Unit 6 Trigonometry

EXERCISE 6.1

1. Find in which quadrant the following angles lie. Write a co-terminal angle for each:
(i) 65° (ii) 135° (iii) -40° (iv) 210° (v) -150°

Solution

(i) 1^{st} (ii) 2^{nd} (iii) 4^{th} (iv) 3^{rd} (v) 3^{rd}

2. Convert the following into degrees, minutes, and seconds:
(i) 123.456° (ii) 58.7891° (iii) 90.5678°

Solution

2(i): $\mathbf{123.456^\circ}$

123

$0.456 \times 60 = 27.36$

$0.36 \times 60 = 21.6$

$123.456^\circ \approx 123^\circ\, 27'\, 22''$

2(ii): $\mathbf{58.7891^\circ}$

58

$0.7891 \times 60 = 47.346$

$0.346 \times 60 = 20.76$

$58.7891^\circ \approx 58^\circ\, 47'\, 21''$

2(iii): $\mathbf{90.5678^\circ}$

90

$0.5678 \times 60 = 34.068$

$0.068 \times 60 = 4.08$

$90.5678^\circ \approx 90^\circ\, 34' 4''$

3. Convert the following into decimal degrees:

(i) 65° 32' 15" (ii) 42° 18' 45" (iii) 78° 45' 36"

Solution

3(i): $\mathbf{65°32'15''}$

$65°32'15'' = 65 + \frac{32}{60} + \frac{15}{60\times60} = 65 + 0.5333 + 0.0042 = 65.5375°$

3(ii): $\mathbf{42°18'45''}$

$42°18'45'' = 42 + \frac{18}{60} + \frac{45}{60\times60} = 42 + 0.3 + 0.0125 = 42.3125°$

3(iii): $\mathbf{78°45'36''}$

$78°45'36'' = 78 + \frac{45}{60} + \frac{36}{60\times60} = 78 + 0.75 + 0.01 = 78.76°$

4. Convert the following into radians:

(i) 36° (ii) 22.5° (iii) 67.5°

Solution

4(i): $\mathbf{36°} = 36 \times \frac{\pi}{180} = \frac{\pi}{5}\text{rad}$

4(ii):$\mathbf{22.5°} = 22.5 \times \frac{\pi}{180} = \frac{\pi}{8}\text{rad}$

4(iii):$\mathbf{67.5°} = 67.5 \times \frac{\pi}{180} = \frac{3\pi}{8}\text{rad}$

5. Convert the following into degrees:

(i) $\frac{\pi}{16}$rad (ii) $\frac{11\pi}{5}$rad (iii) $\frac{7\pi}{6}$rad

Solution

5(i): $\frac{\boldsymbol{\pi}}{\mathbf{16}}\textbf{rad} = \frac{\pi}{16} \times \frac{180°}{\pi} = 11.25°$

5(ii): $\frac{\mathbf{11}\boldsymbol{\pi}}{\mathbf{5}}\textbf{rad} = \frac{11\pi}{5} \times \frac{180°}{\pi} = 396°$

5(iii): $\frac{\mathbf{7}\boldsymbol{\pi}}{\mathbf{6}}\textbf{rad} = \frac{7\pi}{6} \times \frac{180°}{\pi} = 210°$

6. Find the arc length and area of a sector if:

(i) $r = 6$ cm and central angle $\theta = \frac{\pi}{3}$ radians.

(ii) $r = \frac{4.8}{\pi}$ cm and central angle $\theta = \frac{5\pi}{6}$ radians.

Solution

6(i): $\boldsymbol{l} = \text{r}\theta = 6 \times \frac{\pi}{3} = 6.28\text{cm}$

$\boldsymbol{A} = \frac{1}{2}\text{r}^2\theta = \frac{1}{2} \times (6)^2 \times \frac{\pi}{3} = 18.84\text{cm}^2$

6(ii): $\boldsymbol{l} = \text{r}\theta = \frac{4.8}{\pi} \times \frac{5\pi}{6} = 4\text{cm}$

$\boldsymbol{A} = \frac{1}{2}\text{r}^2\theta = \frac{1}{2} \times \left(\frac{4.8}{\pi}\right)^2 \times \frac{5\pi}{6} = 3.06\text{cm}^2$

7. If the central angle of a sector is 60° and the radius of the circle is 12 cm, find the area of the sector and the percentage of the total area of the circle it represents.

Solution

$\theta = 60° = 60 \times \frac{\pi}{180} = \frac{\pi}{3}\text{rad}$

Area of the sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times (12)^2 \times \frac{\pi}{3} = 75.4\text{cm}^2$

Total area of the circle $= \pi r^2 = 3.14159 \times (12)^2 = 452.389\text{cm}^2$

Percentage $= \frac{\text{Area of the sector}}{\text{Total area of the circle}} \times 100\%$

Percentage $= \frac{75.4\text{cm}^2}{452.389\text{cm}^2} \times 100\% = 16.67\%$

8. Find the percentage of the area of sector subtending an angle $\frac{\pi}{8}$ radians.

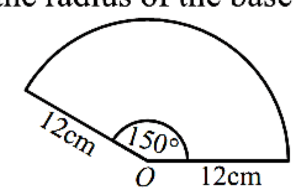
Solution

Percentage $= \frac{\text{Area of the sector}}{\text{Total area of the circle}} \times 100\%$

Percentage $= \frac{\theta}{2\pi} \times 100\% = \frac{\frac{\pi}{8}}{2\pi} \times 100\% = \frac{1}{16} \times 100\% = 6.25\%$

9. A circular sector of radius $r = 12$ cm has an angle of 150°. This sector is cut out and then bent to form a cone. What is the slant height and the radius of the base of this cone?

Hint: Arc length of sector = circumference of cone.



Solution

Radius of the sector $= r = 12\text{cm}$

Angle of the sector $= \theta = 150° = 150 \times \frac{\pi}{180} = \frac{5\pi}{6}\text{rad}$

Arc Length $= l = r\theta = 12 \times \frac{5\pi}{6} = 10\pi\text{cm}$

Now

Circumference of base of the cone $= 2\pi r'$

$10\pi = 2\pi r'$

radius of base $= r' = 5\text{cm}$

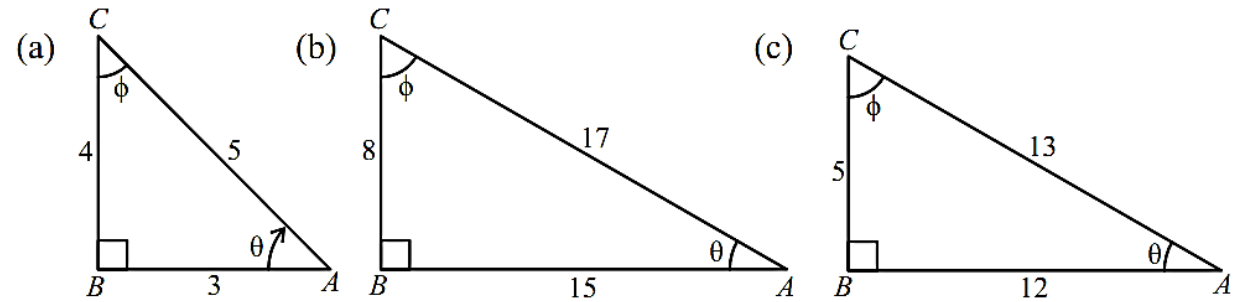
slant height $= l = r = 12\text{cm}$

EXERCISE 6.2

1. For each of the following right-angled triangles, find the trigonometric ratios:

(i) $\sin\theta$ (ii) $\cos\theta$ (iii) $\tan\theta$ (iv) $\sec\theta$ (v) $\operatorname{cosec}\theta$

(vi) $\cot\phi$ (vii) $\tan\phi$ (viii) $\operatorname{cosec}\phi$ (ix) $\sec\phi$ (x) $\cos\phi$



Solution

(a) (i) $\frac{4}{5}$ (ii) $\frac{3}{5}$ (iii) $\frac{4}{3}$ (iv) $\frac{5}{3}$ (v) $\frac{5}{4}$ (vi) $\frac{4}{3}$ (vii) $\frac{3}{4}$ (viii) $\frac{5}{3}$ (ix) $\frac{5}{4}$ (x) $\frac{4}{5}$

(b) (i) $\frac{8}{17}$ (ii) $\frac{15}{17}$ (iii) $\frac{8}{15}$ (iv) $\frac{17}{15}$ (v) $\frac{17}{8}$ (vi) $\frac{8}{15}$ (vii) $\frac{15}{8}$ (viii) $\frac{17}{15}$ (ix) $\frac{17}{8}$ (x) $\frac{8}{17}$

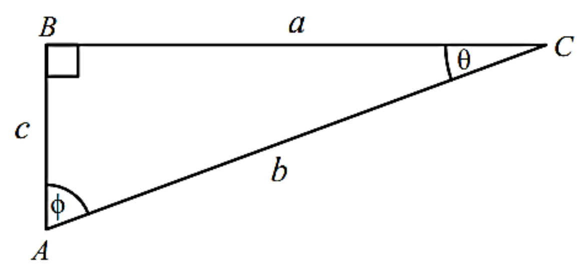
(c) (i) $\frac{5}{13}$ (ii) $\frac{12}{13}$ (iii) $\frac{5}{12}$ (iv) $\frac{13}{5}$ (v) $\frac{13}{12}$ (vi) $\frac{5}{12}$ (vii) $\frac{12}{5}$ (viii) $\frac{13}{12}$ (ix) $\frac{13}{5}$ (x) $\frac{5}{13}$

2. For the following right-angled triangle ABC find the trigonometric ratios for which $m\angle A = \phi$ and $m\angle C = \theta$

(i) $\sin\theta$ (ii) $\cos\theta$

(iii) $\tan\theta$ (iv) $\sin\phi$

(v) $\cos\phi$ (vi) $\tan\phi$



Solution

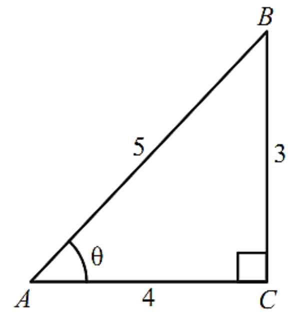
(i) $\frac{c}{b}$ (ii) $\frac{a}{b}$ (iii) $\frac{c}{a}$ (iv) $\frac{a}{b}$ (v) $\frac{c}{b}$ (vi) $\frac{a}{c}$

3. Considering the adjoining triangle ABC, verify that:

(i) $\sin\theta \ \operatorname{cosec}\theta = 1$

(ii) $\cos\theta \ \sec\theta = 1$

(iii) $\tan\theta \ \cot\theta = 1$



Solution

3.(i) $sin\theta cosec\theta = \frac{3}{5} \times \frac{5}{3} = 1$

3.(ii) $cos\theta sec\theta = \frac{4}{5} \times \frac{5}{4} = 1$

3.(iii) $tan\theta cot\theta = \frac{3}{4} \times \frac{4}{3} = 1$

4. Fill in the blanks.

(i) $\sin 30° = \sin(90° - 60°) =$ $cos60°$

(ii) $\cos 30° = \cos(90° - 60°) =$ $sin60°$

(iii) $\tan 30° = \tan(90° - 60°) =$ $cot60°$

(iv) $\tan 60° = \tan(90° - 30°) =$ $cot30°$

(v) $\sin 60° = \sin(90° - 30°) =$ $cos30°$

(vi) $\cos 60° = \cos(90° - 30°) =$ $sin30°$

(vii) $\sin 45° = \sin(90° - 45°) =$ $cos45°$

(viii) $\tan 45° = \tan(90° - 45°) =$ $cot45°$

(ix) $\cos 45° = \cos(90° - 45°) =$ $sin45°$

5. In a right angled triangle ABC, $m\angle B = 90°$ and C is an acute angle of 60º. Alsc $\sin m\angle A = \frac{a}{b}$, then find the following trigonometric ratios:

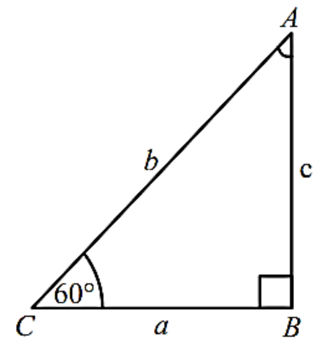
(i) $\frac{m\overline{BC}}{m\overline{AB}}$ (ii) $\cos 60°$

(iii) $\tan 60^{\circ}$ (iv) $\text{cosec}\ \frac{\pi}{3}$

(v) $\cot 60°$ (vi) $\sin 30°$

(vii) $\cos 30°$ (viii) $\tan \frac{\pi}{6}$

(ix) $\sec 30°$ (x) $\cot 30°$



Solution

(i) $\frac{a}{c}$ (ii) $\frac{a}{b}$ (iii) $\frac{c}{a}$ (iv) $\frac{b}{c}$ (v) $\frac{a}{c}$

(vi) $\frac{a}{b}$ (vii) $\frac{c}{b}$ (viii) $\frac{a}{c}$ (ix) $\frac{b}{c}$ (x) $\frac{c}{a}$

EXERCISE 6.3

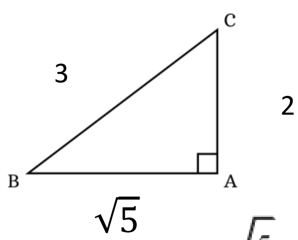
1. If θ lies in first quadrant, find the remaining trigonometric ratios of θ.

(i) $\sin\theta = \frac{2}{3}$ (ii) $\cos\theta = \frac{3}{4}$ (iii) $\tan\theta = \frac{1}{2}$

(iv) $\sec\theta = 3$ (v) $\cot\theta = \sqrt{\frac{3}{2}}$

Solution

1.(i) $\boldsymbol{\sin\theta = \frac{2}{3}}$



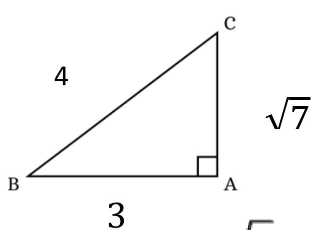
By Pythagoras Formula

$$H^2 = P^2 + B^2 \Rightarrow 3^2 = 2^2 + B^2$$

$$\Rightarrow B^2 = 9 - 4 = 5 \Rightarrow B = \sqrt{5}$$

(i) $\cos\theta = \frac{\sqrt{5}}{3}$, $\tan\theta = \frac{2}{\sqrt{5}}$, $\operatorname{cosec}\theta = \frac{3}{2}$, $\sec\theta = \frac{3}{\sqrt{5}}$, $\cot\theta = \frac{\sqrt{5}}{2}$

1.(ii) $\boldsymbol{\cos\theta = \frac{3}{4}}$



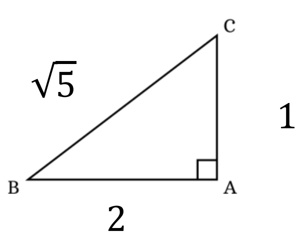
By Pythagoras Formula

$$H^2 = P^2 + B^2 \Rightarrow 4^2 = P^2 + 3^2$$

$$\Rightarrow P^2 = 16 - 9 = 7 \Rightarrow P = \sqrt{7}$$

(ii) $\sin\theta = \frac{\sqrt{7}}{4}$, $\tan\theta = \frac{\sqrt{7}}{3}$, $\operatorname{cosec}\theta = \frac{4}{\sqrt{7}}$, $\sec\theta = \frac{4}{3}$, $\cot\theta = \frac{3}{\sqrt{7}}$

1.(iii) $\boldsymbol{\tan\theta = \frac{1}{2}}$



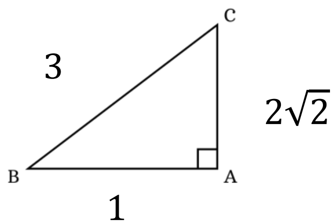
By Pythagoras Formula

$$H^2 = P^2 + B^2 \Rightarrow H^2 = 1^2 + 2^2$$

$$\Rightarrow H^2 = 1 + 4 = 5 \Rightarrow H = \sqrt{5}$$

(iii) $\sin\theta = \frac{1}{\sqrt{5}}$, $\cos\theta = \frac{2}{\sqrt{5}}$, $\operatorname{cosec}\theta = \sqrt{5}$, $\sec\theta = \frac{\sqrt{5}}{2}$, $\cot\theta = 2$

1.(iv) $\boldsymbol{sec\theta = 3 = \frac{3}{1}}$



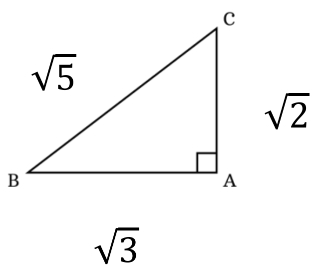
By Pythagoras Formula

$$H^2 = P^2 + B^2 \Rightarrow 3^2 = P^2 + 1^2$$

$$\Rightarrow P^2 = 9 - 1 = 8 \Rightarrow P = 2\sqrt{2}$$

(iv) $\sin\theta = \frac{2\sqrt{2}}{3}$, $\cos\theta = \frac{1}{3}$, $\tan\theta = 2\sqrt{2}$, $\text{cosec}\,\theta = \frac{3}{2\sqrt{2}}$, $\cot\theta = \frac{1}{2\sqrt{2}}$

1.(v) $\boldsymbol{cot\theta = \sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}}}$



By Pythagoras Formula

$$H^2 = P^2 + B^2 \Rightarrow H^2 = \left(\sqrt{2}\right)^2 + \left(\sqrt{3}\right)^2$$

$$\Rightarrow H^2 = 2 + 3 = 5 \Rightarrow H = \sqrt{5}$$

(v) $\sin\theta = \sqrt{\frac{2}{5}}$, $\cos\theta = \sqrt{\frac{3}{5}}$, $\tan\theta = \sqrt{\frac{2}{3}}$, $\text{cosec}\,\theta = \sqrt{\frac{5}{2}}$, $\sec\theta = \sqrt{\frac{5}{3}}$

Prove the Following Trigonometric Identities

2. $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

Solution

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

3. $\frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

Solution

$\frac{\cos\theta}{\sin\theta} = \cot\theta = \frac{1}{\tan\theta}$

$$4.\quad \frac{\sin\theta}{\operatorname{cosec}\theta}+\frac{\cos\theta}{\sec\theta}=1$$

Solution

$$\frac{\sin\theta}{\operatorname{cosec}\theta}+\frac{\cos\theta}{\sec\theta}=\sin\theta\times\frac{1}{\operatorname{cosec}\theta}+\cos\theta\times\frac{1}{\sec\theta}$$
$$\frac{\sin\theta}{\operatorname{cosec}\theta}+\frac{\cos\theta}{\sec\theta}=\sin\theta\times\sin\theta+\cos\theta\times\cos\theta=\sin^2\theta+\cos^2\theta=1$$

$$5.\quad \cos^2\theta-\sin^2\theta=2\cos^2\theta-1$$

Solution

$$\cos^2\theta-\sin^2\theta=\cos^2\theta-(1-\cos^2\theta)=\cos^2\theta-1+\cos^2\theta$$
$$\cos^2\theta-\sin^2\theta=2\cos^2\theta-1$$

$$6.\quad \cos^2\theta-\sin^2\theta=1-2\sin^2\theta$$

Solution

$$\cos^2\theta-\sin^2\theta=(1-\sin^2\theta)-\sin^2\theta=1-\sin^2\theta-\sin^2\theta$$
$$\cos^2\theta-\sin^2\theta=1-2\sin^2\theta$$

$$7.\quad \frac{1-\sin\theta}{\cos\theta}=\frac{\cos\theta}{1+\sin\theta}$$

Solution

$$\frac{1-\sin\theta}{\cos\theta}=\frac{(1-\sin\theta)(1+\sin\theta)}{\cos\theta(1+\sin\theta)}=\frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}=\frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}=\frac{\cos\theta}{1+\sin\theta}$$

$$8.\quad (\sec\theta-\tan\theta)^2=\frac{1-\sin\theta}{1+\sin\theta}$$

Solution

$$(\sec\theta-\tan\theta)^2=\left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)^2=\left(\frac{1-\sin\theta}{\cos\theta}\right)^2=\frac{(1-\sin\theta)^2}{\cos^2\theta}=\frac{(1-\sin\theta)(1-\sin\theta)}{1-\sin^2\theta}$$
$$(\sec\theta-\tan\theta)^2=\frac{(1-\sin\theta)(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}=\frac{1-\sin\theta}{1+\sin\theta}$$

$$9.\quad (\tan\theta+\cot\theta)^2=\sec^2\theta\operatorname{cosec}^2\theta$$

Solution

$$(\tan\theta+\cot\theta)^2=\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)^2=\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)^2=\left(\frac{1}{\cos\theta\sin\theta}\right)^2$$
$$(\tan\theta+\cot\theta)^2=\frac{1}{\cos^2\theta}\times\frac{1}{\sin^2\theta}=\sec^2\theta\operatorname{cosec}^2\theta$$

10. $$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$$

Solution

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$
$$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$
$$= \frac{\tan\theta + \sec\theta - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}$$
$$= \frac{(\tan\theta + \sec\theta)[1 - (\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1}$$
$$= \frac{(\tan\theta + \sec\theta)[1 - \sec\theta + \tan\theta]}{1 - \sec\theta + \tan\theta}$$
$$= \tan\theta + \sec\theta$$

11. $$\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$$

Solution

$$\sin^3\theta - \cos^3\theta$$
$$= (\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)$$
$$= (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$$

12. $$\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$$

Solution

$$\sin^6\theta - \cos^6\theta$$
$$= (\sin^2\theta)^3 - (\cos^2\theta)^3$$
$$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + (\cos^2\theta)^2 + \sin^2\theta\cos^2\theta]$$
$$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta]$$
$$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta]$$
$$= (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$$

EXERCISE 6.4

θ	0	$30° = \frac{\pi}{6}$	$45° = \frac{\pi}{4}$	$60° = \frac{\pi}{3}$	$90° = \frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

1. Find the value of the following trigonometric ratios without using the calculator.

(i) sim 30° (ii) cos 30° (iii) $\tan\frac{\pi}{6}$ (iv) tan 60°

(v) sec 60° (vi) $\cos\frac{\pi}{3}$ (vii) cot 60° (viii) sin 60°

(ix) sec 30° (x) cosec 30° (xi) sin 45° (xii) $\cos\frac{\pi}{4}$

Solution

(i) $\frac{1}{2}$ (ii) $\frac{\sqrt{3}}{2}$ (iii) $\frac{\sqrt{3}}{3}$ (iv) $\sqrt{3}$

(v) 2 (vi) $\frac{1}{2}$ (vii) $\frac{\sqrt{3}}{3}$ (viii) $\frac{\sqrt{3}}{2}$

(ix) $\frac{2\sqrt{3}}{3}$ (x) 2 (xi) $\frac{\sqrt{2}}{2}$ (xii) $\frac{\sqrt{2}}{2}$

2. Evaluate:

(i) $2 \sin 60° \cos 60°$ (ii) $2 \cos \frac{\pi}{6} \sin \frac{\pi}{6}$

(iii) $2 \sin 45° + 2\cos 45°$ (iv) $\sin 60° \cos 30° + \cos 60° \sin 30°$

(v) $\cos 60° \cos 30° - \sin 60° \sin 30°$ (vi) $\sin 60° \cos 30° - \cos 60° \sin 30°$

(vii) $\cos 60° \cos 30° + \sin 60° \sin 30°$ (viii) $\tan \frac{\pi}{6} \cot \frac{\pi}{6} + 1$

Solution

2(i): $2\sin60°\cos60° = 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$

2(ii): $2\cos\frac{\pi}{6}\sin\frac{\pi}{6} = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

2(iii): $2\sin45° + 2\cos45° = 2 \times \frac{1}{\sqrt{2}} + 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

2(iv): $\sin60°\cos30° + \cos60°\sin30° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

2(v): $\cos60°\cos30° - \sin60°\sin30° = \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

2(vi): $\sin60°\cos30° - \cos60°\sin30° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

2(vii): $\cos60°\cos30° + \sin60°\sin30° = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

2(viii): $\tan\frac{\pi}{6}\cot\frac{\pi}{6} + 1 = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{1} + 1 = 1 + 1 = 2$

3. If $\sin \frac{\pi}{4}$ and $\cos \frac{\pi}{4}$ equal to $\frac{1}{\sqrt{2}}$ each, then find the value of the followings:

(i) $2 \sin 45° - 2 \cos 45°$ (ii) $3 \cos 45° + 4 \sin 45°$

(iii) $5 \cos 45° - 3 \sin 45°$

Solution

3(i): $2\sin45° - 2\cos45° = 2 \times \frac{1}{\sqrt{2}} - 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} - \sqrt{2} = 0$

3(ii): $3\cos45° + 4\sin45° = 3 \times \frac{1}{\sqrt{2}} + 4 \times \frac{1}{\sqrt{2}} = \frac{7}{\sqrt{2}}$

3(iii): $5\cos45° - 3\sin45° = 5 \times \frac{1}{\sqrt{2}} - 3 \times \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{\sqrt{2}\times\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

EXERCISE 6.5

1. Find the values of *x*, *y* and *z* from the following right angled triangles.

1(i) $m\angle A = 30°, y = 4cm$

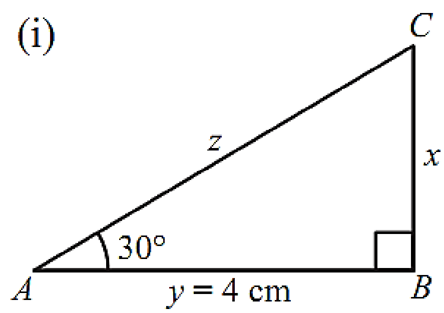
Solution

$m\angle C = m\angle B - m\angle A = 90° - 30°$

$m\angle C = 60°$

$\frac{x}{y} = \tan 30°$ $\frac{x}{4} = \frac{1}{\sqrt{3}}$ $x = \frac{4}{\sqrt{3}}$	$\frac{y}{z} = \cos 30°$ $\frac{4}{z} = \frac{\sqrt{3}}{2}$ $z = 4 \times \frac{2}{\sqrt{3}} = \frac{8}{\sqrt{3}}$

(i)



1(ii) $m\angle A = 45°, y = \sqrt{3}cm$

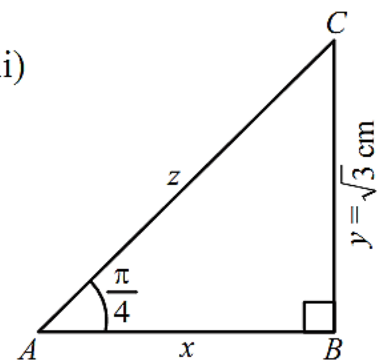
Solution

$m\angle C = m\angle B - m\angle A = 90° - 45°$

$m\angle C = 45°$

$\frac{y}{x} = \tan 45°$ $\frac{\sqrt{3}}{x} = 1$ $x = \sqrt{3}$	$\frac{y}{z} = \sin 45°$ $\frac{\sqrt{3}}{z} = \frac{1}{\sqrt{2}}$ $z = \sqrt{3} \times \sqrt{2} = \sqrt{6}$

(ii)



1(iii) $m\angle C = 60°, z = 2cm$

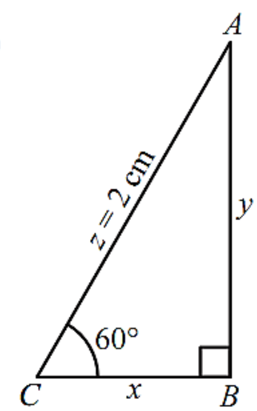
Solution

$m\angle A = m\angle B - m\angle C = 90° - 60°$

$m\angle A = 30°$

$\frac{x}{z} = \cos 60°$ $\frac{x}{2} = \frac{1}{2}$ $x = \frac{2}{2}$ $x = 1$	$\frac{y}{z} = \sin 60°$ $\frac{y}{2} = \frac{\sqrt{3}}{2}$ $y = \frac{2 \times \sqrt{3}}{2}$ $y = \sqrt{3}$

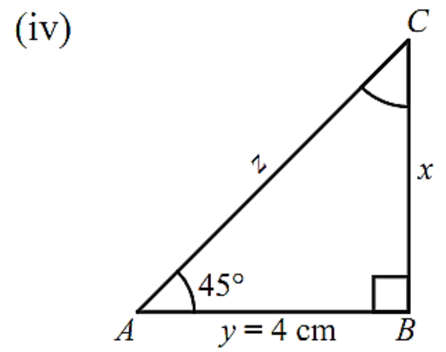
(iii)



1(iv) $m\angle A = 45°, y = 4cm$
Solution

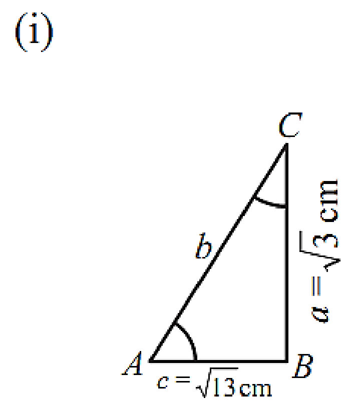
$m\angle C = m\angle B - m\angle A = 90° - 45°$
$m\angle C = 45°$

$\frac{x}{y} = \tan 45°$ $\frac{x}{4} = 1$ $x = 4$	$\frac{y}{z} = \cos 45°$ $\frac{4}{z} = \frac{1}{\sqrt{2}}$ $z = 4\sqrt{2}$



2. Find the unknown side and angles of the following triangles.

2(i)
By Pythagoras Formula



$b^2 = a^2 + c^2 \Rightarrow b^2 = (\sqrt{3})^2 + (\sqrt{13})^2$

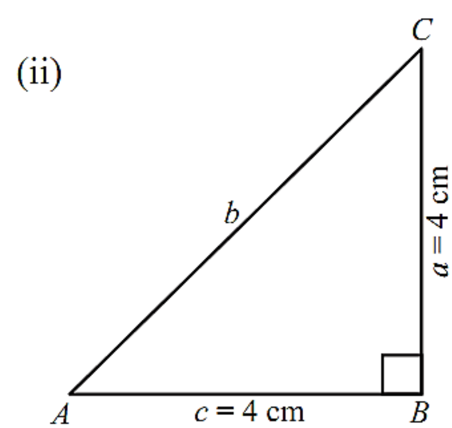
$\Rightarrow b^2 = 3 + 13 = 16 \Rightarrow b = 4$

$\sin A = \frac{a}{b} = \frac{\sqrt{3}}{4} = 0.4330$

$A = \sin^{-1}(0.4330) = 25.64°$

$m\angle C = m\angle B - m\angle A = 90° - 25.64°$
$m\angle C = 64.36°$

2(ii)
By Pythagoras Formula



$b^2 = a^2 + c^2 \Rightarrow b^2 = (4)^2 + (4)^2$

$\Rightarrow b^2 = 16 + 16 = 32 \Rightarrow b = 4\sqrt{2}$

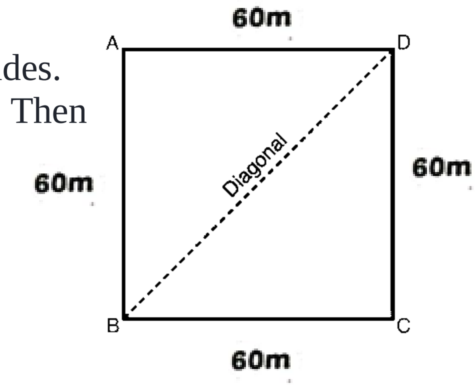
$\cos A = \frac{c}{b} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.7071$

$A = \cos^{-1}(0.7071) = 45°$

$m\angle C = m\angle B - m\angle A = 90° - 45°$
$m\angle C = 45°$

3. Each side of a square field is 60 m long. Find the lengths of the diagonals of the field.

Solution



A square's diagonal forms a right-angled triangle with two sides.
If 'a' and 'b' are the sides of the square and 'c' is the diagonal. Then
Using Pythagorean Theorem:
$c^2 = a^2 + b^2$
In this case, a = b = 60m.
Therefore, $c^2 = 60^2 + 60^2$
$c^2 = 3600 + 3600 = 7200$
$c = \sqrt{7200} = \sqrt{3600 \times 2} = 60\sqrt{2}$m

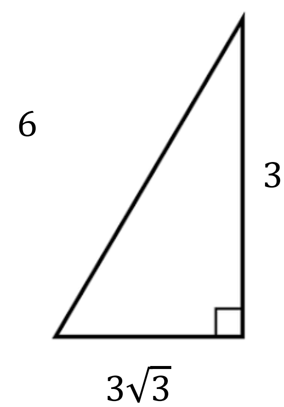
Solve the following triangles when $m\angle B = 90°$:

4. $m\angle C = 60°,\ \ c = 3\sqrt{3}$ cm

Solution

$m\angle C = 60°, c = 3\sqrt{3}\text{cm}$
$m\angle A = m\angle B - m\angle C = 90° - 60°$
$m\angle A = 30°$

$\frac{c}{b} = \sin 60°$ $\frac{3\sqrt{3}}{b} = \frac{\sqrt{3}}{2}$ $b = \frac{2\times 3\sqrt{3}}{\sqrt{3}}$ $b = 6\text{cm}$	$\frac{a}{b} = \sin 30°$ $\frac{a}{6} = \frac{1}{2}$ $a = \frac{6}{2}$ $a = 3\text{cm}$



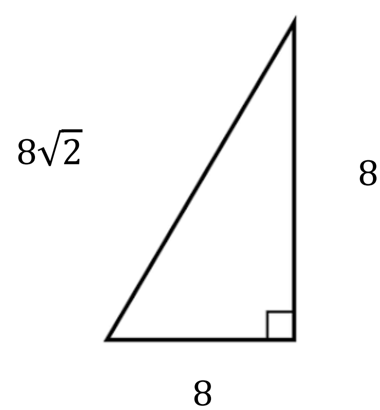
Solve the following triangles when $m\angle B = 90°$:

5. $m\angle C = 45°,\ \ a = 8$ cm

Solution

$m\angle C = 45°, a = 8\text{cm}$
$m\angle A = m\angle B - m\angle C = 90° - 45°$
$m\angle A = 45°$

$\frac{a}{b} = \sin 45°$ $\frac{8}{b} = \frac{1}{\sqrt{2}}$ $b = 8\sqrt{2}\text{cm}$	$\frac{c}{b} = \cos 45°$ $\frac{c}{8\sqrt{2}} = \frac{1}{\sqrt{2}}$ $c = \frac{8\sqrt{2}}{\sqrt{2}}$ $c = 8\text{cm}$



Solve the following triangles when $m\angle B = 90°$:

6. $a = 12$ cm, $c = 6$ cm

Solution

By Pythagoras Formula

$b^2 = a^2 + c^2 \Rightarrow b^2 = (12)^2 + (6)^2$

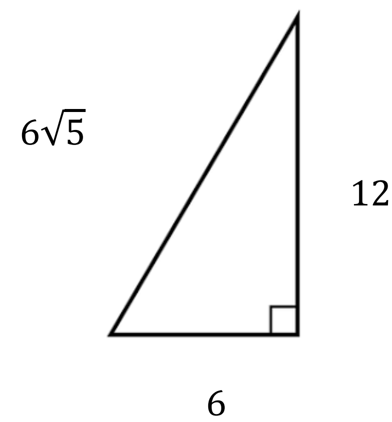
$\Rightarrow b^2 = 144 + 36 = 180 \Rightarrow b = 6\sqrt{5}$

$\mathrm{sinA} = \frac{\mathrm{a}}{\mathrm{b}} = \frac{12}{6\sqrt{5}} = 0.8944$

$\mathrm{A} = \sin^{-1}(0.8944) = 63.4°$

$\mathrm{m}\angle\mathrm{C} = \mathrm{m}\angle\mathrm{B} - \mathrm{m}\angle\mathrm{A} = 90° - 63.4°$

$\mathrm{m}\angle\mathrm{C} = 26.6°$



Solve the following triangles when $m\angle B = 90°$:

7. $m\angle A = 60°$, $c = 4$ cm

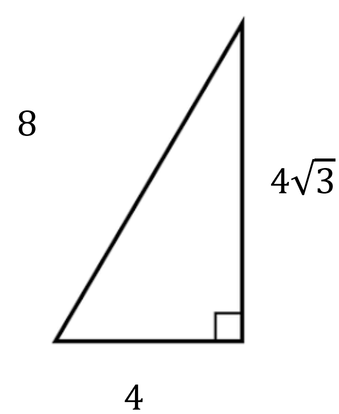
Solution

$\mathrm{m}\angle\mathrm{A} = 60°, \mathrm{c} = 4\mathrm{cm}$

$\mathrm{m}\angle\mathrm{C} = \mathrm{m}\angle\mathrm{B} - \mathrm{m}\angle\mathrm{C} = 90° - 60°$

$\mathrm{m}\angle\mathrm{C} = 30°$

$\frac{\mathrm{c}}{\mathrm{b}} = \cos 60°$ $\frac{4}{\mathrm{b}} = \frac{1}{2}$ $\mathrm{b} = 4 \times 2$ $\mathrm{b} = 8\mathrm{cm}$	$\frac{\mathrm{a}}{\mathrm{b}} = \sin 60°$ $\frac{\mathrm{a}}{8} = \frac{\sqrt{3}}{2}$ $\mathrm{a} = \frac{8\sqrt{3}}{2}$ $\mathrm{a} = 4\sqrt{3}\mathrm{cm}$



Solve the following triangles when $m\angle B = 90°$:

8. $m\angle A = 30°, c = 4cm$

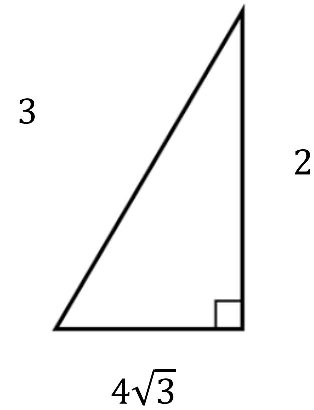
Solution

$m\angle A = 30°, c = 4cm$

$m\angle C = m\angle B - m\angle C = 90° - 30°$

$m\angle C = 60°$

$\frac{c}{b} = \cos 60°$ $\frac{4}{b} = \frac{1}{2}$ $b = 4 \times 2$ $b = 8cm$	$\frac{a}{b} = \sin 60°$ $\frac{a}{8} = \frac{\sqrt{3}}{2}$ $a = \frac{8\sqrt{3}}{2}$ $a = 4\sqrt{3}cm$



Solve the following triangles when $m\angle B = 90°$:

9. $b = 10$ cm, $a = 6$ cm

Solution

By Pythagoras Formula

$b^2 = c^2 + a^2 \Rightarrow (10)^2 = c^2 + (6)^2$

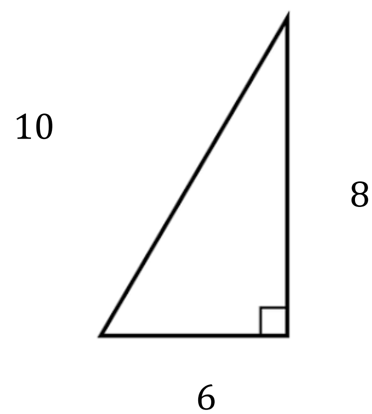
$\Rightarrow c^2 = 100 - 36 = 64 \Rightarrow c = 8$

$\sin C = \frac{c}{b} = \frac{8}{10} = 0.8$

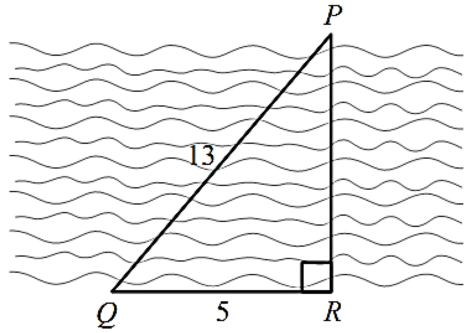
$C = \sin^{-1}(0.8) = 53.1°$

$m\angle A = m\angle B - m\angle C = 90° - 53.1°$

$m\angle A = 36.9°$



10. Let Q and R be the two points on the same bank of a canal. The point P is placed on the other bank straight to point R. Find the width of the canal and the angle PQR.



Solution
By Pythagoras Formula

$|PQ|^2 = |PR|^2 + |QR|^2$

$(13)^2 = |PR|^2 + (5)^2$

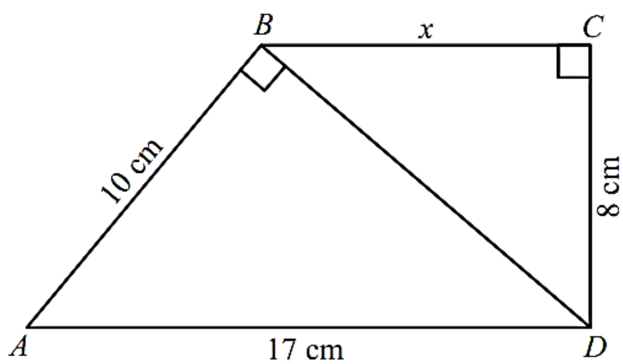
$|PR|^2 = 169 - 25 = 144$

$|PR| = 12km$

$\tan(\angle \mathrm{PQR}) = \frac{\mathrm{PR}}{\mathrm{QR}} = \frac{12}{5} = 2.4$

$\angle \mathrm{PQR} = \tan^{-1}(2.4) = 67.38°$

11. Calculate the length x in the adjoining figure.



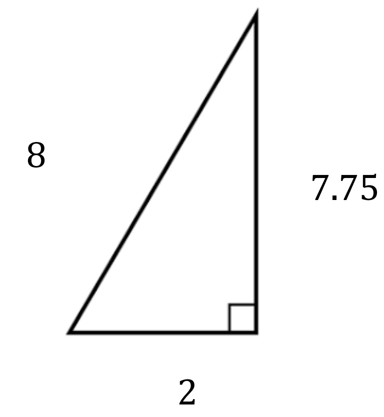
Solution

Applying Pythagoras Formula For ΔABD $\|AD\|^2 = \|BD\|^2 + \|AB\|^2$ $(17)^2 = \|BD\|^2 + (10)^2$ $\|BD\|^2 = 289 - 100 = 189$ $\|BD\| = 3\sqrt{21}$	Again applying Pythagoras Formula ForΔBCD $\|BD\|^2 = \|BC\|^2 + \|CD\|^2$ $\left(3\sqrt{21}\right)^2 = x^2 + (8)^2$ $x^2 = 189 - 64 = 125$ $x = 5\sqrt{5}$

12. If the ladder is placed along the wall such that the foot of the ladder is 2 m away from the wall. If the length of the ladder is 8 m, find the height of the wall.

Solution

By Pythagoras Formula



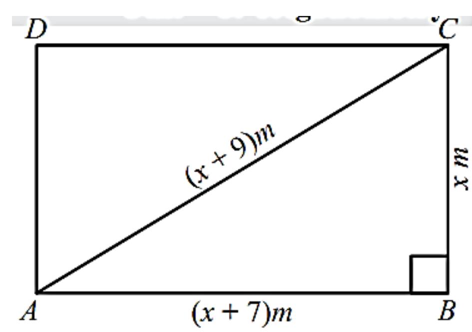
$8^2 = H^2 + 2^2$

$64 = H^2 + 4$

$H^2 = 64 - 4 = 60$

$H = 7.75m$

13. The diagonal of a rectangular field $ABCD$ is $(x + 9)m$ and the sides are $(x + 7)$m and x m. Find the value of x.



Solution

By Pythagoras Formula

$(x + 9)^2 = (x + 7)^2 + x^2$

$x^2 + 18x + 81 = x^2 + 14x + 49 + x^2$

$x^2 + 18x + 81 = 2x^2 + 14x + 49$

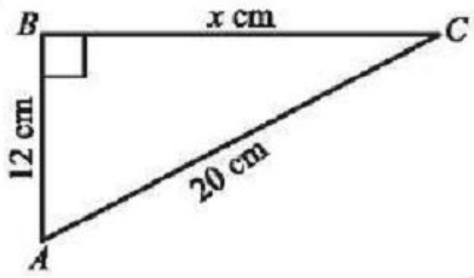
$x^2 - 4x - 32 = 0$

$(x - 8)(x + 4) = 0$

$x = 8$ or $x = -4$

Since x cannot be negative, therefore $x = 8$

14. Calculate the value of 'x' in each case.



Solution
By Pythagoras Formula

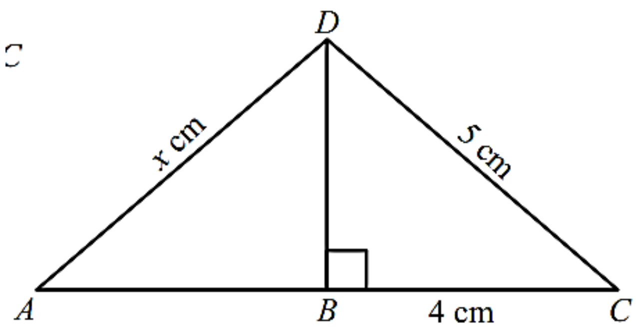
$|AC|^2 = |BC|^2 + |AB|^2$

$(20)^2 = x^2 + (12)^2$

$x^2 = 400 - 144 = 256$

$x = 16cm$

14. Calculate the value of 'x' in each case.



$m\overline{AB} = m\overline{BC}$

Solution

Applying Pythagoras Formula For ΔDBC $\|DC\|^2 = \|DB\|^2 + \|BC\|^2$ $(5)^2 = \|DB\|^2 + (4)^2$ $\|DB\|^2 = 25 - 16 = 9$ $\|DB\| = 3cm$	Again applying Pythagoras Formula For ΔDBA $\|AD\|^2 = \|DB\|^2 + \|AB\|^2$ $x^2 = (3)^2 + (4)^2$ $x^2 = 9 + 16 = 25$ $x = 5\ cm$

EXERCISE 6.6

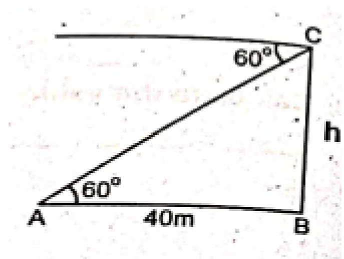
1. The angle of elevation of the top of a flag post from a point on the ground level 40 m away from the flag post is 60°. Find the height of the post.

Solution

$\tan(60°) = \frac{h}{40}$

$h = 40 \times \tan(60°)$

$h = 40 \times \sqrt{3}$



$h \approx 40 \times 1.732050807$

$h \approx 69.28$ meters

So, the height of the flag post is approximately 69.28 meters.

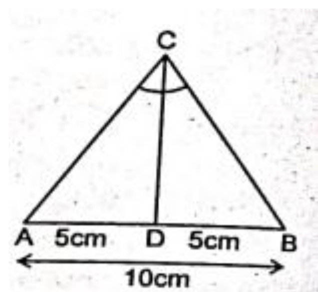
2. An isosceles triangle has a vertical angle of 120° and a base 10 cm long. Find the length of its altitude.

Solution

$\tan 60° = \frac{5}{h}$

$h = \frac{5}{\tan 60°}$

$h = \frac{5}{\sqrt{3}}$



$h \approx 2.89$ cm

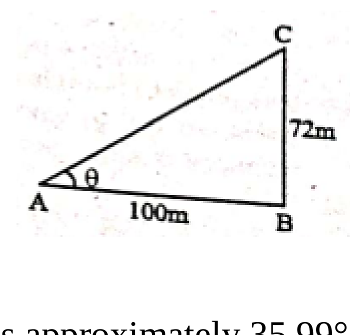
So, the length of the altitude is approximately 2.89 cm.

3. A tree is 72 m high. Find the angle of elevation of its top from a point 100 m away on the ground level.

Solution

$\tan(\theta) = \frac{h}{d}$

$\tan(\theta) = \frac{72}{100}$



$\theta = \arctan\left(\frac{72}{100}\right)$

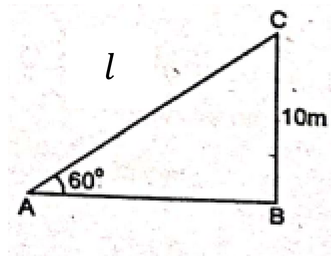
$\theta \approx 35.99°$

So, the angle of elevation of the top of the tree is approximately 35.99°.

4. A ladder makes an angle of 60° with the ground and reaches a height of 10m along the wall. Find the length of the ladder.

Solution

$\sin(\theta) = \frac{h}{l}$



$\sin(60°) = \frac{10}{l}$

$\frac{\sqrt{3}}{2} = \frac{10}{l}$

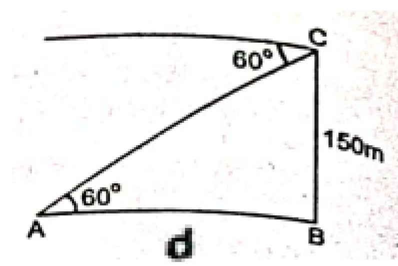
$l = \frac{20}{\sqrt{3}} \approx 11.55$ meters

So, the length of the ladder is approximately 11.55 meters.

5. A light house tower is 150 m high from the sea level. The angle of depression from the top of the tower to a ship is 60°. Find the distance between the ship and the tower.

Solution

$\tan(\theta) = \frac{h}{d}$



$\tan(60°) = \frac{150}{d}$

$d = \frac{150}{\tan(60°)}$

$d = \frac{150}{\sqrt{3}} \approx 86.60$ meters

So, the distance between the ship and the tower is approximately 86.60 meters.

6. Measure of an angle of elevation of the top of a pole is 15° from a point on the ground, in walking 100 m towards the pole the measure of angle is found to be 30°. Find the height of the pole.

Solution

Initial distance from the pole (d_1) = x + 100 meters

Initial angle of elevation (θ_1) = 15°

Final distance from the pole (d_2) = x meters (after walking 100 meters)

Final angle of elevation (θ_2) = 30°

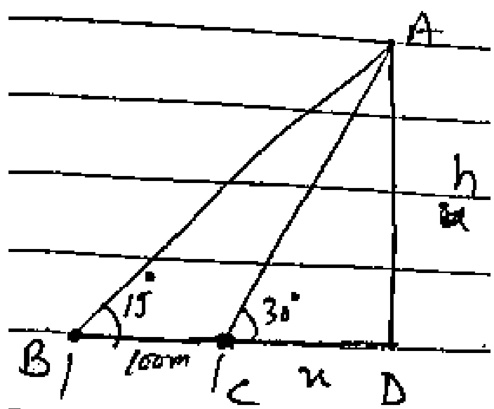
We can use the tangent function to relate the angles, distances, and height (h) of the pole:

$$\tan(\theta_1) = \frac{h}{d_1}$$

$$\tan(15°) = \frac{h}{(x + 100)}$$

h = (x + 100) × tan(15°)

$$\tan(\theta_2) = \frac{h}{d_2}$$



$$\tan(30°) = \frac{h}{x}$$

h = x × tan(30°)

Equating the two expressions:

(x + 100) × tan(15°) = x × tan(30°)

(x + 100) × 0.2679 = x × 0.5773

0.2679x + 26.79 = 0.5773x

26.79 = 0.3094x

x ≈ 86.73 meters

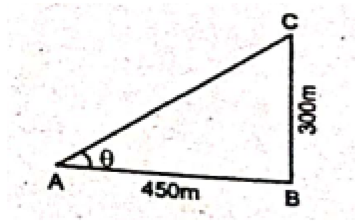
Now that we have x, we can find the height (h) of the pole:

h = x × tan(30°) = 86.73 × 0.5773 ≈ 50 meters

So, the height of the pole is approximately 50 meters.

7. Find the measure of an angle of elevation of the Sun, if a tower 300 m high casts a shadow 450 m long.

Solution



$\tan(\theta) = \frac{h}{s}$

$\tan(\theta) = \frac{300}{450}$

$\theta = \arctan\left(\frac{300}{450}\right)$

$\theta \approx 33.69°$

So, the measure of the angle of elevation of the sun is approximately 33.69°.

8. Measure of angle of elevation of the top of a cliff is 25°, on walking 100 metres towards the cliff, measure of angle of elevation of the top is 45°. Find the height of the cliff.

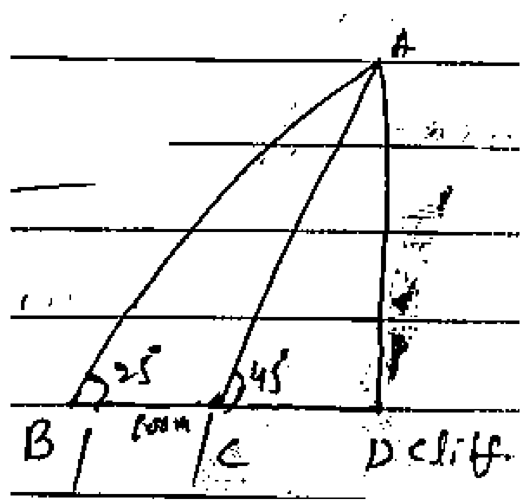
Solution

Initial distance from the cliff (d_1) = x + 100 meters

Initial angle of elevation (θ_1) = 25°

Final distance from the cliff (d_2) = x meters

Final angle of elevation (θ_2) = 45°



$\tan(\theta_1) = \frac{\mathrm{h}}{\mathrm{d}_1}$

$\tan(25°) = \frac{\mathrm{h}}{(\mathrm{x}+100)}$

$\mathrm{h} = (\mathrm{x} + 100) \times \tan(25°)$

$\tan(\theta_2) = \frac{\mathrm{h}}{\mathrm{d}_2}$

$\tan(45°) = \frac{\mathrm{h}}{\mathrm{x}}$

$\mathrm{h} = \mathrm{x} \times \tan(45°)$

Equating the two expressions:

(x + 100) × tan(25°) = x × tan(45°)

(x + 100) × 0.4663 = x × 1

0.4663x + 46.63 = x

46.63 = 0.5337x

x ≈ 87.32 meters

Now that we have x, we can find the height (h) of the cliff:

h = x × tan(45°) = 87.32 × 1 ≈ 87.32 meters

So, the height of the cliff is approximately 87.32 meters.

9. From the top of a hill 300 m high, the measure of the angle of depression of a point on the nearer shore of the river is 70° and measure of the angle of depression of a point, directly across the river is 50°. Find the width of the river How far is the river from the foot of the hill?

Solution

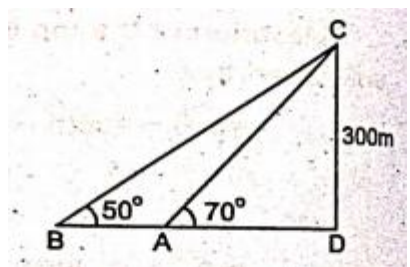
Distance to the nearer shore:

tan(70°) = 300 / x

x = 300 / tan(70°) ≈ 300 / 2.748 ≈ 109.2 meters

Distance to the point across the river:

tan(50°) = 300 / (x + w)



where w is the width of the river.

1.192 = 300 / (x + w)

x + w ≈ 300 / 1.192

x + w ≈ 251.7 meters

109.2 + w ≈ 251.7

w ≈ 142.5 meters

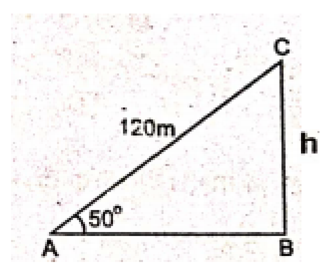
So, the width of the river is approximately 142.5 meters.

The distance from the foot of the hill to the river is approximately 109.2 meters.

10. A kite has 120 m of string attached to it when at an angle of elevation of 50°. How far is it above the hand holding it? (Assume that the string is stretched.

Solution

$\sin(\theta) = \frac{h}{l}$



$\sin(50°) = \frac{h}{120}$

h = 120 × sin(50°)

h ≈ 120 × 0.766 ≈ 91.92 meters

So, the kite is approximately 91.92 meters above the hand holding it.

REVIEW EXERCISE 6

1. Four options are given against each statement. Encircle the correct one.

(i) The value of $\tan^{-1} 2$ in radians is:

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) 1.11π (d) ✓ 1.11

(ii) In a right triangle, the hypotenuse is 13 units and one of the angles is $\theta = 30°$. The length of the opposite side is:

(a) 6.5 units (b) ✓ 7.5 units (c) 6 units (d) 5 units

(iii) A person standing 50 m away from a building sees the top of the building at an angle of elevation of 45°. Height of the building is:

(a) ✓ 50 m (b) 25 m (c) 35 m (d) 70 m

(iv) $\sec^2\theta - \tan^2\theta =$ __________.

(a) $\sin^2 \theta$ (b) ✓ 1 (c) $\cos^2 \theta$ (d) $\cot^2 \theta$

(v) If $\sin \theta = \frac{3}{5}$ and θ is an acute angle, $\cos^2 \theta =$ __________.

(a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) ✓ $\frac{16}{25}$ (d) $\frac{4}{25}$

(vi) $\frac{5\pi}{24}$ rad = __________ degrees.

(a) 30° (b) ✓ 37.5° (c) 45° (d) 52.5°

(vii) 292.5° = __________ rad.

(a) $\frac{17\pi}{6}$ (b) $\frac{17\pi}{4}$ (c) $1.6\,\pi$ (d) ✓ $1.625\,\pi$

(viii) Which of the following is a valid identity?

(a) ✓ $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\ \theta$ (b) $\cos\left(\frac{\pi}{2}-\theta\right)=\cos\ \theta$

(c) $\cos\left(\frac{\pi}{2}-\theta\right)=\sec\ \theta$ (d) $\cos\left(\frac{\pi}{2}-\theta\right)=\text{cosec}\ \theta$

ix. $\sin 60° =$ ________.

(a) 1 (b) $\frac{1}{2}$ (c) $\sqrt{(3)^2}$ (d) ✓ $\frac{\sqrt{3}}{2}$

x. $\cos^2 100\pi + \sin^2 100\pi =$ _________.

(a) ✓ 1 (b) 2 (c) 3 (d) 4

2. Convert the given angles from:

(a) degrees to radians giving answer in terms of π.

(i) 255° (ii) 75° 45′ (iii) 142.5°

Solution

2(i): $\mathbf{255°} = 255 \times \frac{\pi}{180} = \frac{17\pi}{2}\text{rad}$

2(ii): $\mathbf{75°45'} = \left(75+\frac{45}{60}\right)° = 75.75° \times \frac{\pi}{180} = \frac{101\pi}{240}\text{rad}$

2(iii): $\mathbf{142.5°} = 142.5 \times \frac{\pi}{180} = \frac{19\pi}{24}\text{rad}$

2. Convert the given angles from:

(b) radians to degrees giving answer in degrees and minutes.

(i) $\frac{17\pi}{24}$ (ii) $\frac{7\pi}{12}$ (iii) $\frac{11\pi}{16}$

Solution

2(i): $\frac{\mathbf{17\pi}}{\mathbf{24}}\textbf{rad} = \frac{17\pi}{24} \times \frac{180°}{\pi} = 127.5° = 127°30'$

2(ii): $\frac{\mathbf{7\pi}}{\mathbf{12}}\textbf{rad} = \frac{7\pi}{12} \times \frac{180°}{\pi} = 105°$

2(iii): $\frac{\mathbf{11\pi}}{\mathbf{16}}\textbf{rad} = \frac{11\pi}{16} \times \frac{180°}{\pi} = 123°45'$

3. Prove the following trigonometric identities:

(i) $$\frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$$

Solution

$$\frac{\sin\theta}{1-\cos\theta} = \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta} = \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta} = \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta} = \frac{1+\cos\theta}{\sin\theta}$$

3. Prove the following trigonometric identities:

(ii) $$\sin\theta\,(\operatorname{cosec}\theta - \sin\theta) = \frac{1}{\sec^2\theta}$$

Solution

$$\sin\theta(\operatorname{cosec}\theta - \sin\theta) = \sin\theta \times \frac{1}{\sin\theta} - \sin^2\theta = 1 - \sin^2\theta = \cos^2\theta = \frac{1}{\sec^2\theta}$$

3. Prove the following trigonometric identities:

(iii) $$\frac{\operatorname{cosec}\theta - \sec\theta}{\operatorname{cosec}\theta + \sec\theta} = \frac{1-\tan\theta}{1+\tan\theta}$$

Solution

$$\frac{\operatorname{cosec}\theta - \sec\theta}{\operatorname{cosec}\theta + \sec\theta} = \frac{\frac{1}{\sin\theta}-\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}+\frac{1}{\cos\theta}} = \frac{\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}}{\frac{\cos\theta+\sin\theta}{\sin\theta\cos\theta}} = \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \frac{\cos\theta\left(1-\frac{\sin\theta}{\cos\theta}\right)}{\cos\theta\left(1+\frac{\sin\theta}{\cos\theta}\right)} = \frac{1-\tan\theta}{1+\tan\theta}$$

3. Prove the following trigonometric identities:

(iv) $$\tan\theta + \cot\theta = \frac{1}{\sin\theta\,\cos\theta}$$

Solution

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

3. Prove the following trigonometric identities:

(v) $$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{2}{1-2\sin^2\theta}$$

Solution

$$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$
$$=\frac{(\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}$$
$$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$$
$$=\frac{1+1}{1-\sin^2\theta-\sin^2\theta}=\frac{2}{1-2\sin^2\theta}$$

3. Prove the following trigonometric identities:

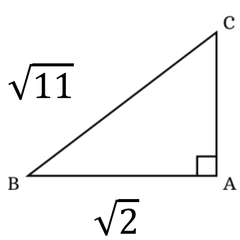
(vi) $$\frac{1+\cos\theta}{1-\cos\theta}=(\operatorname{cosec}\theta+\cot\theta)^2$$

Solution

$$(\operatorname{cosec}\theta+\cot\theta)^2$$
$$=\left(\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}\right)^2=\frac{(1+\cos\theta)^2}{\sin^2\theta}=\frac{(1+\cos\theta)^2}{1-\cos^2\theta}=\frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}=\frac{1+\cos\theta}{1-\cos\theta}$$

4. If $\tan\theta=\frac{3}{\sqrt{2}}$ then find the remaining trigonometric ratios when θ lies in first quadrant.

Solution



3

By Pythagoras Formula

$$H^2=P^2+B^2 \Rightarrow H^2=3^2+\left(\sqrt{2}\right)^2$$
$$\Rightarrow H^2=9+2=11 \Rightarrow H=\sqrt{11}$$

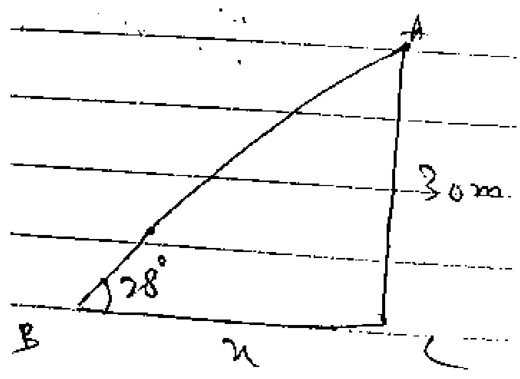
(i) $\sin\theta=\frac{3}{\sqrt{11}}$ (ii) $\cos\theta=\frac{\sqrt{2}}{\sqrt{11}}$ (iii) $\cot\theta=\frac{\sqrt{2}}{3}$

(iv) $cosec\theta=\frac{\sqrt{11}}{3}$ (v) $\sec\theta=\frac{\sqrt{11}}{\sqrt{2}}$

5. From a point on the ground, the angle of elevation to the top of a 30 m high building is 28°. How far is the point from the base of the building?

Solution
Let's denote the distance from the point to the base of the building as x.

We know the angle of elevation (θ) is 28 degrees, and the height of the building (h) is 30 meters.



$$\tan(\theta) = \frac{h}{x}$$

$$\tan(28°) = \frac{30}{x}$$

$$x = \frac{30}{\tan(28°)}$$

$$x \approx \frac{30}{0.53170943}$$

x ≈ 56.42 meters

So, the point is approximately 56.42 meters away from the base of the building.

6. A ladder leaning against a wall forms an angle of 65° with the ground. If the ladder is 10 m long, how high does it reach on the wall?

Solution
Let's denote the height the ladder reaches on the wall as h.

We know the angle between the ladder and the ground (θ) is 65 degrees, and the length of the ladder (l) is 10 meters.

$$\sin(\theta) = \frac{h}{l}$$

$$\sin(65°) = \frac{h}{10}$$

$$h = 10 \times \sin(65°)$$

$$h \approx 10 \times 0.906307787$$

$$h \approx 9.06 \text{ meters}$$

So, the ladder reaches approximately 9.06 meters high on the wall.