(10, -10)$ (iii) $(1, 15)$

Solution

(i) Distance of $\left(\sqrt{176},7\right)$ from origin $=\sqrt{\left(\sqrt{176}-0\right)^2+\left(7-0\right)^2}$

$=\sqrt{(176)+(49)}$

$=\sqrt{(176)+(49)} \quad =\sqrt{225} \quad =15$

$\Rightarrow$ the point $\left(\sqrt{176},7\right)$ is at 15 unit away from origin.

(ii) Distance of $(10,-10)$ from origin $=\sqrt{(10-0)^2+(-10-0)^2}$

$=\sqrt{100+100} \quad =\sqrt{200}$

$=\sqrt{100\times 2} \quad =10\sqrt{2}\neq 15$

$\Rightarrow$ the point $(10,-10)$ is not at distance of 15 unit from origin.

(iii) Distance of (1,15) from origin $=\sqrt{(1-0)^2+(15-0)^2}$

$=\sqrt{1+225}=\sqrt{226}$

the point(1,15) from is not at distance of 15 unit from origin

5. Show that:

(i) the points $A(0, 2)$, $B(\sqrt{3}, 1)$ and $C(0, -2)$ are vertices of a right triangle.

Solution

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow AB = \sqrt{(\sqrt{3} - 0)^2 + (1 - 2)^2}$$

$$= \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4}$$

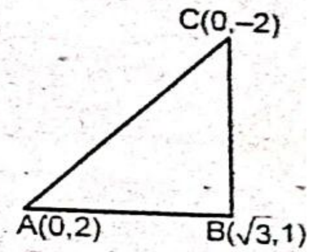
$$= 2$$

$$\text{And } BC = \sqrt{(0 - \sqrt{3})^2 + (-2 - 1)^2}$$

$$= \sqrt{(-\sqrt{3})^2 + (-3)^2} = \sqrt{12}$$

$$\text{Also } AC = \sqrt{(0 - 0)^2 + (-2 - 2)^2}$$

$$= \sqrt{0 + (-4)^2} = 4$$



By pythagoras theorem

$(AC)^2 = (AB)^2 + (BC)^2$

$(4)^2 = (2)^2 + (\sqrt{12})^2$

$16 = 4 + 12 = 16$

Therefore, the given points are vertices of a at right triangle.

5. Show that:

(ii) the points $A(3, 1)$, $B(-2, -3)$ and $C(2, 2)$ are vertices of an isosceles triangle.

Solution

Given: $A(3,1)$, $B(-2,-3)$ and $C(2,2)$

$$|AB| = \sqrt{(-2-3)^2 + (-3-1)^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$$

$$|BC| = \sqrt{(2-(-2))^2 + (2-(-3))^2} = \sqrt{(4)^2 + (5)^2} = \sqrt{16+25} = \sqrt{41}$$

$$|CA| = \sqrt{(3-2)^2 + (1-2)^2} = \sqrt{(1)^2 + (-1)^2}$$

$$= \sqrt{1+1} = \sqrt{2}$$

$\because |AB| = |BC| \Rightarrow A, B$ & C are vertices of an isosceles triangle.

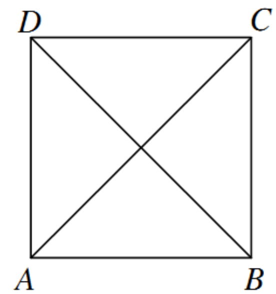
5. Show that:

(iii) the points $A(5, 2)$, $B(-2, 3)$, $C(-3, -4)$ and $D(4, -5)$ are vertices of a parallelogram.

Solution

Given: $A(5,2)$, $B(-2,3)$ & $C(-3,-4)$ and $D(4,-5)$

$$|AB| = \sqrt{(-2-5)^2 + (3-2)^2} = \sqrt{(-7)^2 + (1)^2}$$
$$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$
$$|BC| = \sqrt{(-3+2)^2 + (-4-3)^2} = \sqrt{(-1)^2 + (-7)^2}$$
$$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$
$$|CD| = \sqrt{(4+3)^2 + (-5+4)^2} = \sqrt{(7)^2 + (-1)^2}$$
$$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$
$$|DA| = \sqrt{(5-4)^2 + (2+5)^2} = \sqrt{(1)^2 + (7)^2}$$
$$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$



$\because$ $|AB| = |CD|$ and $|BC| = |DA|$ $\Rightarrow$ A, B, C and D are vertices of parallelogram.

6. Find h such that the points $A(\sqrt{3}, -1)$, $B(0, 2)$ and $C(h, -2)$ are vertices of a right triangle with right angle at the vertex A.

Solution

Since ABC is a right triangle therefore by Pythagoras theorem

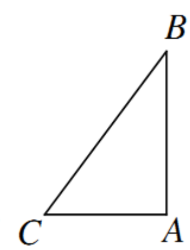
$$|AB|^2 + |CA|^2 = |BC|^2$$

$$\Rightarrow \left[\left(0-\sqrt{3}\right)^2 + (2+1)^2\right] + \left[\left(\sqrt{3}-h\right)^2 + (-1+2)^2\right] = (h-0)^2 + (-2-2)^2$$

$$\Rightarrow [3+9] + \left[3 - 2\sqrt{3}h + h^2 + 1\right] = h^2 + 16$$

$$\Rightarrow 12 + 4 - 2\sqrt{3}\,h + h^2 = h^2 + 16$$

$$\Rightarrow -2\sqrt{3}\,h = h^2 + 16 - 12 - 4 - h^2 \quad \Rightarrow -2\sqrt{3}\,h = 0 \quad \Rightarrow \boxed{h = 0}.$$



7. Find h such that $A(-1, h)$, $B(3, 2)$ and $C(7, 3)$ are collinear.

Solution

Points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Since given points are collinear therefore

$$\begin{vmatrix} -1 & h & 1 \\ 3 & 2 & 1 \\ 7 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow -1(2-3) - h(3-7) + 1(9-14) = 0 \quad \Rightarrow -1(-1) - h(-4) + 1(-5) = 0$

$\Rightarrow 1 + 4h - 5 = 0 \quad \Rightarrow 4h - 4 = 0 \quad \Rightarrow 4h = 4 \quad \Rightarrow \boxed{h = 1}$

8. The points $A(-5, -2)$ and $B(5, -4)$ are ends of a diameter of a circle. Find the centre and radius of the circle.

Solution

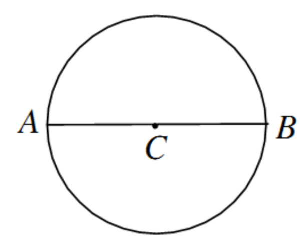
The centre of the circle is mid point of AB

i.e. centre 'C' $= \left(\dfrac{-5+5}{2}, \dfrac{-2-4}{2}\right) = \left(\dfrac{0}{2}, \dfrac{-6}{2}\right) = (0, -3)$

Now radius $= |AC|$

$= \sqrt{(0+5)^2 + (-3+2)^2}$

$= \sqrt{25+1} \quad = \sqrt{26}$



9. Find h such that the points $A(h, 1)$, $B(2, 7)$ and $C(-6, -7)$ are vertices of a right triangle with right angle at the vertex A.

Solution

According to the given condition

$|AB|^2 + |AC|^2 = |BC|^2$ ……(i)

$|\overline{AB}| = \sqrt{40 - 4h + h^2}$; $|\overline{BC}| = \sqrt{260}$; $|\overline{AC}| = \sqrt{h^2 + 12h + 100}$

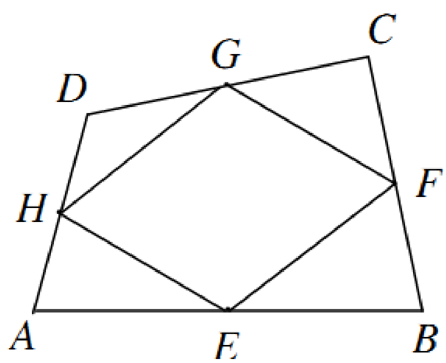
Putting in (i) we have $h^2 + 4h - 60 = 0$

$\Rightarrow (h+10)(h-6) = 0 \Rightarrow h + 10 = 0$; $h - 6 = 0$

$\Rightarrow h = -10$ or $h = 6$

10. A quadrilateral has the points $A(9, 3)$, $B(-7, 7)$, $C(-3, -7)$ and $D(5, -5)$ as its vertices. Find the midpoints of its sides. Show that the figure formed by joining the midpoints consecutively is a parallelogram.

Solution



Given: $A(9,3)$, $B(-7,7)$, $C(-3,-7)$ and $D(5,-5)$

Let E, F, G and H be the mid-points of sides of quadrilateral

Coordinate of $E = \left(\frac{9-7}{2}, \frac{3+7}{2}\right) = \left(\frac{2}{2}, \frac{10}{2}\right) = (1,5)$

Coordinate of $F = \left(\frac{-7-3}{2}, \frac{7-7}{2}\right) = \left(\frac{-10}{2}, \frac{0}{2}\right) = (-5,0)$

Coordinate of $G = \left(\frac{-3+5}{2}, \frac{-7-5}{2}\right) = \left(\frac{2}{2}, \frac{-12}{2}\right) = (1,-6)$

Coordinate of $H = \left(\frac{9+5}{2}, \frac{3-5}{2}\right) = \left(\frac{14}{2}, \frac{-2}{2}\right) = (7,-1)$

Now $|EF| = \sqrt{(-5-1)^2 + (0-5)^2} = \sqrt{36+25} = \sqrt{61}$

$|FG| = \sqrt{(1+5)^2 + (-6-0)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

$|GH| = \sqrt{(7-1)^2 + (-1+6)^2} = \sqrt{36+25} = \sqrt{61}$

$|HE| = \sqrt{(1-7)^2 + (5+1)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

Since $|EF| = |GH|$ and $|FG| = |HE|$

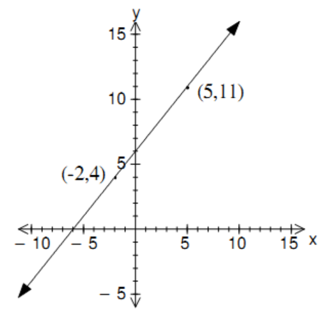
Therefore $EFGH$ is a parallelogram.

EXERCISE 7.2

1. Find the slope and inclination of the line joining the points:
(i) (–2, 4) ; (5, 11) (ii) (3, –2) ; (2, 7) (iii) (4, 6) ; (4, 8)

Solution

(i) $(-2,4)\quad;\quad(5,11)$

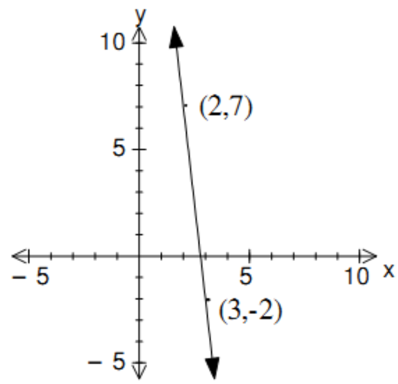


Slope $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{11-4}{5+2}=\dfrac{7}{7}=1$

Since $\tan\alpha=m=1$

$\Rightarrow\ \alpha=\tan^{-1}(1)=45^\circ$

(ii) $(3,-2)\quad;\quad(2,7)$



Slope $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{7+2}{2-3}=\dfrac{9}{-1}=-9$

Since $\tan\alpha=m=-9$

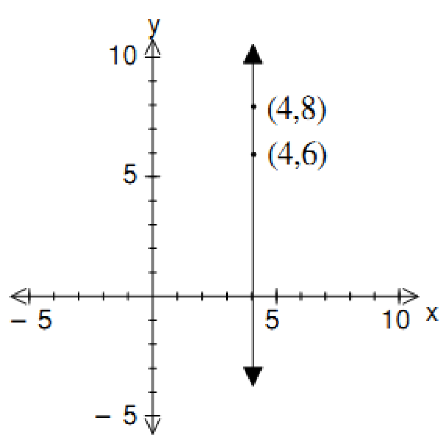
$\Rightarrow\ -\tan\alpha=9\ \Rightarrow\ \tan(180-\alpha)=9$

$\Rightarrow\ 180-\alpha=\tan^{-1}(9)$

$\Rightarrow\ 180-\alpha=83^\circ 40'$

$\Rightarrow\ \alpha=180-83^\circ 40'=96^\circ 20'$

(iii) $(4,6)\quad;\quad(4,8)$



Slope $m=\dfrac{y_2-y_1}{x_2-x_1}$

$=\dfrac{8-6}{4-4}=\dfrac{2}{0}=\infty$

Since $\tan\alpha=m=\infty$

$\Rightarrow\ \alpha=\tan^{-1}(\infty)$

$=90^\circ$

2. By means of slopes, show that the following points lie on the same line:

(i) $A(-1,-3)$; $B(1, 5)$; $C(2, 9)$ (ii) $P(4, -5)$; $Q(7, 5)$; $R(10, 15)$

(iii) $L(-4, 6)$; $M(3, 8)$; $N(10, 10)$ (iv) $X(a, 2b)$; $Y(c, a+b)$; $Z(2c-a, 2a)$

Solution

(i) Let $A(-1,-3), B(1,5), C(2,9)$ be given points

Slope of AB $= \frac{5-(-3)}{1-(-1)} = \frac{5+3}{1+1} = \frac{8}{2} = 4$

Slope of BC $= \frac{9-5}{2-1} = \frac{4}{1} = 4$

Slope of AB = Slope of BC

Therefore A,B and C lie on the same line.

(ii) Let $P(4,-5), Q(7,5), R(10,15)$ be given points

Slope of PQ $= \frac{5-(-5)}{7-4} = \frac{5+5}{7-4} = \frac{10}{3}$

Slope of QR $= \frac{15-5}{10-7} = \frac{10}{3}$

Slope of PQ = Slope of QR

Therefore P,Q and R lie on the same line.

(iii) Let $L(-4,6), M(3,8), N(10,10)$ be given points

Slope of LM $= \frac{8-6}{3-(-4)} = \frac{8-6}{3+4} = \frac{2}{7}$

Slope of MN $= \frac{10-8}{10-3} = \frac{2}{7}$

Slope of LM = Slope of MN

Therefore P,Q and R lie on the same line.

(iv) Let $X(a, 2b), Y(c, a+b), Z(2c-a, 2a)$ be given points

Slope of XY $= \frac{(a+b)-2b}{c-a} = \frac{a-b}{c-a}$

Slope of YZ $= \frac{2a-(a+b)}{(2c-a)-c} = \frac{a-b}{c-a}$

Slope of XY = Slope of YZ

Therefore X,Y and Z lie on the same line.

3. Find k so that the line joining $A(7, 3)$; $B(k, -6)$ and the line joining $C(-4, 5)$; $D(-6, 4)$ are:

(i) parallel (ii) perpendicular.

Solution

Since $A(7,3)$, $B(k,-6)$, $C(-4,5)$ and $D(-6,4)$

$$\text{Therefore slope of } AB = m_1 = \frac{-6-3}{k-7} = \frac{-9}{k-7}$$

$$\text{Slope of } CD = m_2 = \frac{4-5}{-6+4} = \frac{-1}{-2} = \frac{1}{2}$$

(i) If AB and CD are parallel then $m_1 = m_2$

$$\Rightarrow \frac{-9}{k-7} = \frac{1}{2} \quad \Rightarrow -18 = k-7$$

$$\Rightarrow k = -18+7 \quad \Rightarrow \boxed{k=-11}$$

(ii) If AB and CD are perpendicular then $m_1 m_2 = -1$

$$\Rightarrow \left(\frac{-9}{k-7}\right)\left(\frac{1}{2}\right) = -1 \quad \Rightarrow -9 = -2(k-7)$$

$$\Rightarrow 9 = 2k-14 \quad \Rightarrow 2k = 9+14 = 23$$

$$\Rightarrow \boxed{k = \frac{23}{2}}$$

4. Using slopes, show that the triangle with its vertices $A(6, 1)$, $B(2, 7)$ and $C(-6, -7)$ is a right triangle.

Solution

Since $A(6,1), B(2,7)$ and $C(-6,-7)$ are vertices of triangle therefore

$$\text{Slope of } \overline{AB} = m_1 = \frac{7-1}{2-6} = \frac{6}{-4} = -\frac{3}{2}$$

$$\text{Slope of } \overline{BC} = m_2 = \frac{-7-7}{-6-2} = \frac{-12}{-8} = \frac{7}{4}$$

REMEMBER
The symbols
(i) $\parallel$ stands for 'parallel"
(ii) $\nparallel$ stands for "not parallel"
(iii) $\perp$ stands for "perpendicular"

$$\text{Slope of } \overline{CA} = m_3 = \frac{1+7}{6+6} = \frac{8}{12} = \frac{2}{3}$$

$$\text{Since} \quad m_1 m_3 = \left(-\frac{3}{2}\right)\left(\frac{2}{3}\right) = -1$$

$\Rightarrow$ The triangle ABC is a right triangle with $m\angle A = 90^\circ$

5. Two pairs of points are given. Find whether the two lines determined by these points are:
(i) parallel (ii) perpendicular (iii) none.
(a) (1, –2), (2, 4) and (4, 1), (–8, 2)
(b) (–3, 4), (6, 2) and (4, 5), (–2, –7)

Solution

(a) Slope of line joining $(1,-2)$ and $(2,4) = m_1 = \frac{4+2}{2-1} = \frac{6}{1} = 6$

Slope of line joining $(4,1)$ and $(-8,2) = m_2 = \frac{2-1}{-8-4} = \frac{1}{-12}$

Since $m_1 \neq m_2$

Also $m_1 m_2 = 6 \cdot \frac{1}{-12} = -\frac{1}{2} \neq -1$

$\Rightarrow$ lines are neither parallel nor perpendicular.

(b) Slope of line joining $(-3,4)$ and $(6,2) = m_1 = \frac{2-4}{6-(-3)} = -\frac{2}{9}$

Slope of line joining $(4,5)$ and $(-2,-7) = m_2 = \frac{-7-5}{-2-4} = \frac{-12}{-6} = 2$

Since $m_1 \neq m_2$

Also $m_1 m_2 = \left(-\frac{2}{9}\right)(2) = -\frac{4}{9} \neq -1$

⇒ Lines are neither parallel nor perpendicular.

6. Find an equation of:
   - (a) the horizontal line through (7, –9)
   - (b) the vertical line through (–5, 3)
   - (c) through $A(-6, 5)$ having slope 7
   - (d) through (8, –3) having slope 0
   - (e) through (–8 , 5) having slope undefined
   - (f) through (–5, –3) and (9, –1)
   - (g) y-intercept: –7 and slope: –5
   - (h) x-intercept: –3 and y-intercept: 4
   - (i) x-intercept: –9 and slope: –4

Solution

(a) Since slope of horizontal line $= m = 0$

$\& \ (x_1, y_1) = (7, -9)$

therefore equation of line:

$$y - (-9) = 0(x - 7)$$

$$\Rightarrow \ x + 9 = 0$$

(b) Since slope of vertical line $m = \infty = \frac{1}{0}$

$\& \ (x_1, y_1) = (-5, 3)$

therefore required equation of line

$$y - 3 = \infty\left(x - (-5)\right)$$

$$\Rightarrow \ y - 3 = \frac{1}{0}(x + 5) \qquad \Rightarrow \ 0(y - 3) = 1(x + 5)$$

$$\Rightarrow \ x + 5 = 0$$

(c) $\because \quad (x_1, y_1) = (-6, 5)$

and slope of line $= m = 7$

so required equation

$$y - 5 = 7(x - (-6))$$

$$\Rightarrow \quad y - 5 = 7(x + 6) \qquad \Rightarrow \quad y - 5 = 7x + 42$$

$$\Rightarrow \quad 7x + 42 - y + 5 = 0 \qquad \Rightarrow \quad 7x - y + 47 = 0$$

(d) Slope $= m = 0$ and Point $= (x_1, y_1) = (8, -3)$

Equation of line is $y - y_1 = m(x - x_1)$

$$y - (-3) = 0(x - 8)$$

$$y + 3 = 0$$

(e) $\because \quad (x_1, y_1) = (-8, 5)$

and slope of line $= m = \infty$

So required equation

$$y - 5 = \infty(x - (-8))$$

$$\Rightarrow \quad y - 5 = \frac{1}{0}(x + 8) \qquad \Rightarrow \quad 0(y - 5) = 1(x + 8)$$

$$\Rightarrow \quad x + 8 = 0$$

(f) The line through $(-5, -3)$ and $(9, -1)$ is

$$y - (-3) = \frac{-1 - (-3)}{9 - (-5)}(x - (-5)) \qquad \Rightarrow \quad y + 3 = \frac{2}{14}(x + 5)$$

$$\Rightarrow \quad y + 3 = \frac{1}{7}(x + 5) \qquad \Rightarrow \quad 7y + 21 = x + 5$$

$$\Rightarrow \quad x + 5 - 7y - 21 = 0 \qquad \Rightarrow \quad x - 7y - 16 = 0$$

(g) $\because \quad y -$ intercept $= -7$

$\Rightarrow \quad (0, -7)$ lies on a required line

Also slope $= m = -5$

So required equation

$$y - (-7) = -5(x - 0)$$

$$\Rightarrow \quad y + 7 = -5x \qquad \Rightarrow \quad 5x + y + 7 = 0$$

(h) $x-\text{intercept } = a = -3$

$y-\text{intercept } = b = 4$

Using two-intercept form of equation line

$$\frac{x}{a}+\frac{y}{b}=1 \quad \Rightarrow \frac{x}{-3}+\frac{y}{4}=0$$

$$\Rightarrow 4x-3y=-12 \qquad \times\text{ing by } -12$$

$$\Rightarrow 4x-3y+12=0$$

(*i*) Slope $= m = -4$ and Point $= (x_1, y_1) = (-9,0)$

Equation of line is $y - 0 = -4\big(x-(-9)\big)$

$$y = -4(x+9)$$

$$4x + y + 36 = 0$$

7. Find an equation of the perpendicular bisector of the segment joining the points $A(3\ ,5)$ and $B(9,\ 8)$.

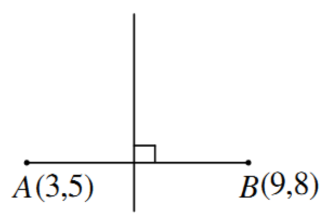
Solution

Given points $A(3,5)$ and $B(9,8)$

Midpoint of $\overline{AB} = \left(\frac{3+9}{2}, \frac{5+8}{2}\right) = \left(\frac{12}{2}, \frac{13}{2}\right) = \left(6, \frac{13}{2}\right)$

Slope of $\overline{AB} = m = \frac{8-5}{9-3} = \frac{3}{6} = \frac{1}{2}$

Slope of line $\perp$ to $\overline{AB} = -\frac{1}{m} = -\frac{1}{1/2} = --2$



Now equation of $\perp$ bisector having slope -2 through $\left(6, \frac{13}{2}\right)$

$$\Rightarrow y - \frac{13}{2} = -2(x-6)$$

$$\Rightarrow y - \frac{13}{2} = -2x + 12 \qquad \Rightarrow y - \frac{13}{2} + 2x - 12 = 0$$

$$\Rightarrow 2x + y - \frac{37}{2} = 0 \qquad \Rightarrow 4x + 2y - 37 = 0$$

8. Find an equation of the line through (–4, –6) and perpendicular to a line having slope $\frac{-3}{2}$.

Solution

Here $(x_1, y_1) = (-4, -6)$

Slope of given line $= m = \frac{-3}{2}$

$\because$ required line is $\perp$ to given line

$\therefore$ slope of required line $= -\frac{1}{m} = -\frac{1}{-3/2} = \frac{2}{3}$

Now equation of line having slope $\frac{2}{3}$ passing through $(-4, -6)$

$$y - (-6) = \frac{2}{3}(x - (-4))$$

$$\Rightarrow 3(y+6) = 2(x+4) \qquad \Rightarrow 3y + 18 = 2x + 8$$

$$\Rightarrow 2x + 8 - 3y - 18 = 0 \qquad \Rightarrow 2x - 3y - 10 = 0$$

9. Find an equation of the line through (11, –5) and parallel to a line with slope –24.

Solution

Here $(x_1, y_1) = (11, -5)$

Slope of given line $= m = -24$

$\because$ required line is || to given line

$\therefore$ slope of required line $= m = -24$

Now equation of line having slope -24 passing through $(11, -5)$

$$y - (-5) = -24(x - 11)$$

$$\Rightarrow y + 5 = -24x + 264 \qquad \Rightarrow 24x - 264 + y + 5 = 0$$

$$\Rightarrow 24x + y - 259 = 0$$

10. Convert each of the following equations into slope intercept form, two intercept form and normal form:

(a) $2x-4y+11=0$ (b) $4x+7y-2=0$ (c) $15y-8x+3=0$

Solution

(a) $2x-4y+11=0$

(i) - Slope-intercept form

$$\because\ 2x-4y+11=0$$

$$\Rightarrow\ 4y=2x+11 \quad \Rightarrow\ y=\frac{2x+11}{4}$$

$$\Rightarrow\ y=\frac{1}{2}x+\frac{11}{4}$$

is the intercept form of equation of line with $m=\frac{1}{2}$ and $c=\frac{11}{4}$

(ii) - Two-intercept form

$$\because\ 2x-4y+11=0 \quad \Rightarrow\ 2x-4y=-11$$

$$\Rightarrow\ \frac{2}{-11}x-\frac{4}{-11}y=1 \quad \Rightarrow\ \frac{x}{-11/2}+\frac{y}{11/4}=1$$

is the two-point form of equation of line with $a=-\frac{11}{2}$ and $b=\frac{11}{4}$.

(iii) - Normal form

$$\because\ 2x-4y+11=0 \quad \Rightarrow\ 2x-4y=-11$$

Dividing above equation by $\sqrt{(2)^2+(-4)^2}=\sqrt{20}=2\sqrt{5}$

$$\frac{2x}{2\sqrt{5}}-\frac{4y}{2\sqrt{5}}=\frac{-11}{2\sqrt{5}} \quad \Rightarrow\ \frac{x}{\sqrt{5}}-\frac{2y}{\sqrt{5}}=\frac{-11}{2\sqrt{5}}$$

$$\Rightarrow\ -\frac{x}{\sqrt{5}}+\frac{2y}{\sqrt{5}}=\frac{11}{2\sqrt{5}} \qquad \times\text{ing by } -1.$$

Suppose $\cos\alpha=-\frac{1}{\sqrt{5}}<0$ and $\sin\alpha=\frac{2}{\sqrt{5}}>0$

$\Rightarrow\ \alpha$ lies in 2nd quadrant and $\alpha=\cos^{-1}\left(-\frac{1}{\sqrt{5}}\right)=116.57^\circ$

Hence the normal form is

$$x\cos(116.57^\circ)+y\sin(116.57^\circ)=\frac{11}{2\sqrt{5}}$$

And length of perpendicular from $(0,0)$ to line $=p=\frac{11}{2\sqrt{5}}$

(b) $4x + 7y - 2 = 0$

(i) - Slope-intercept form

$$\because \ 4x+7y-2=0$$

$$\Rightarrow \ 7y=-4x+2 \qquad \Rightarrow \ y=\frac{-4x+2}{7}$$

$$\Rightarrow \ y=-\frac{4}{7}x+\frac{2}{7}$$

is the intercept form of equation of line with $m=-\frac{4}{7}$ and $c=\frac{2}{7}$

(ii) - Two-intercept form

$$\because \ 4x+7y-2=0 \qquad \Rightarrow \ 4x+7y=2$$

$$\Rightarrow \ 2x+\frac{7}{2}y=1 \qquad \div\text{ing by } 2$$

$$\Rightarrow \ \frac{x}{1/2}+\frac{y}{2/7}=1$$

is the two-point form of equation of line with $a=\frac{1}{2}$ and $b=\frac{2}{7}$.

(iii) - Normal form

$$\because \ 4x+7y-2=0$$
$$\Rightarrow \ 4x+7y=2$$

Dividing above equation by $\sqrt{(4)^2+(7)^2}=\sqrt{16+49}=\sqrt{65}$

$$\Rightarrow \ \frac{4}{\sqrt{65}}x+\frac{7}{\sqrt{65}}y=\frac{2}{\sqrt{65}} \quad .$$

Suppose $\cos\alpha=\frac{4}{\sqrt{65}}>0$ and $\sin\alpha=\frac{7}{\sqrt{65}}>0$

$\Rightarrow$ α lies in first quadrant and $\alpha=\cos^{-1}\left(\frac{4}{\sqrt{65}}\right)=60.26^\circ$

Hence the normal form is

$$x\cos\left(60.26^\circ\right)+y\sin\left(60.26^\circ\right)=\frac{2}{\sqrt{65}}$$

And length of perpendicular from $(0,0)$ to line $=p=\frac{2}{\sqrt{65}}$

(c) $15y - 8x + 3 = 0$

(i) - Slope-intercept form

$$\because\ 15y - 8x + 3 = 0$$

$$\Rightarrow 15y = 8x - 3 \qquad \Rightarrow y = \frac{8x-3}{15}$$

$$\Rightarrow y = \frac{8}{15}x - \frac{3}{15} \qquad \Rightarrow y = \frac{8}{15}x - \frac{1}{5}$$

is the intercept form of equation of line with $m = \frac{8}{15}$ and $c = -\frac{1}{5}$

(ii) - Two-intercept form

$$\because\ 15y - 8x + 3 = 0 \qquad \Rightarrow -8x + 15y = -3$$

$$\Rightarrow \frac{8x}{3} - 5y = 1 \qquad \Rightarrow \frac{x}{3/8} + \frac{y}{-1/5} = 1$$

is the two-point form of equation of line with $a = \frac{3}{8}$ and $b = -\frac{1}{5}$.

(iii) - Normal form

$$\because\ 15y - 8x + 3 = 0$$

$$\Rightarrow 8x - 15y = 3$$

Dividing above equation by $\sqrt{(8)^2 + (-15)^2} = \sqrt{64+225} = \sqrt{289} = 17$

$$\Rightarrow \frac{8}{17}x - \frac{15}{17}y = \frac{3}{17} \quad .$$

Suppose $\cos\alpha = \frac{8}{17} > 0$ and $\sin\alpha = -\frac{15}{17} < 0$

$\Rightarrow$ α lies in 4^{th} quadrant and $\alpha = \cos^{-1}\left(\frac{8}{17}\right) = 298.07^\circ$

Hence the normal form is

$$x\cos\left(298.07^\circ\right) + y\sin\left(298.07^\circ\right) = \frac{3}{17}$$

And length of perpendicular from $(0,0)$ to line $= p = \frac{3}{17}$

> $\alpha = \cos^{-1}\left(\frac{8}{17}\right)$
> $= 61.93^\circ ,\ 298.07^\circ$
> Taking value that lies in 4^{th} quadrant.

11. In each of the following check whether the two lines are

(i) parallel (ii) perpendicular (iii) neither parallel nor perpendicular

(a) $2x+y-3=0$; $4x+2y+5=0$

(b) $3y=2x+5$; $3x+2y-8=0$

(c) $4y+2x-1=0$; $x-2y-7=0$

Solution

(a) Let $l_1:\ 2x+y-3=0$

$l_2:\ 4x+2y+5=0$

Slope of $l_1=m_1=-\dfrac{2}{1}=-2$

Slope of $l_2=m_2=-\dfrac{4}{2}=-2$

Since $m_1=m_2$ therefore l_1 and l_2 are parallel.

(b) Let $l_1:\ 3y=2x+5 \Rightarrow 2x-3y+5=0$

$l_2:\ 3x+2y-8=0$

Slope of $l_1=m_1=-\dfrac{2}{-3}=\dfrac{2}{3}$

Slope of $l_2=m_2=-\dfrac{3}{2}=$

Since $m_1m_2=\left(\dfrac{2}{3}\right)\left(-\dfrac{3}{2}\right)=-1 \Rightarrow l_1$ and l_2 are perpendicular.

(c) Let $l_1:\ 4y+2x-1=0 \Rightarrow 2x+4y-1=0$

$l_2:\ x-2y-7=0$

Slope of $l_1=m_1=-\dfrac{2}{4}=-\dfrac{1}{2}$

Slope of $l_2=m_2=-\dfrac{1}{-2}=\dfrac{1}{2}$

Since $m_1\neq m_2$ and $m_1m_2=\left(\dfrac{1}{2}\right)\left(-\dfrac{1}{2}\right)=-\dfrac{1}{4}\neq -1$

$\Rightarrow$ l_1 and l_2 are neither parallel nor perpendicular.

12. Find an equation of the line through (–4, 7) and parallel to the line $2x - 7y + 4 = 0$

Solution

Let $l: \ 2x - 7y + 4 = 0$

Slope of $l = m = -\frac{2}{-7} = \frac{2}{7}$

Since required line is parallel to l

REMEMBER

If $l: \ ax + by + c = 0$

then slope of $l = -\frac{a}{b}$

Therefore slope of required line $= m = \frac{2}{7}$

Now equation of line having slope $\frac{2}{7}$ passing through $(-4, 7)$

$$y - 7 = \frac{2}{7}(x - (-4))$$

$$\Rightarrow \ 7(y - 7) = 2(x + 4)$$

$$\Rightarrow \ 7y - 49 = 2x + 8 \quad \Rightarrow \ 2x + 8 - 7y + 49 = 0$$

$$\Rightarrow \ 2x - 7y + 57 = 0$$

13. Find an equation of the line through (5, –8) and perpendicular to the join of $A(-15, -8)$, $B(10, 7)$.

Solution

Given: $A(-15, -18)$, $B(10, 7)$ and $(5, 8)$

Slope of $\overline{AB} = m = \frac{7 - (-18)}{10 - (-15)}$

$$= \frac{7 + 18}{10 + 15} = \frac{25}{25} = 1$$

Since required line is perpendicular to $\overline{AB}$

Therefore slope of required line $= -\frac{1}{m} = -\frac{1}{1} = -1$

Now equation of line having slope -1 through $(5, -8)$

$$y - (-8) = -1(x - 5)$$

$$\Rightarrow \ y + 8 = -x + 5$$

$$\Rightarrow \ x + y + 8 - 5 = 0 \quad \Rightarrow \ x + y + 3 = 0$$

Exercise 7.3

1. If the houses of two friends are represented by coordinates (2, 6) and (9, 12) on a grid. Find the straight line distance between their houses if the grid units represent kilometres?

Solution

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{distance} = \sqrt{(9-2)^2 + (12-6)^2} = \sqrt{(7)^2 + (6)^2} = \sqrt{49+36}$$

$$\text{distance} = \sqrt{85} \approx 9.22 \text{ km}$$

2. Consider a straight trail (represented by coordinate plane) that starts at point (5, 7) and ends at point (15, 3). What is the coordinate of the midpoint?

Solution

$$\text{Mid Point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\text{Mid Point} = \left(\frac{5+15}{2}, \frac{7+3}{2}\right) = \left(\frac{20}{2}, \frac{10}{2}\right)$$

$$\text{Mid Point} = (10,5)$$

3. An architect is designing a park with two buildings located at (10, 8) and (4, 3) on the grid. Calculate the straight-line distance between the buildings. Assume the coordinates are in meters.

Solution

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{distance} = \sqrt{(4-10)^2 + (3-8)^2} = \sqrt{(-6)^2 + (-5)^2} = \sqrt{36+25}$$

$$\text{distance} = \sqrt{61} \approx 7.81 \text{ m}$$

4. A delivery driver needs to calculate the distance between two delivery locations. One location is at (7, 2) and the other at (12, 10) on the city grid map, where each unit represents kilometres. What is the distance between the two locations?

Solution

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{distance} = \sqrt{(12-7)^2 + (10-2)^2} = \sqrt{(5)^2 + (8)^2} = \sqrt{25 + 64}$$

$$\text{distance} = \sqrt{89} \approx 9.43 \text{ km}$$

5. The start and end points of a race track are given by coordinates (3, 9) and (9, 13). What is the midpoint of the track.

Solution

$$\text{Mid Point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\text{Mid Point} = \left(\frac{3+9}{2}, \frac{9+13}{2}\right) = \left(\frac{12}{2}, \frac{22}{2}\right)$$

$$\text{Mid Point} = (6,11)$$

6. The coordinates of two points on a road are *A*(3, 4) and *B*(7, 10). Find the midpoint of the road.

Solution

$$\text{Mid Point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\text{Mid Point} = \left(\frac{3+7}{2}, \frac{4+10}{2}\right) = \left(\frac{10}{2}, \frac{14}{2}\right)$$

$$\text{Mid Point} = (5,7)$$

7. A ship is navigating from port *A* located at (12° N, 65° W) to port *B* at (20 ° N, 45 ° W). If the ship travels along the shortest path on the surface of the Earth, calculate the straight line distance between the points.

Solution

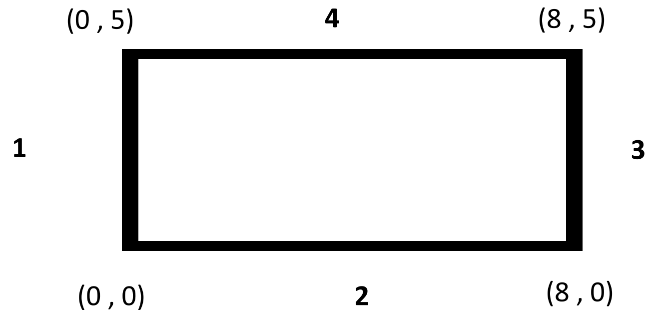
$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{distance} = \sqrt{(20-12)^2 + (45-65)^2} = \sqrt{(8)^2 + (-20)^2} = \sqrt{64 + 400}$$

$$\text{distance} = \sqrt{464} = \sqrt{16 \times 29} = 4\sqrt{29} \approx 21.5 \text{ unit}$$

8. Farah is fencing around a rectangular field with corners at (0,0), (0,5), (8, 5) and (8, 0). How much fencing material will she need to cover the entire perimeter of the field?

Solution



Length of side 1 $= 5 - 0 = 5$

Length of side 2 $= 8 - 0 = 8$

Length of side 3 $= 5 - 0 = 5$

Length of side 4 $= 8 - 0 = 8$

Perimeter = Side 1 + Side 2 + Side 3 + Side 4

Perimeter = 5 + 8 + 5 + 8 = 26 units

9. An airplane is flying from city *X* at (40° N, 100 ° W) to city *Y* at (50 ° N, 80° W). Use coordinate geometry, calculate the shortest distance between these two cities.

Solution

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\text{distance} = \sqrt{(50 - 40)^2 + (80 - 100)^2} = \sqrt{(-10)^2 + (-20)^2} = \sqrt{100 + 400}$$
$$\text{distance} = \sqrt{500} \approx 22.4 \text{ unit}$$

10. A land surveyor is marking out a rectangular plot of land with corners at (3, 1), (3, 6), (8, 6), and (8, 1). Calculate the perimeter.

Solution

Length of side 1 $= 6 - 1 = 5$

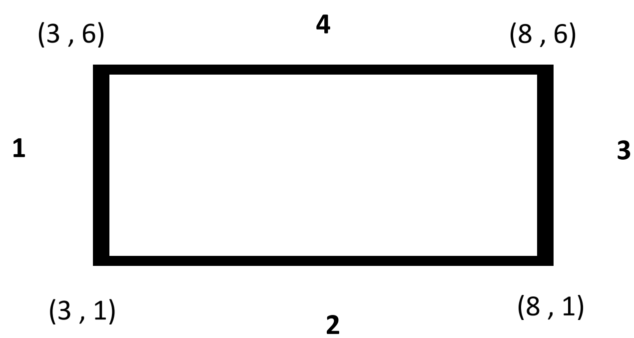
Length of side 2 $= 8 - 3 = 5$

Length of side 3 $= 6 - 1 = 5$

Length of side 4 $= 8 - 3 = 5$

Perimeter = Side 1 + Side 2 + Side 3 + Side 4

Perimeter = 5 + 5 + 5 + 5 = 20 units



11. A landscaper needs to install a fence around a rectangular garden. The garden has its corners at the coordinates: $A(0, 0)$, $B(5, 0)$, $C(5, 3)$, and $D(0, 3)$. How much fencing is required?

Solution

Solution

Length of side 1 = 5 − 0 = 5

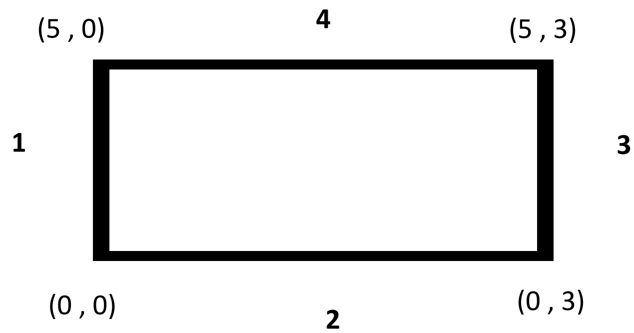
Length of side 2 = 3 − 0 = 3

Length of side 3 = 5 − 0 = 5

Length of side 4 = 3 − 0 = 3

Perimeter = Side 1 + Side 2 + Side 3 + Side 4

Perimeter = 5 + 3 + 5 + 3 = 16 units



REVIEW EXERCISE 7

1. Four options are given against each statement. Encircle the correct option.

(i) The equation of a straight line in the slope-intercept form is written as:
- (a) $y = m(x + c)$
- (b) $y - y_1 = m(x - x_1)$
- (c) ✓ $y = c + mx$
- (d) $ax + by + c = 0$

(ii) The gradients of two parallel lines are:
- (a) ✓ equal
- (b) zero
- (c) negative reciprocals of each other
- (d) always undefined

(iii) If the product of the gradients of two lines is −1, then the lines are:
- (a) Parallel
- (b) ✓ perpendicular
- (c) Collinear
- (d) coincident

(iv) Distance between two points $P(1, 2)$ and $Q(4, 6)$ is:
- (a) ✓ 5
- (b) 6
- (c) $\sqrt{13}$
- (d) 4

(v) The midpoint of a line segment with endpoints (–2, 4) and (6, –2) is:
- (a) (4, 2)
- (b) ✓ (2, 1)
- (c) (1, 1)
- (d) (0, 0)

(vi) A line passing through points (1, 2) and (4, 5) is:
- (a) ✓ $y = x + 1$
- (b) $y = 2x + 3$
- (c) $y = 3x - 2$
- (d) $y = x + 2$

(vii) The equation of a line in point-slope form is:

(a) $y = m(x + c)$ (b) ✓ $y - y_1 = m(x - x_1)$

(c) $y = c + mx$ (d) $ax + by + c = 0$

(viii) $2x + 3y - 6 = 0$ in the slope-intercept form is:

(a) ✓ $y = \frac{-2}{3}x + 2$ (b) $y = \frac{2}{3}x - 2$

(c) $y = \frac{2}{3}x + 1$ (d) $y = \frac{-2}{3}x - 2$

(ix) The equation of a line in symmetric form is:

(a) $\frac{x}{a} + \frac{y}{b} = 1$ (b) $\frac{x - x_1}{1} + \frac{y - y_1}{m} = \frac{z - z_1}{1}$

✓(c) $\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha} = r$ (d) $y - y_1 = m(x - x_1)$

(x) The equation of a line in normal form is:

(a) $y = mx + c$ (b) $\frac{x}{a} + \frac{y}{b} = 1$

(c) $\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha}$ (d) ✓ $x\cos\alpha + y\sin\alpha = p$

2. Find the distance between two points $A(2, 3)$ and $B(7, 8)$ on a coordinate plane.

Solution

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{distance} = \sqrt{(7 - 2)^2 + (8 - 3)^2} = \sqrt{(5)^2 + (5)^2} = \sqrt{25 + 25}$$

$$\text{distance} = \sqrt{50} = 5\sqrt{2}$$

3. Find the midpoint of the line segment joining the points (4, –2) and (–6, 3).

Solution

$$\text{Mid Point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{Mid Point} = \left(\frac{-6 + 4}{2}, \frac{3 - 2}{2}\right) = \left(\frac{-2}{2}, \frac{1}{2}\right)$$

$$\text{Mid Point} = \left(-1, \frac{1}{2}\right)$$

4. Calculate the gradient (slope) of the line passing through the points (1, 2) and (4, 6).

Solution

$$\text{Gradient(Slope)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{4-1} = \frac{4}{3}$$

5. Find the equation of the line in the form $y = mx + c$ that passes through the points (3, 7) and (5, 11).

Solution

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-7}{5-3} = \frac{4}{2} = 2$$

Equation of line through (3,7):

$$y - y_1 = m(x - x_1)$$
$$y - 7 = 2(x - 3)$$
$$y - 7 = 2x - 6$$
$$y = 2x + 1$$

6. If two lines are parallel and one line has a gradient of $\frac{2}{3}$, what is the gradient of the other line?

Solution

Gradient (Slope) of one line $= m_1 = \frac{2}{3}$

For parallel lines $m_1 = m_2$

Gradient (Slope) of other line $= m_2 = \frac{2}{3}$

7. An airplane needs to fly from city A to coordinates (12, 5) to city B at coordinates (8, –4). Calculate the straight-line distance between these two cities.

Solution

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{distance} = \sqrt{(8-12)^2 + (-4-5)^2} = \sqrt{(-4)^2 + (-9)^2} = \sqrt{16+81}$$

$$\text{distance} = \sqrt{97} \approx 9.85 \text{ units}$$

8. In a landscaping project, the path starts at (2, 3) and ends at (10, 7). Find the midpoint.

Solution

$\text{Mid Point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\text{Mid Point} = \left(\frac{10+2}{2}, \frac{3+7}{2}\right) = \left(\frac{12}{2}, \frac{10}{2}\right)$

$\text{Mid Point} = (6,5)$

9. A drone is flying from point (2, 3) to point (10, 15) on the grid. Calculate the gradient of the line along which the drone is flying and the total distance travelled.

Solution

$\text{Gradient(Slope)} = \frac{y_2-y_1}{x_2-x_1} = \frac{15-3}{10-2} = \frac{12}{8} = \frac{3}{2}$

$\text{distance} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$\text{distance} = \sqrt{(10-2)^2 + (15-3)^2} = \sqrt{(8)^2 + (12)^2} = \sqrt{64+144}$

$\text{distance} = \sqrt{208} = \sqrt{16 \times 13} = 4\sqrt{13} \approx 14.4 \text{ units}$

10. For a line with a gradient of –3 and a y-intercept of 2, write the equation of the line in:

(a) Slope-intercept form
(b) Point-slope form using the point (1, 2)
(c) Two-point form using the points (1, 2) and (4, –7)
(d) Intercepts form
(e) Symmetric form
(f) Normal form

Solution

(a) Slope-Intercept Form

$y = mx + c$, where $m = -3$ and $c = 2$

$y = -3x + 2$

(b) Point-Slope Form with $m = -3, P(1, 2)$

$y - y_1 = m(x - x_1)$
$y - 2 = -3(x - 1)$

(c) Two-Point Form Using points (1, 2) and (4, −7)

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$
$\frac{y - 2}{-7 - 2} = \frac{x - 1}{4 - 1}$

(d) Intercept Form

y = −3x + 2

y + 3x = 2

$\frac{y}{2} + \frac{x}{\frac{2}{3}} = 1$

(e) Symmetric Form

y = −3x + 2

y + 3x = 2

Dividing $\sqrt{(3)^2 + (1)^2} = \sqrt{10}$ on both sides

$\frac{y}{\sqrt{10}} + \frac{3x}{\sqrt{10}} = \frac{2}{\sqrt{10}}$

(f) Normal Form

Slope = $tan\alpha = -3$

$\alpha = \tan^{-1}(-3) = -71.56°$

x cos α + y sin α = p where p = $\frac{2}{\sqrt{10}}$

x cos($-71.56°$) + ysin($-71.56°$) = $\frac{2}{\sqrt{10}}$