Chapter # 5

Linear Equations and Inequalities

Exercise # 5.1

Question # 1: Solve and represent the solution on a real line.

(i) $\mathbf{12x + 30 = -6}$ ________ (A)

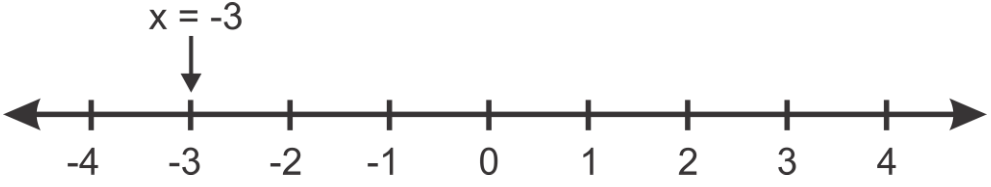
$$12x = -6 - 30$$
$$12x = -36$$
$$x = \frac{\cancel{-36}^{\,3}}{\cancel{12}}$$
$$x = -3$$

Check:

put $x = -3$ in equation (A)

$$12(-3) + 30 = -6$$
$$-36 + 30 = -6$$
$$-6 = -6$$

S.S $= \{-3\}$



(ii) $\mathbf{\frac{x}{3} + 6 = -12}$ ________ (A)

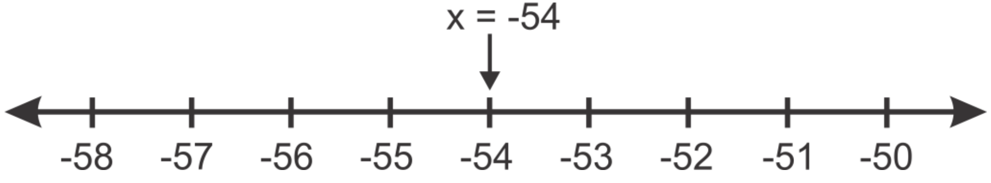
$$\frac{x}{3} = -12 - 6$$
$$\frac{x}{3} = -18$$
$$x = -18 \times 3$$
$$x = -54$$

Check:

put $x = -54$ in equation (A)

$$\frac{-54}{3} + 6 = -12$$
$$\frac{\cancel{-36}^{\,12}}{\cancel{3}} = -12$$
$$-12 = -12$$

S.S $= \{-54\}$



(iii) $\mathbf{\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}}$ ________ (A)

Multiply by '12' on both sides

$$\overset{6}{\cancel{12}} \times \frac{x}{\cancel{2}} - \overset{3}{\cancel{12}} \times \frac{3x}{\cancel{4}} = \overset{1}{\cancel{12}} \times \frac{1}{\cancel{12}}$$
$$6x - 9x = 1$$
$$-3x = 1$$
$$x = -\frac{1}{3}$$

2	2,4,12
2	1,2,6
3	1,1,3
	1,1,1

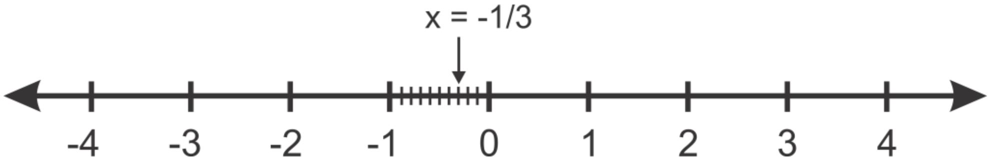
Check:

put $x = -\frac{1}{3}$ in equation (A)

$$\frac{-\frac{1}{3}}{2} - \frac{3\left(-\frac{1}{3}\right)}{4} = \frac{1}{12}$$
$$-\frac{1}{3} \times \frac{1}{2} + \frac{1}{4} = \frac{1}{12}$$
$$-\frac{1}{6} + \frac{1}{4} = \frac{1}{12}$$
$$\frac{-2+3}{12} = \frac{1}{12}$$
$$\frac{1}{12} = \frac{1}{12}$$

S.S $= \left\{-\frac{1}{3}\right\}$

2	6,4
2	3,2
3	3,1
	1,1



(iv) $\mathbf{2 = 7(2x+4) + 12x}$ ________ (A)

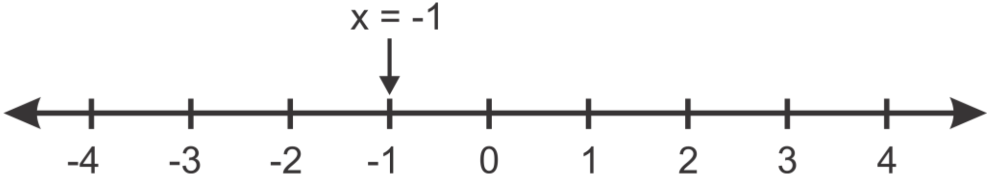
$$2 = 14x + 28 + 12x$$
$$2 - 28 = 26x$$
$$-26 = 26x$$
$$\frac{\cancel{-26}^{\,1}}{\cancel{26}} = x$$
$$x = -1$$

Check:

put $x = -1$ in equation (A)

$$2 = 7[2(-1) + 4] + 12(-1)$$
$$2 = 7(-2 + 4) - 12$$
$$2 = 7(2) - 12$$
$$2 = 14 - 12$$
$$2 = 2$$

S.S = $\{-1\}$



(v) $\mathbf{\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}}$ ________ (A)

Multiply by ‘12’ on both sides

$$\overset{4}{\cancel{12}} \times \frac{2x-1}{\cancel{3}} - \overset{3}{\cancel{12}} \times \frac{3x}{\cancel{4}} = \overset{2}{\cancel{12}} \times \frac{5}{\cancel{6}}$$
$$4(2x - 1) - 3 \times 3x = 2 \times 5$$
$$8x - 4 - 9x = 10$$
$$8x - 9x = 10 + 4$$
$$-x = 14$$
$$x = -14$$

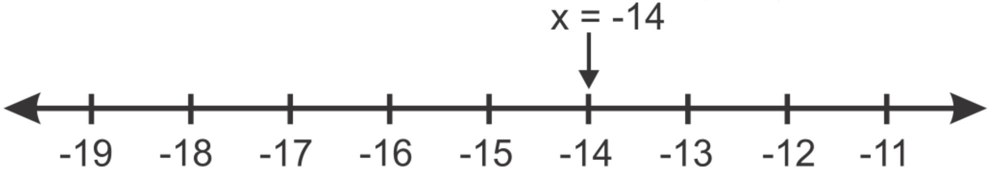
2	3,4,6
2	3,2,3
3	3,1,3
	1,1,1

Check:

put $x = -14$ in equation (A)

$$\frac{2(-14)-1}{3} - \frac{3(-14)}{4} = \frac{5}{6}$$
$$\frac{-28-1}{3} + \frac{42}{4} = \frac{5}{6}$$
$$\frac{-29}{3} + \frac{42}{4} = \frac{5}{6}$$
$$\frac{-116+126}{12} = \frac{5}{6}$$
$$\frac{{}^{5}\cancel{10}}{{}^{6}\cancel{12}} = \frac{5}{6}$$
$$\frac{5}{6} = \frac{5}{6}$$

S.S = $\{-14\}$



(vi) $\mathbf{-\frac{5x}{10} = 9 - \frac{10}{5}x}$ ________ (A)

Multiply by ‘10’ on both sides

$$-\frac{5x}{\cancel{10}} \times \overset{1}{\cancel{10}} = 9 \times 10 - \frac{10}{\cancel{5}}x \times \overset{2}{\cancel{10}}$$
$$-5x = 90 - 20x$$
$$-5x + 20x = 90$$
$$15x = 90$$
$$x = \frac{\cancel{90}^{\,6}}{\cancel{15}}$$
$$x = 6$$

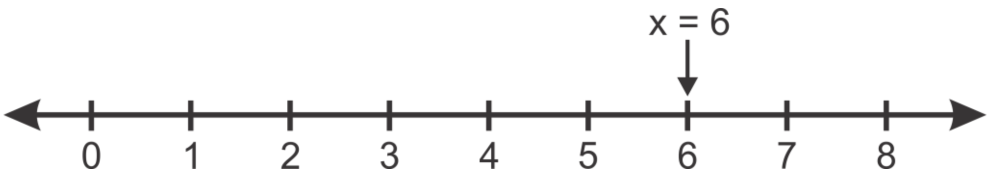
2	10,5
5	5,5
	1,1

Check:

put $x = 6$ in equation (A)

$$-\frac{5(6)}{10} = 9 - \frac{10}{5}(6)$$
$$-\frac{\cancel{30}^{\,3}}{\cancel{10}} = 9 - \frac{\cancel{60}^{\,12}}{\cancel{5}}$$
$$-3 = 9 - 12$$
$$-3 = -3$$

S.S = $\{6\}$

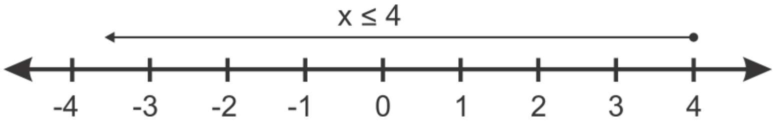


Question # 2: Solve each inequality and represent on a real line.

(i) $\boldsymbol{x - 6 \le -2}$

$x \le -2 + 6$

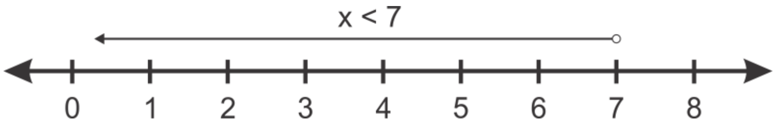
$x \le 4$



(ii) $\boldsymbol{-9 > -16 + x}$

$-9 + 16 > x$

$7 > x$



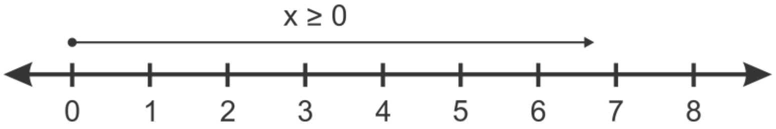
(iii) $\boldsymbol{3 + 2x \ge 3}$

$2x \ge 3 - 3$

$2x \ge 0$

$x \ge \frac{0}{2}$

$x \ge 0$



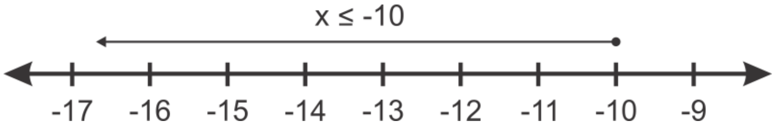
(iv) $\boldsymbol{6(x + 10) \le 0}$

$6x + 60 \le 0$

$6x \le -60$

$x \le -\frac{\cancel{60}^{\,10}}{\cancel{6}}$

$x \le -10$



(v) $\boldsymbol{\frac{5}{3}x - \frac{3}{4} < \frac{-1}{12}}$

2	3,4,12
2	3,2,6
3	3,1,3
	1,1,1

Multiply by '12' on both sides

$\overset{4}{\cancel{12}} \times \frac{5}{\cancel{3}}x - \overset{3}{\cancel{12}} \times \frac{3}{\cancel{4}} < \overset{1}{\cancel{12}} \times \frac{-1}{\cancel{12}}$

$20x - 9 < -1$

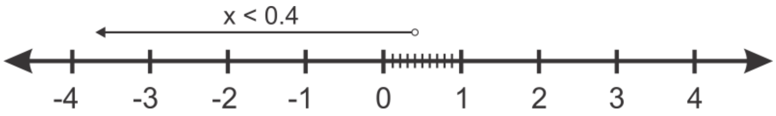
$20x < -1 + 9$

$20x < 8$

$x < \frac{\cancel{8}^{\,2}}{\cancel{20}_{\,5}}$

$x < \frac{2}{5}$

$x < 0.4$



(vi) $\boldsymbol{\frac{1}{4}x - \frac{1}{2} \le -1 + \frac{1}{2}x}$

2	4,2,2
2	2,1,1
	1,1,1

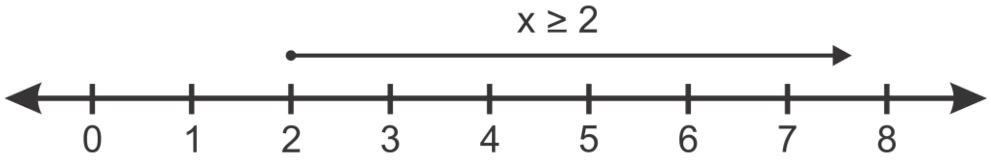
Multiply by '4' on both sides

$\overset{1}{\cancel{4}} \times \frac{1}{\cancel{4}}x - \overset{2}{\cancel{4}} \times \frac{1}{\cancel{2}} \le -4 \times 1 + \overset{2}{\cancel{4}} \times \frac{1}{\cancel{2}}x$

$x - 2 \le -4 + 2x$

$-2 + 4 \le 2x - x$

$2 \le x$ OR $x \ge 2$



Question # 3: Shade the solution region for the following linear inequalities in xy-plane.

(i) $\mathbf{2x + y \leq 6}$

(a) Associated Equation

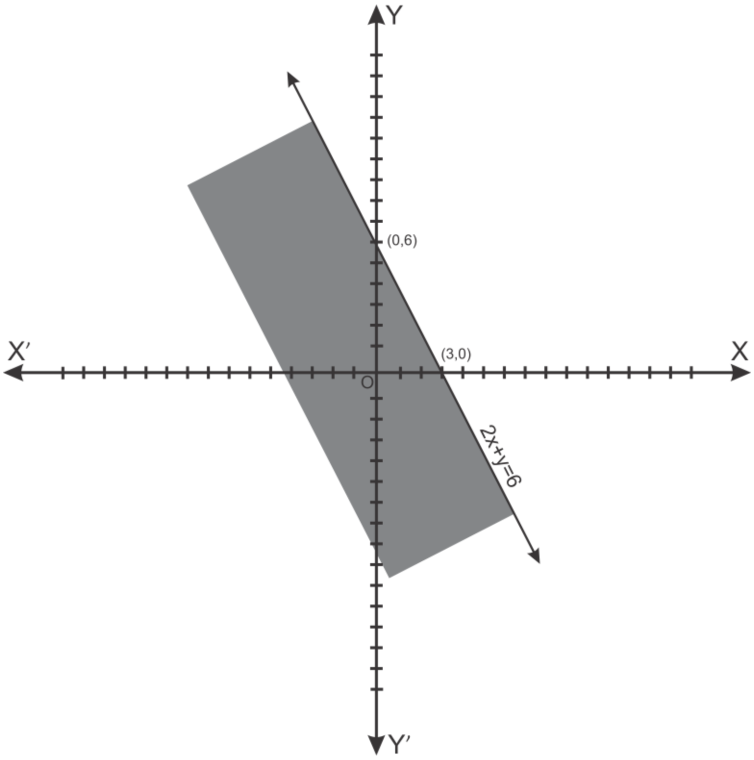
$2x + y = 6$ ______(A)

(b) x – Intercept

put $y = 0$ in equation (A)

$$2x + 0 = 6$$
$$2x = 6$$
$$x = \frac{6}{2}$$
$$x = 3$$

P₁ $(3,0)$



(c) y – Intercept

put $x = 0$ in equation (A)

$$2(0) + y = 6$$
$$y = 6$$

P₂ $(0,6)$

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequality

$$2(0) + 0 \leq 6$$
$$0 \leq 6 \text{ (True)}$$

Solution Region lies towards the Origin

(ii) $\mathbf{3x + 7y \geq 21}$

(a) Associated Equation

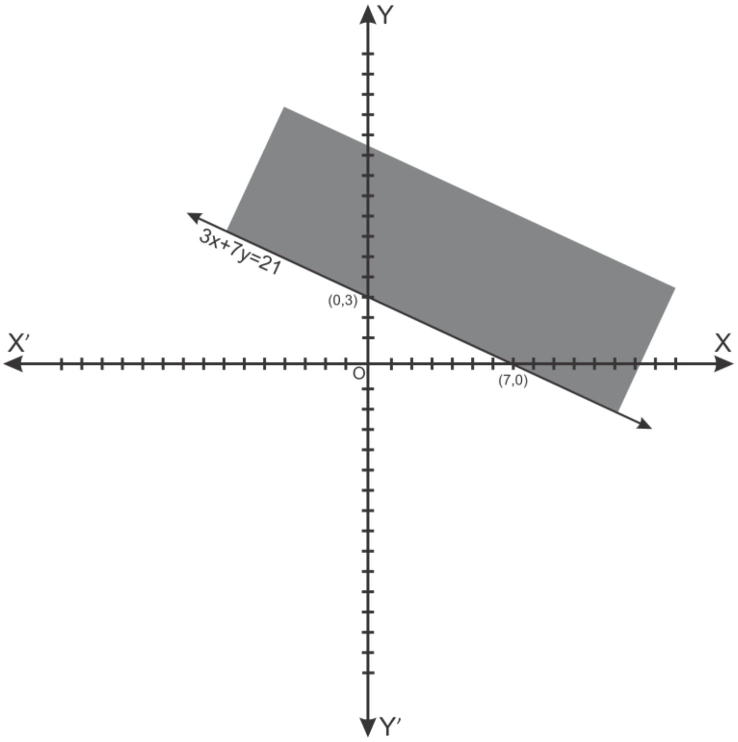
$3x + 7y = 21$ ______(A)

(b) x – Intercept

put $y = 0$ in equation (A)

$$3x + 7(0) = 21$$
$$3x = 21$$
$$x = \frac{21}{3}$$
$$x = 7$$

P₁ $(7,0)$



(c) y – Intercept

put $x = 0$ in equation (A)

$$2(0) + 7y = 21$$
$$7y = 21$$
$$y = \frac{21}{7}$$
$$y = 3$$

P₂ $(0,3)$

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequality

$$3(0)+7(0)\geq 21$$
$$0\geq 21 \text{ (False)}$$

Solution Region lies away from Origin

(iii) $\boldsymbol{3x-2y\geq 6}$

(a) Associated Equation

$3x-2y=6$ ______(A)

(b) x – Intercept

put $y=0$ in equation (A)

$$3x-2(0)=6$$
$$3x=6$$
$$x=\frac{6}{3}$$
$$x=2$$

P_1 (2,0)

(c) y – Intercept

put $x=0$ in equation (A)

$$3(0)-2y=6$$
$$-2y=6$$
$$y=\frac{6}{-2}$$
$$y=-3$$

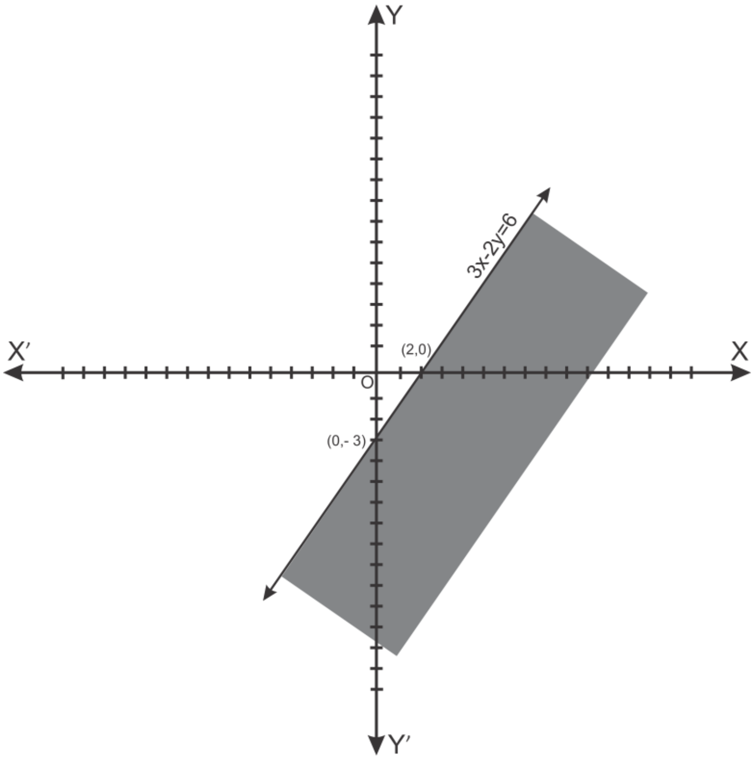
P_2 $(0,-3)$

(d) Test Point (0,0)

put $x=0, y=0$ in given inequality

$$3(0)-2(0)\geq 6$$
$$0\geq 6 \text{ (False)}$$

Solution Region lies away from Origin



(iv) $\boldsymbol{5x-4y\leq 20}$

(a) Associated Equation

$5x-4y=20$ ______(A)

(b) x – Intercept

put $y=0$ in equation (A)

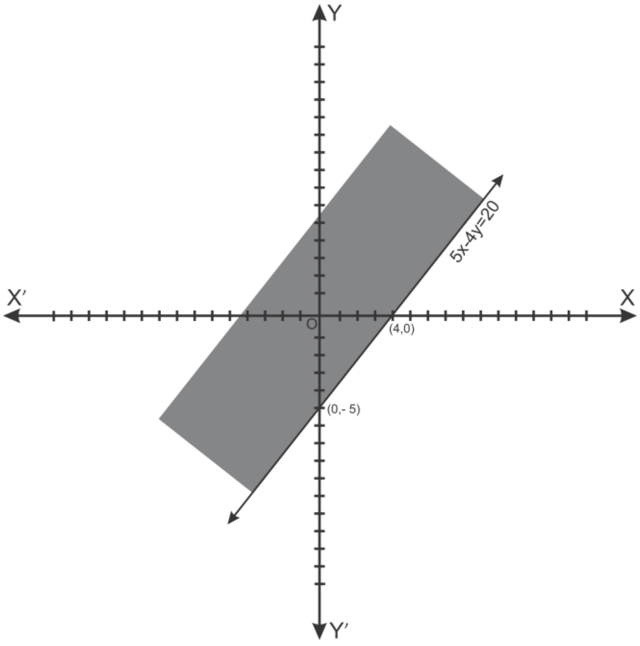
$$5x-4(0)=20$$
$$5x=20$$
$$x=\frac{20}{5}$$
$$x=4$$

P_1 (4,0)

(c) y – Intercept

put $x=0$ in equation (A)

$$5(0)-4y=20$$
$$-4y=20$$
$$y=\frac{20}{-4}$$



$$y = -5$$
$$P_2\ (0, -5)$$

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequality

$$5(0) - 4(0) \leq 20$$
$$0 \leq 20 \text{ (True)}$$

Solution Region lies towards the Origin

(v) $\mathbf{2x + 1 \geq 0}$

(a) Associated Equation

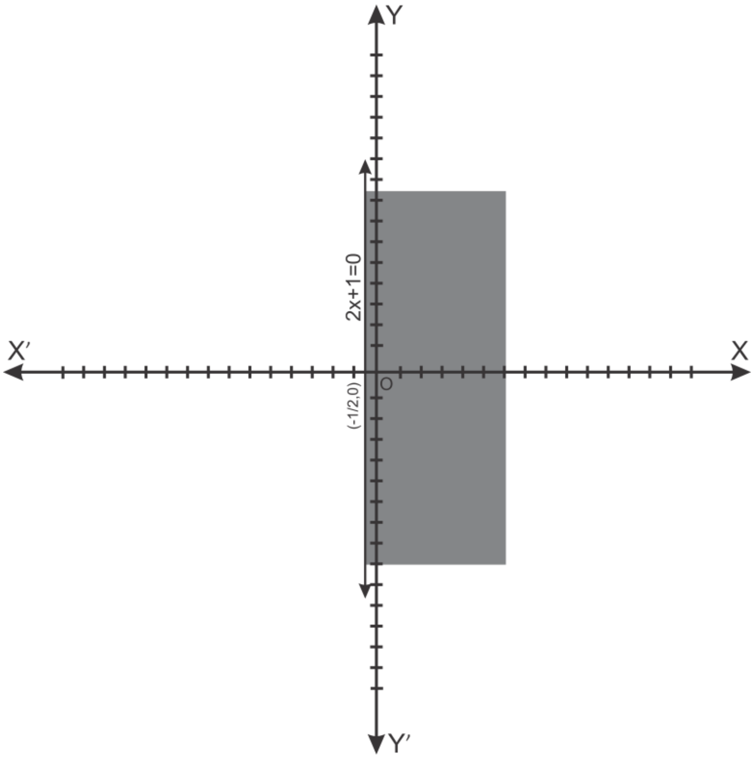
$$2x + 1 = 0$$
$$2x = -1$$
$$x = -\frac{1}{2}$$
$$x = -0.5$$
$$P\ (-0.5,0)$$

(b) Test Point

put $x = 0$ in given inequality

$$2(0) + 1 \geq 0$$
$$1 \geq 0 \text{ (True)}$$

Solution Region lies towards the Origin



(vi) $\mathbf{3y - 4 \leq 0}$

(a) Associated Equation

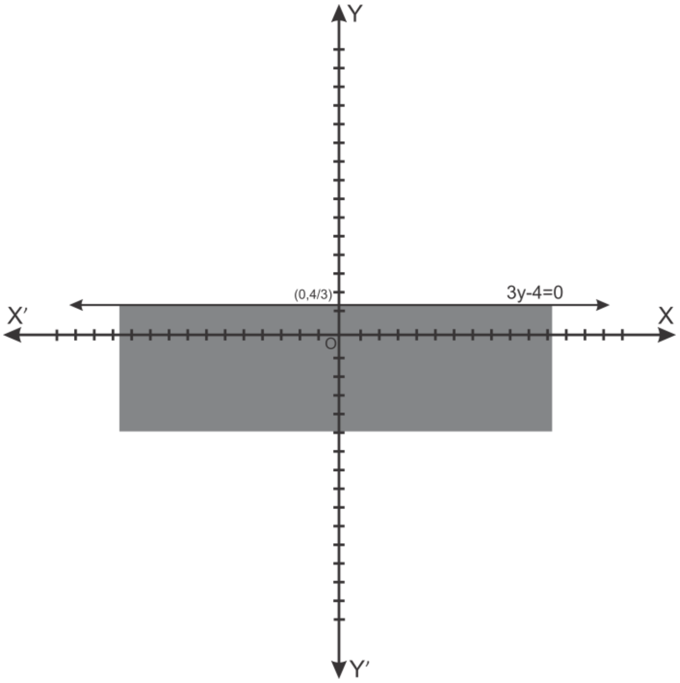
$$3y - 4 = 0$$
$$3y = 4$$
$$y = \frac{4}{3}$$
$$x = 1.3$$
$$P\ (1.3,0)$$

(b) Test Point

put $y = 0$ in given inequality

$$3(0) - 4 \leq 0$$
$$-4 \leq 0 \text{ (True)}$$

Solution Region lies towards the Origin



Question # 4: Indicate the solution region of the following linear inequalities by shading.

(i) $\mathbf{2x-3y\le 6}$; $\mathbf{2x+3y\le 12}$

(a) Associated Equations

$2x-3y=6$ ______(A)

$2x+3y=12$ _____(B)

(b) x – Intercept

put $y=0$ in equations (A) and (B)

$$2x-3(0)=6$$
$$2x=6$$
$$x=\frac{6}{2}$$
$$x=3$$

P₁ (3,0)

$$2x+3(0)=12$$
$$2x=12$$
$$x=\frac{12}{2}$$
$$x=6$$

P₃ (6,0)

(c) y – Intercept

put $x=0$ in equations (A) and (B)

$$2(0)-3y=6$$
$$-3y=6$$
$$y=\frac{6}{-3}$$
$$y=-2$$

P₂ $(0,-2)$

$$2(0)+3y=12$$
$$3y=12$$
$$y=\frac{12}{3}$$

P₄ (0,4)

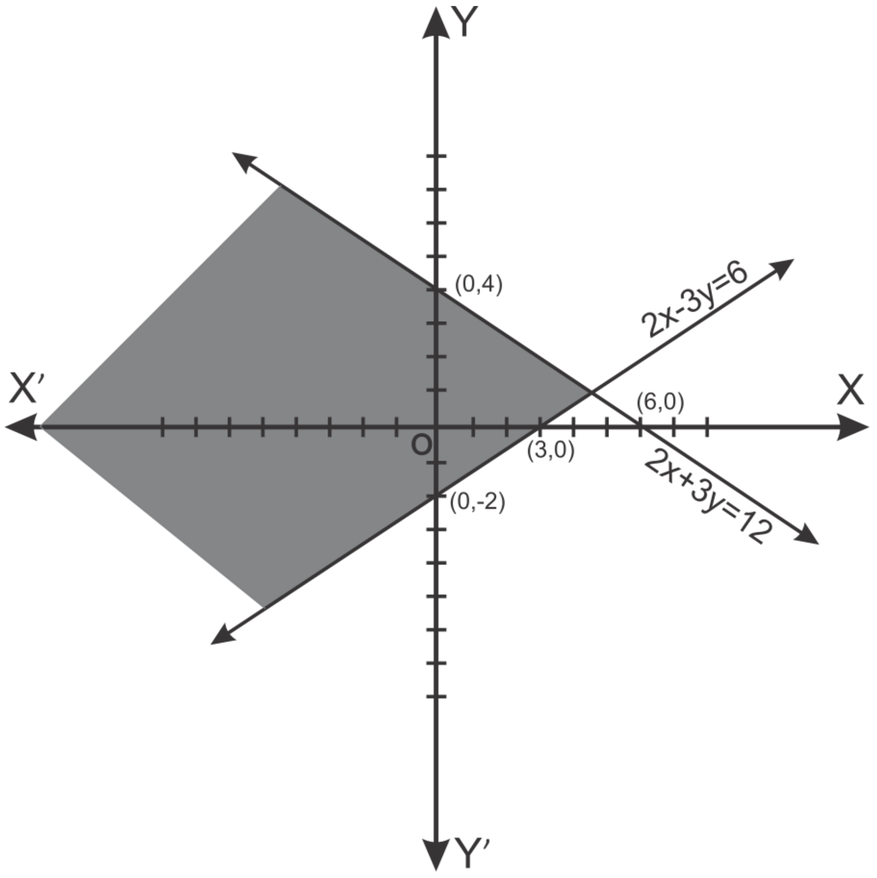
(d) Test Point (0,0)

put $x=0, y=0$ in given inequalities

$$2(0)-3(0)\le 6$$
$0\le 6$ (True)

$$2(0)+3(0)\le 12$$
$0\le 12$ (True)

Solution Regions lie towards the origin



(ii) $\boldsymbol{x+y \geq 5 \quad ; \quad -y+x \leq 1}$

(a) Associated Equations

$x+y=5$ ______(A) $\qquad$ $-y+x=1$ _____(B)

(b) x – Intercept

put $y=0$ in equations (A) and (B)

$x+0=5$	$-0+x=1$
$x=5$	$x=1$
P_1 (5,0)	P_3 (1,0)

(c) y – Intercept

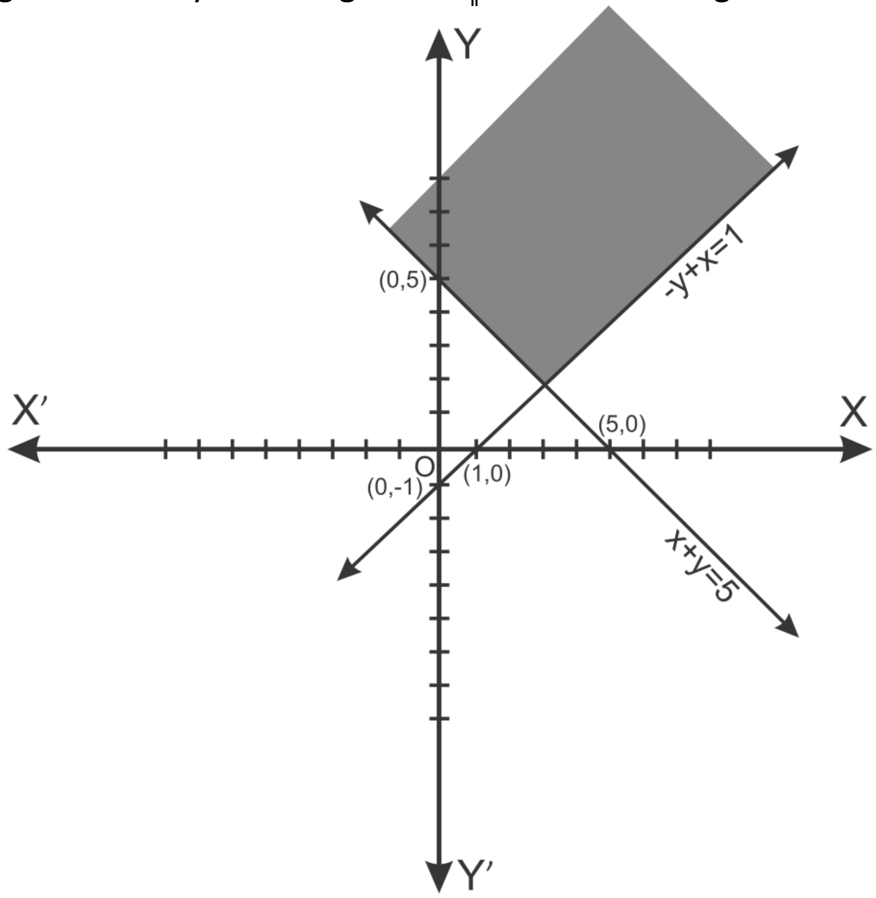
put $x=0$ in equations (A) and (B)

$0+y=5$	$-y+0=1$
$y=5$	$-y=1$
P_2 (0,5)	$y=-1$
	P_4 $(0,-1)$

(d) Test Point (0,0)

put $x=0, y=0$ in given inequalities

$0+0 \geq 5$	$-0+0 \leq 1$
$0 \geq 5$(False)	$0 \leq 1$ (True)
Solution region lies away from origin	Solution region lies towards origin



(iii) $\mathbf{3x+7y \geq 21 \quad ; \quad x-y \leq 2}$

(a) Associated Equations

$3x+7y=21$ ______(A)

$x-y=2$ _____(B)

(b) x – Intercept

put $y=0$ in equations (A) and (B)

$$3x+7(0)=21$$
$$3x=21$$
$$x=\frac{21}{3}$$
$$x=7$$

P₁ (7,0)

$$x-0=2$$
$$x=2$$

P₃ (2,0)

(c) y – Intercept

put $x=0$ in equations (A) and (B)

$$3(0)+7y=21$$
$$7y=21$$
$$y=\frac{21}{7}$$
$$y=3$$

P₂ (0,3)

$$0-y=2$$
$$-y=2$$
$$y=-2$$

P₄ $(0,-2)$

(d) Test Point (0,0)

put $x=0, y=0$ in given inequalities

$$3(0)+7(0) \geq 21$$

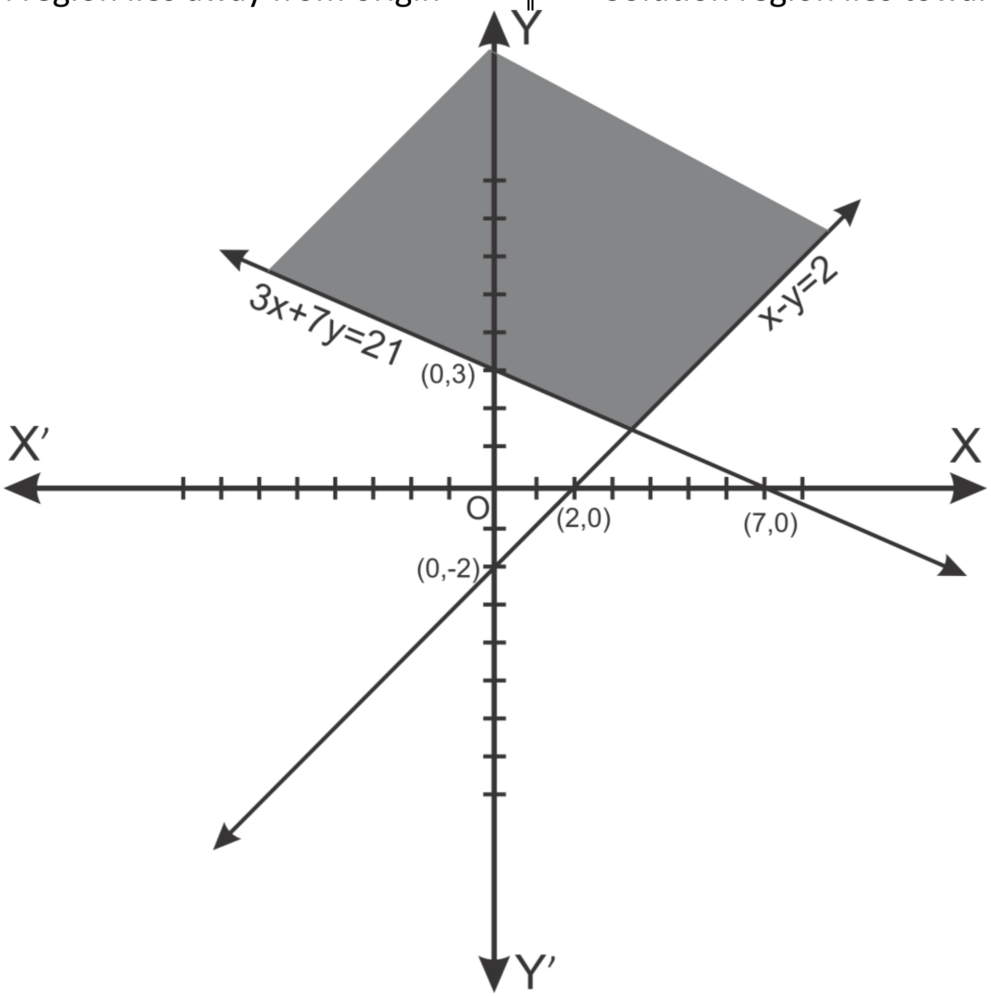
$0 \geq 21$ (False)

Solution region lies away from origin

$$0-0 \leq 2$$

$0 \leq 2$ (True)

Solution region lies towards origin



(iv) $\mathbf{4x - 3y \leq 12}$; $\mathbf{x \geq -\frac{3}{2}}$

(a) Associated Equations

$4x - 3y = 12$ ______(A)

$x = -\frac{3}{2}$

(b) x – Intercept

put $y = 0$ in equations (A)

$$4x - 3(0) = 12$$
$$4x = 12$$
$$x = \frac{12}{4}$$
$$x = 3$$

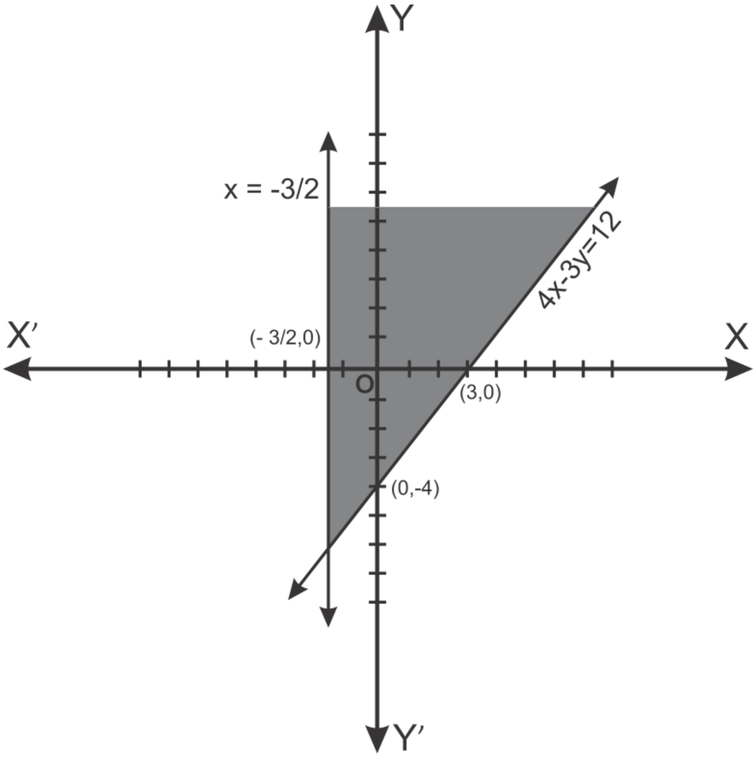
$P_1\ (3,0)$

(c) y – Intercept

put $x = 0$ in equations (A)

$$4(0) - 3y = 12$$
$$-3y = 12$$
$$y = \frac{12}{-3}$$
$$y = -4$$

$P_2\ (0,-4)$



(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$$4(0) - 3(0) \leq 12$$
$$0 \leq 12 \text{ (True)}$$

$$0 \geq -\frac{3}{2}$$
$$0 \geq -\frac{3}{2} \text{ (True)}$$

Solution Regions lie towards the origin

(v) $\mathbf{3x + 7y \geq 21}$; $\mathbf{y \leq 4}$

(a) Associated Equations

$3x + 7y = 21$ ______(A)

$y = 4$

(b) x – Intercept

put $y = 0$ in equations (A)

$$3x + 7(0) = 21$$
$$3x = 21$$
$$x = \frac{21}{3}$$
$$x = 7$$

$P_1\ (7,0)$

(c) y – Intercept

put $x = 0$ in equations (A)

$$3(0) + 7y = 21$$
$$7y = 21$$
$$y = \frac{21}{7}$$
$$y = 3$$

$P_2\ (0,3)$

(d) Test Point (0,0)

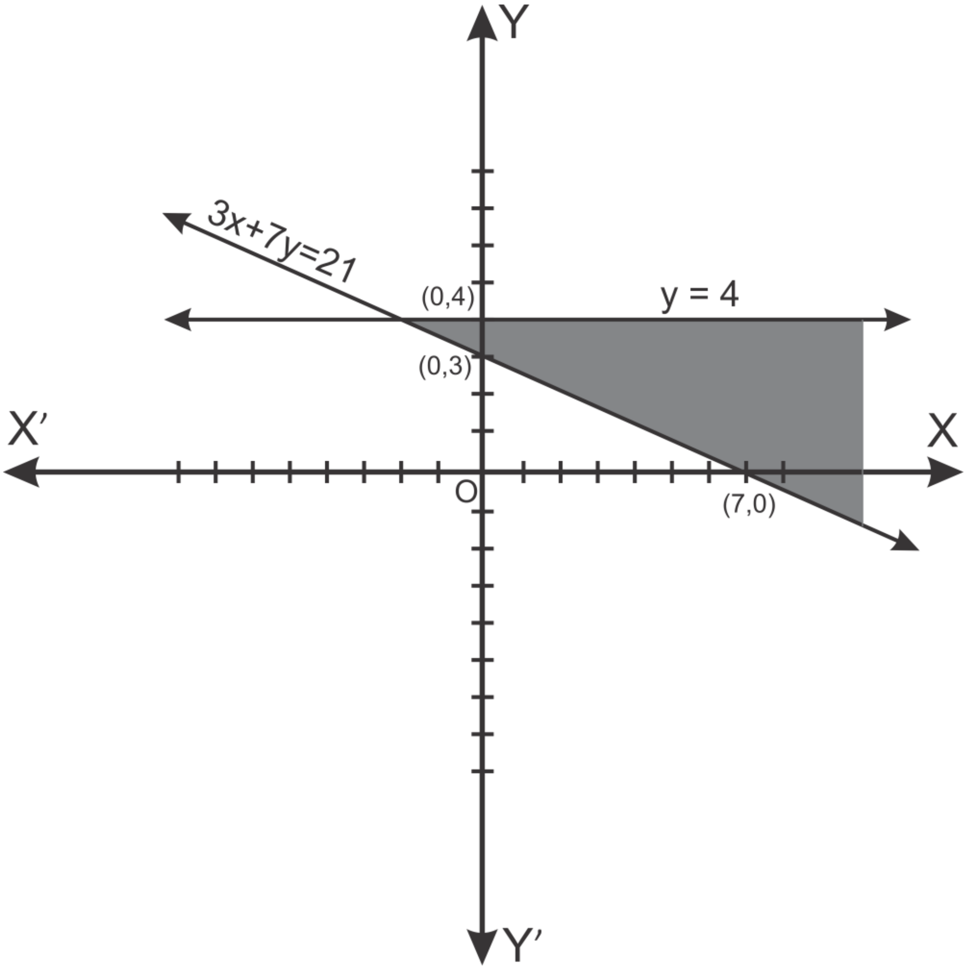
put $x = 0, y = 0$ in given inequalities

$$3(0) + 7(0) \geq 21$$
$$0 \geq 21 \text{ (False)}$$

Solution Region lies away from origin

$$0 \leq 4$$
$$0 \leq 4 \text{ (True)}$$

Solution region lies towards origin



(vi) $\mathbf{5x+7y \le 35 \quad ; \quad x-2y \le 2}$

(a) Associated Equations

$5x+7y=35$ ______(A) || $x-2y=2$ _____(B)

(b) x – Intercept

put $y=0$ in equations (A) and (B)

$$5x+7(0)=35$$
$$5x=35$$
$$x=\frac{35}{5}$$
$$x=7$$

P₁ (7,0)

$$x-2(0)=2$$
$$x=2$$

P₃ (2,0)

(c) y – Intercept

put $x=0$ in equations (A) and (B)

$$5(0)+7y=35$$
$$7y=35$$
$$y=\frac{35}{7}$$
$$y=5$$

P₂ (0,5)

$$0-2y=2$$
$$-2y=2$$
$$y=\frac{2}{-2}$$

P₄ $(0,-1)$

(d) Test Point (0,0)

put $x=0, y=0$ in given inequalities

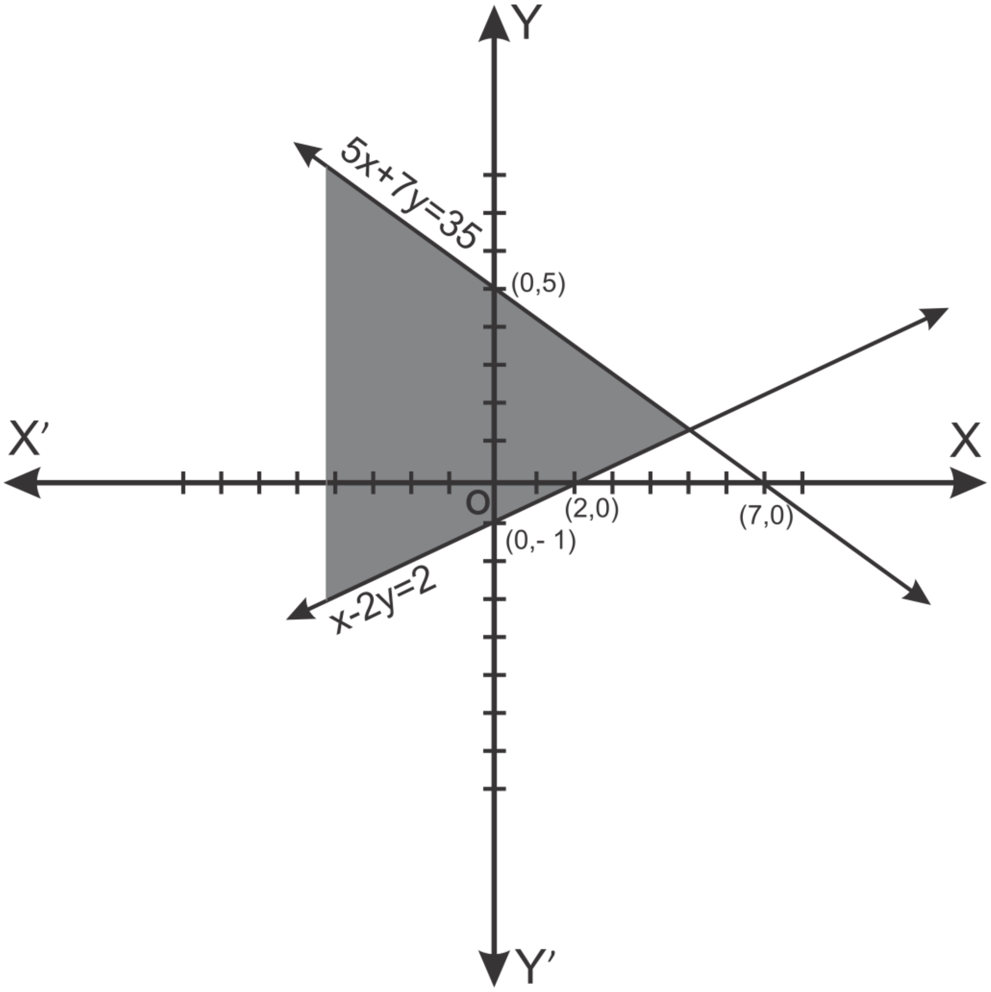
$5(0)+7(0) \le 35$

$0 \le 35$ (True)

$0-2(0) \le 2$

$0 \le 2$ (True)

Solution regions lie towards origin


Y
X'
X
Y'
O
5x+7y=35
x-2y=2
(0,5)
(2,0)
(7,0)
(0,- 1)

Chapter # 5

Linear Equations and Inequalities

Exercise # 5.2

Question # 1: Maximize $f(x, y) = 2x + 5y$; subject to the constraints:

$$2y - x \leq 8 \quad ; \quad x - y \leq 4 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$2y - x = 8$ ______(A)

$x - y = 4$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$2(0) - x = 8$

$-x = 8$

$x = -8$

$P_1\ (-8,0)$

$x - 0 = 4$

$x = 4$

$P_3\ (4,0)$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$2y - 0 = 8$

$2y = 8$

$y = \frac{8}{2}$

$y = 4$

$P_2\ (0,4)$

$0 - y = 4$

$-y = 4$

$y = -4$

$P_4\ (0, -4)$

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$2(0) - 0 \leq 8$

$0 \leq 8$ (True)

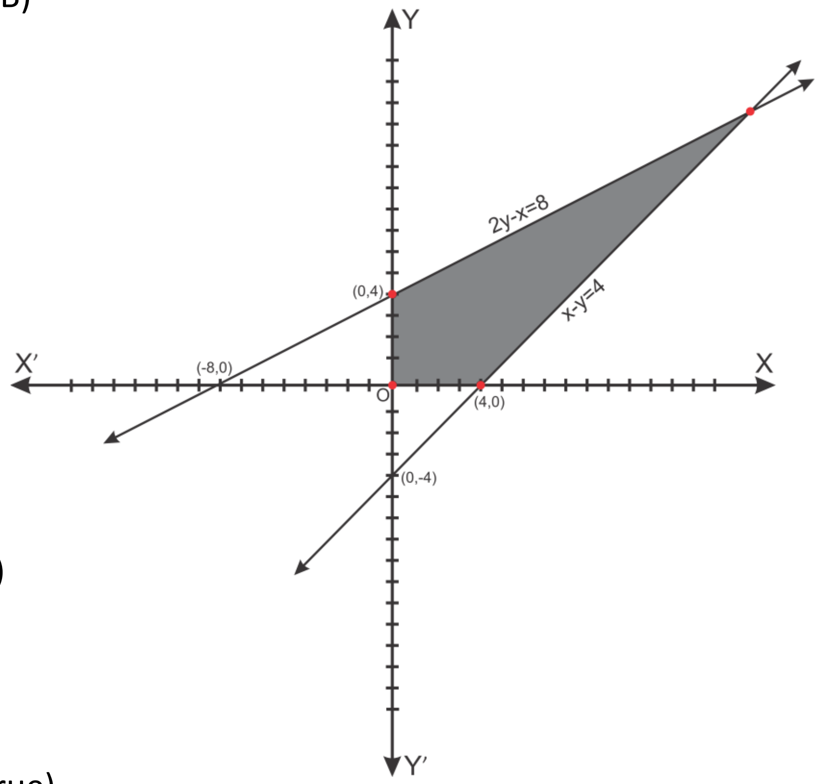
$0 - 0 \leq 4$

$0 \leq 4$ (True)

Solution Regions lie towards the Origin

$\because x \geq 0, y \geq 0$

$\therefore$ **Solution region will be feasible (I-Quadrant)**



(e) Point of Intersection

$$2y - x = 8 \_\_\_\_\_ \text{(A)}$$

$$x - y = 4 \_\_\_\_\_ \text{(B)}$$

(A) + (B)

$$-\cancel{x} + 2y = 8$$

$$\cancel{x} - y = 4$$

$$y = 12$$

Put in equation (B)

$$x - 12 = 4$$

$$x = 4 + 12$$

$$x = 16$$

(f) Corner Points

$(0,0), (0,4), (4,0), (16,12)$

$\because f(x, y) = 2x + 5y$

put $x = 0, y = 0$

$$f(0,0) = 2(0) + 5(0) = 0 + 0 = 0$$

put $x = 0, y = 4$

$$f(0,4) = 2(0) + 5(4) = 0 + 20 = 20$$

put $x = 4, y = 0$

$$f(4,0) = 2(4) + 5(0) = 8 + 0 = 8$$

put $x = 16, y = 12$

$$f(16,12) = 2(16) + 5(12) = 32 + 60 = 92$$

Hence, $f(x, y)$ is maximized at $(16,12)$

Question # 2: Maximize $f(x, y) = x + 3y$; subject to the constraints:

$$2x + 5y \leq 30 \quad ; \quad 5x + 4y \leq 20 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$2x + 5y = 30 \_\_\_\_\_\_$(A)

$5x + 4y = 20 \_\_\_\_\_$(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$$2x + 5(0) = 30$$

$$2x = 30$$

$$x = \frac{30}{2}$$

$$x = 15$$

P_1 $(15,0)$

$$5x + 4(0) = 20$$

$$5x = 20$$

$$x = \frac{20}{5}$$

$$x = 4$$

P_3 $(4,0)$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$2(0)+5y=30$	$5(0)+4y=20$
$5y=30$	$4y=20$
$y=\frac{30}{5}$	$y=\frac{20}{4}$
$y=6$	$y=5$
P_2 (0,6)	P_4 (0,5)

(d) Test Point (0,0)

put $x=0, y=0$ in given inequalities

$2(0)+5(0)\le 30$	$5(0)+4(0)\le 20$
$0\le 30$ (True)	$0\le 20$ (True)

Solution Regions lie towards the Origin

(e) Corner Points

$(0,0),(4,0),(0,5)$

$\because f(x,y)=x+3y$

put $x=0, y=0$

$f(0,0)=0+3(0)=0+0=0$

put $x=4, y=0$

$f(4,0)=4+3(0)=4+0=4$

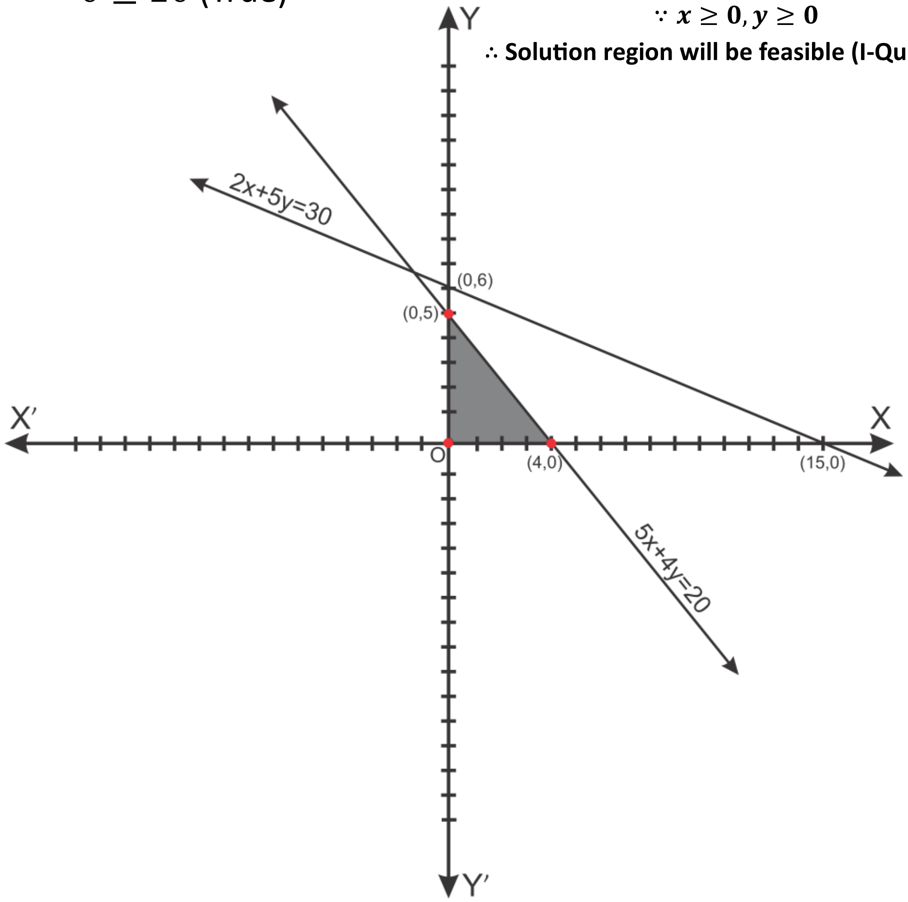
put $x=0, y=5$

$f(0,5)=0+3(5)=0+15=15$

Hence, $f(x,y)$ is maximized at $(0,5)$

$\because x\ge 0, y\ge 0$

$\therefore$ Solution region will be feasible (I-Quadrant)



Question # 3: Maximize $z = 2x + 3y$; subject to the constraints:

$$2x + y \leq 4 \quad ; \quad 4x - y \leq 2 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$2x + y = 4$ ______(A)

$4x - y = 2$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$$2x + 0 = 4$$
$$2x = 4$$
$$x = \frac{4}{2}$$
$$x = 2$$

P₁ (2,0)

$$4x - 0 = 2$$
$$4x = 2$$
$$x = \frac{2}{4}$$
$$x = \frac{1}{2}$$

P₃ $(\frac{1}{2}, 0)$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$$2(0) + y = 4$$
$$0 + y = 4$$
$$y = 4$$

P₂ $(0,4)$

$$4(0) - y = 2$$
$$-y = 2$$
$$y = -2$$

P₄ $(0, -2)$

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

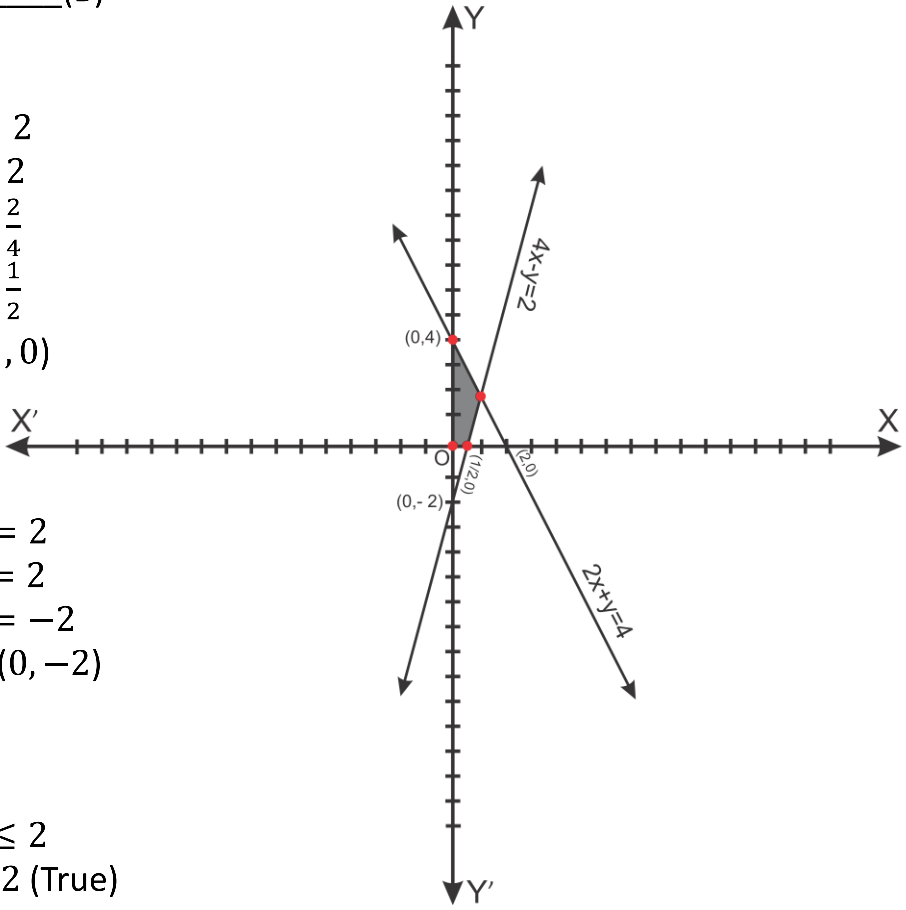
$$2(0) + 0 \leq 4$$
$0 \leq 4$ (True)

$$4(0) - 0 \leq 2$$
$0 \leq 2$ (True)

Solution Regions lie towards the Origin

$\because x \geq 0, y \geq 0$

$\therefore$ Solution region will be feasible (I-Quadrant)



(e) Point of Intersection

$2x + \cancel{y} = 4$ _____ (A)

$4x - \cancel{y} = 2$ _____ (B)

(A) + (B)

$$\begin{array}{r} 2x + y = 4 \\ 4x - y = 2 \\ \hline 6x = 6 \end{array}$$

$x = \frac{6}{6}$

$x = 1$

Put in equation (A)

$2(1) + y = 4$

$2 + y = 4$

$y = 4 - 2$

$y = 2$

(f) Corner Points

$(0,0), (0,4), (1,0), (1,2)$

$\because z = 2x + 3y$

put $x = 0, y = 0$

$z = 2(0) + 3(0) = 0 + 0 = 0$

put $x = 0, y = 4$

$z = 2(0) + 3(4) = 0 + 12 = 12$

put $x = 1, y = 0$

$z = 2(1) + 3(0) = 2 + 0 = 2$

put $x = 16, y = 12$

$z = 2(1) + 3(2) = 2 + 6 = 8$

Hence, z is maximized at $(0,4)$

Question # 4: Maximize $z = 2x + y$; subject to the constraints:

$$x + y \geq 3 \quad ; \quad 7x + 5y \leq 35 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$x + y = 3$ ______(A)

$7x + 5y = 35$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$x + 0 = 3$

$x = 3$

P_1 (3,0)

$7x + 5(0) = 35$

$7x = 35$

$x = \frac{35}{7}$

$x = 5$

P_3 (5,0)

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$$0 + y = 3$$
$$y = 4$$
$P_2\ (0,4)$

$$7(0) + 5y = 35$$
$$5y = 35$$
$$y = \frac{35}{5}$$
$$y = 7$$
$P_4\ (0,7)$

(d) Test Point (0,0)

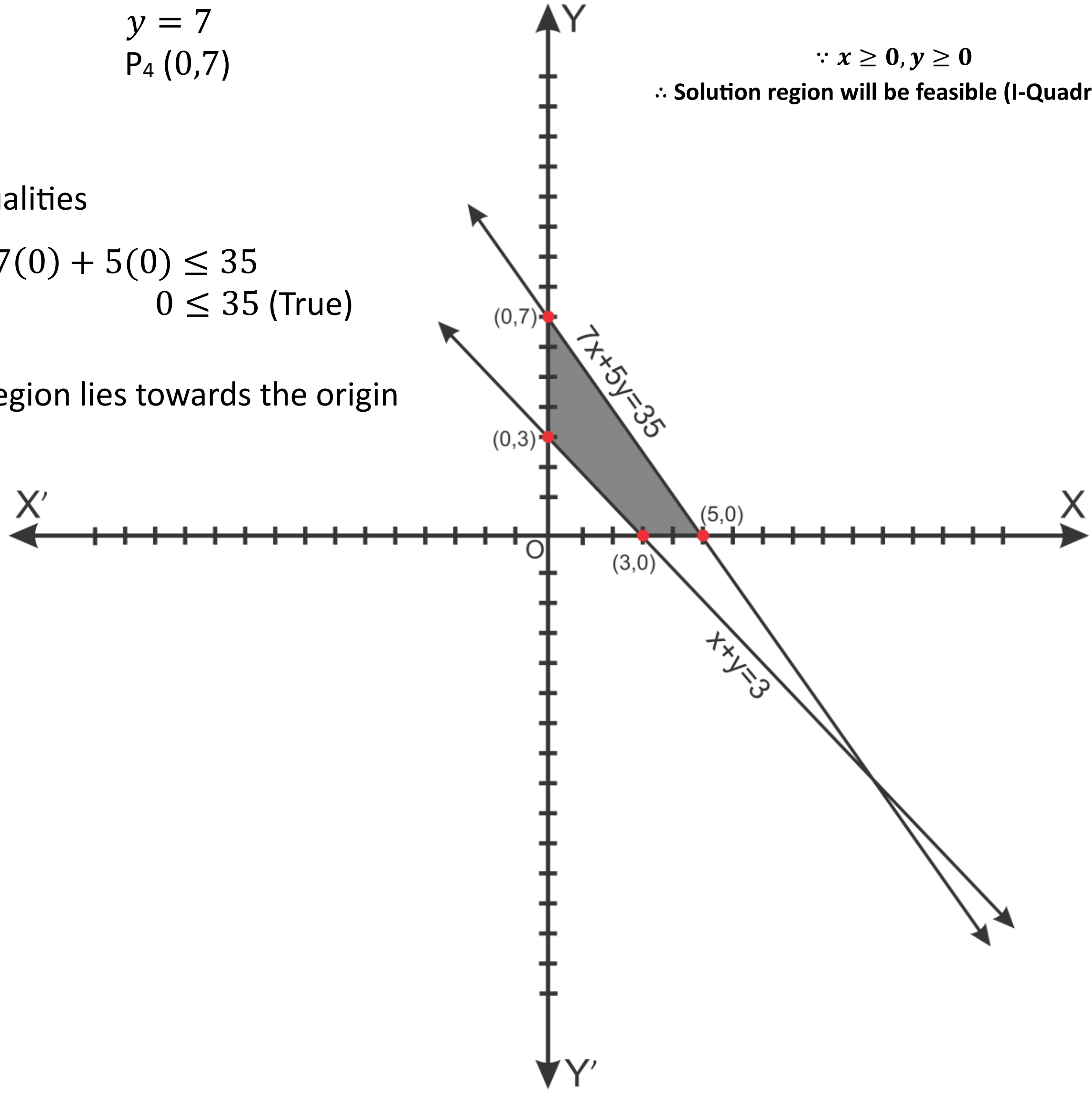
put $x = 0, y = 0$ in given inequalities

$$0 + 0 \geq 3$$
$$0 \geq 4 \text{ (False)}$$

Solution region lies away from the origin

$$7(0) + 5(0) \leq 35$$
$$0 \leq 35 \text{ (True)}$$

Solution region lies towards the origin



$\because x \geq 0, y \geq 0$

∴ Solution region will be feasible (I-Quadrant)

(e) Corner Points

$(3,0), (0,3), (5,0), (0,7)$

$$\because z = 2x + y$$

put $x = 3, y = 0$

$$z = 2(3) + 0 = 6 + 0 = 6$$

put $x = 0, y = 3$

$$z = 2(0) + 3 = 0 + 3 = 3$$

put $x = 5, y = 0$

$$z = 2(5) + 0 = 10 + 0 = 10$$

put $x = 0, y = 7$

$$z = 2(0) + 7 = 0 + 7 = 7$$

Hence, z is minimized at $(0,3)$

Question # 5: Maximize the function defined as: $f(x,y) = 2x + 3y$; subject to the constraints:

$$\mathbf{2x + y \le 10 \quad ; \quad x + 2y \le 14 \quad ; \quad x \ge 0; y \ge 0}$$

(a) Associated Equations

$2x + y = 10$ ______(A)

$x + 2y = 14$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$$2x + 0 = 10$$
$$2x = 10$$
$$x = \frac{10}{2}$$
$$x = 5$$

P_1 (5,0)

$$x + 2(0) = 14$$
$$x + 0 = 14$$
$$x = 14$$

P_3 (5,0)

(c) y – Intercept

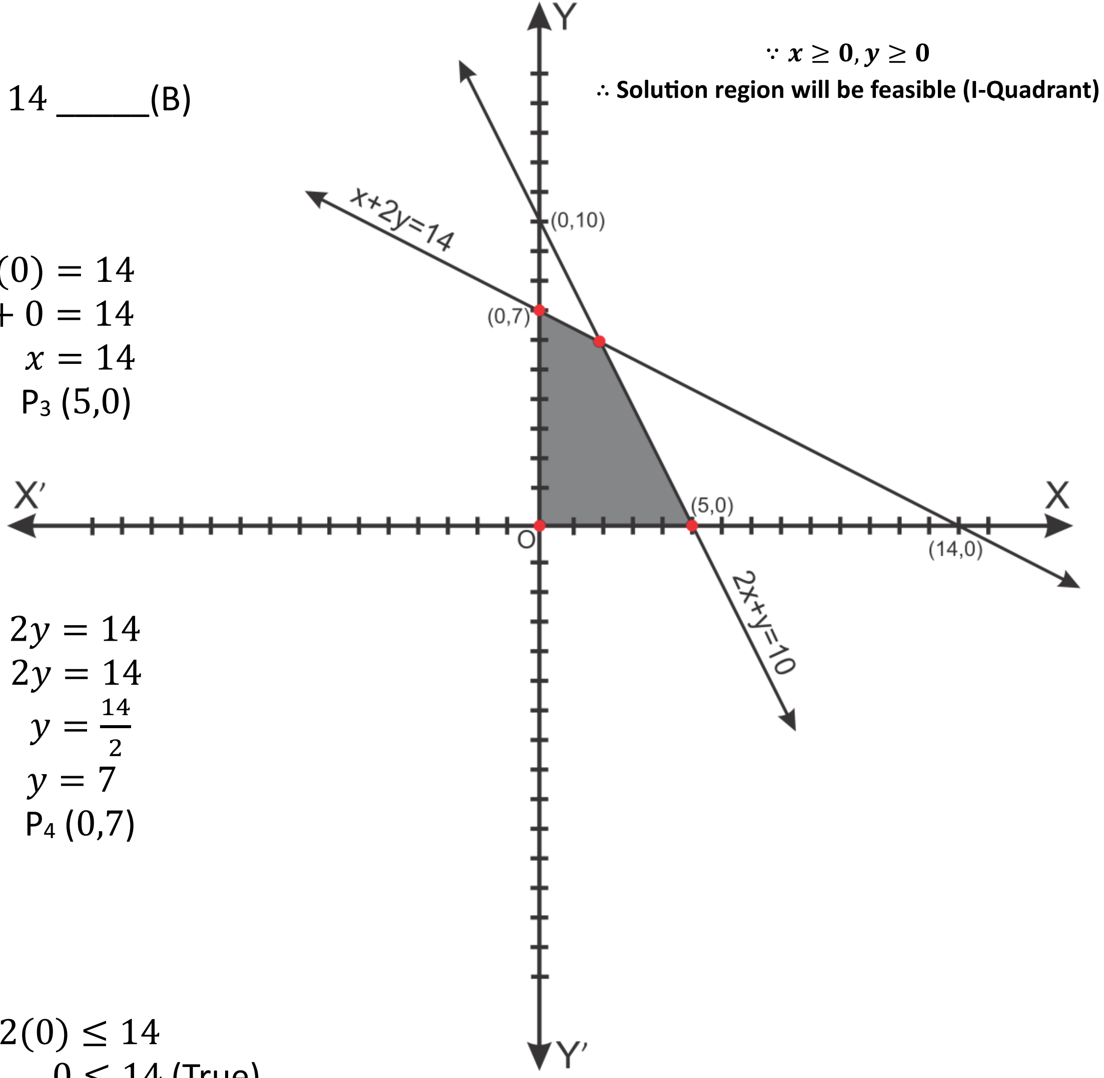
put $x = 0$ in equations (A) and (B)

$$2(0) + y = 10$$
$$0 + y = 10$$
$$y = 10$$

P_2 (0,10)

$$0 + 2y = 14$$
$$2y = 14$$
$$y = \frac{14}{2}$$
$$y = 7$$

P_4 (0,7)



$\because x \ge 0, y \ge 0$

$\therefore$ **Solution region will be feasible (I-Quadrant)**

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$$2(0) + 0 \le 10$$
$0 \le 10$ (True)

$$0 + 2(0) \le 14$$
$0 \le 14$ (True)

Solution Regions lie towards the origin

(e) Point of Intersection

$2x + y = 10\ ; x + 2y = 14$ ______ (D)

Multiply by '2'

$4x + 2y = 20$ ________ (C)

(C) − (D)

$$\begin{array}{r} 4x + \cancel{2y} = \ \ 20 \\ \pm x \pm \cancel{2y} = \pm 14 \\ \hline 3x = 6 \end{array}$$

$x = \frac{6}{3}$

$x = 2$

Put in equation (D)

$2 + 2y = 14$

$2y = 14 - 2$

$2y = 12$

$y = \frac{12}{2}$

$y = 6$

(f) Corner Points

$(0,0), (5,0), (0,7), (2,6)$

$\because f(x, y) = 2x + 3y$

put $x = 0, y = 0$

$f(0,0) = 2(0) + 3(0) = 0 + 0 = 0$

put $x = 5, y = 0$

$f(5,0) = 2(5) + 3(0) = 10 + 0 = 10$

put $x = 0, y = 7$

$f(0,7) = 2(0) + 3(7) = 0 + 21 = 21$

put $x = 2, y = 6$

$f(2,6) = 2(2) + 3(6) = 4 + 18 = 22$

Hence, z is maximized at $(2,6)$

Question # 6: Find minimum and maximum values of $z = 3x + y$; subject to the constraints:

$$3x + 5y \geq 15 \quad ; \quad x + 3y \leq 9 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$3x + 5y = 15$ ______(A)

$x + 3y = 9$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$3x + 5(0) = 15$

$3x = 15$

$x = \frac{15}{3}$

$x = 5$

$P_1\ (5,0)$

$x + 3(0) = 9$

$x + 0 = 9$

$x = 9$

$P_3\ (9,0)$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$$3(0) + 5y = 15$$
$$5y = 15$$
$$y = \frac{15}{5}$$
$$y = 3$$
$P_2\,(0,3)$

$$0 + 3y = 9$$
$$3y = 9$$
$$y = \frac{9}{3}$$
$$y = 3$$
$P_4\,(0,3)$

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$$3(0) + 5(0) \geq 15$$
$0 \geq 15$ (False)

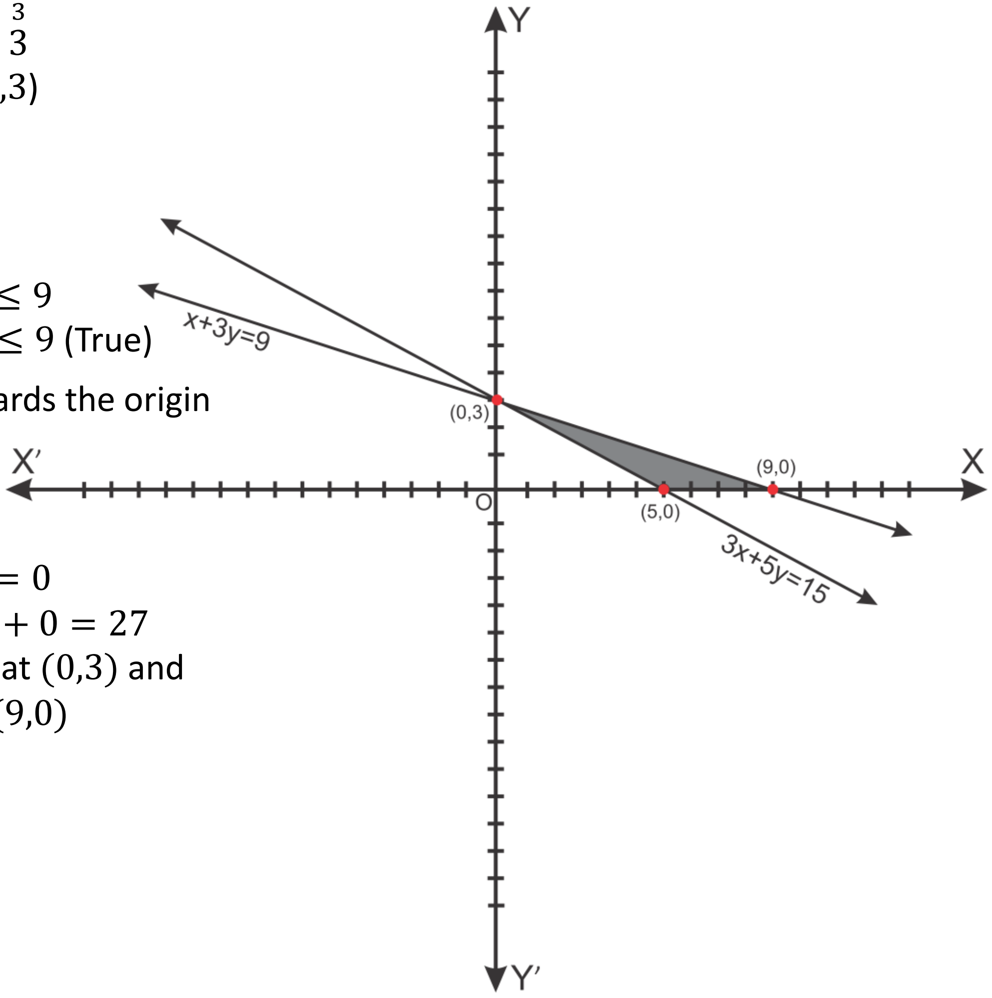
Solution region lies away from the origin

$$0 + 3(0) \leq 9$$
$0 \leq 9$ (True)

Solution region lies towards the origin

$\because x \geq 0, y \geq 0$

$\therefore$ Solution region will be feasible (I-Quadrant)



(e) (Croner Points)

$(0,3), (5,0), (9,0)$

$\because z = 3x + y$

put $x = 0, y = 3$

$$z = 3(0) + 3 = 0 + 3 = 3$$

put $x = 5, y = 0$

$$z = 3(5) + 0 = 15 + 0 = 15$$

put $x = 9, y = 0$

$$z = 3(9) + 0 = 27 + 0 = 27$$

Hence, z is minimized at $(0,3)$ and maximized at $(9,0)$

Chapter # 5

Linear Equations and Inequalities

Review Exercise # 5

Question # 1: Four options are given against each statement. Encircle the correct one.

#	Answer	#	Answer
i	C	**vi**	B
ii	C	**vii**	B
iii	C	**viii**	C
iv	D	**ix**	B
v	B	**x**	B

Question # 2: Solve and represent their solution on real line.

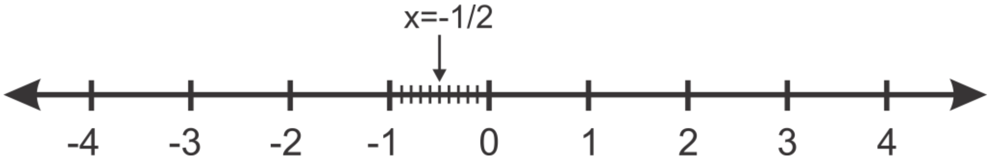
(i) $\boldsymbol{\frac{x+5}{3} = 1 - x}$ __________ (A)

$$x + 5 = 3(1 - x)$$
$$x + 5 = 3 - 3x$$
$$x + 3x = 3 - 5$$
$$4x = -2$$
$$x = \frac{-\cancel{2}^{1}}{\cancel{4}_{2}}$$
$$x = \frac{-1}{2}$$

Check:

put $x = \frac{-1}{2}$ in equation (A)

$$\frac{\frac{-1}{2}+5}{3} = 1 - \left(-\frac{1}{2}\right)$$
$$\frac{9/2}{3} = 1 + \frac{1}{2}$$
$$\frac{^{3}\cancel{9}}{2} \times \frac{1}{\cancel{3}_{1}} = \frac{3}{2}$$
$$\frac{3}{2} = \frac{3}{2}$$
$$\text{S.S} = \left\{-\frac{1}{2}\right\}$$



(ii) $\boldsymbol{\frac{2x+1}{3} + \frac{1}{2} = 1 - \frac{x-1}{3}}$ __________(A)

Multiply by ‘6’ on both sides

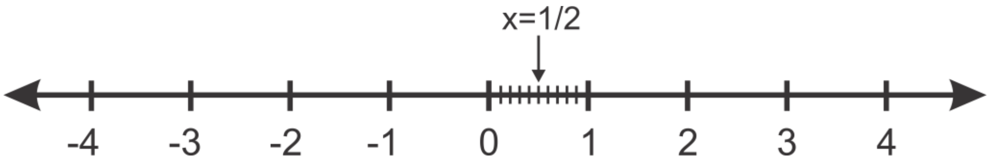
$$^{2}\cancel{6} \times \frac{2x+1}{\cancel{3}} + {}^{3}\cancel{6} \times \frac{1}{\cancel{2}} = 6 \times 1 - {}^{2}\cancel{6} \times \frac{x-1}{\cancel{3}}$$
$$2(2x + 1) + 3 = 6 - 2(x - 1)$$
$$4x + 2 + 3 = 6 - 2x + 2$$
$$4x + 5 = 8 - 2x$$
$$4x + 2x = 8 - 5$$
$$6x = 3$$

Check:

put $x = \frac{1}{2}$ in equation (A)

$$\frac{2\left(\frac{1}{2}\right)+1}{3} = 1 - \frac{\frac{1}{2}-1}{3}$$
$$\frac{1+1}{3} + \frac{1}{2} = 1 - \frac{-\frac{1}{2}}{3}$$
$$\frac{2}{3} + \frac{1}{2} = 1 + \frac{1}{2} \times \frac{1}{3}$$
$$\frac{4+3}{6} = 1 + \frac{1}{6}$$

$x = \frac{\cancel{3}^{1}}{\cancel{6}_{2}}$	$\frac{7}{6} = \frac{7}{6}$
$x = \frac{1}{2}$	S.S $= \left\{\frac{1}{2}\right\}$



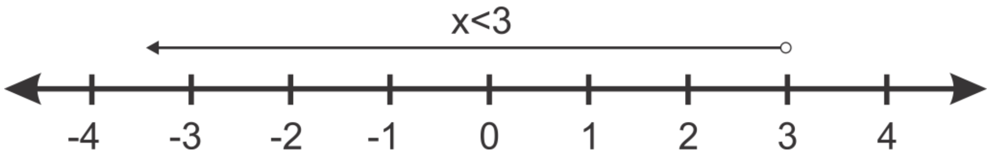
(iii) $\mathbf{3x + 7 < 16}$

$3x < 16 - 7$

$3x < 9$

$x < \frac{\cancel{9}^{3}}{\cancel{3}}$

$x < 3$



(iv) $\mathbf{5(x - 3) \geq 26x - (10x + 4)}$

$5x - 15 \geq 26x - 10x - 4$

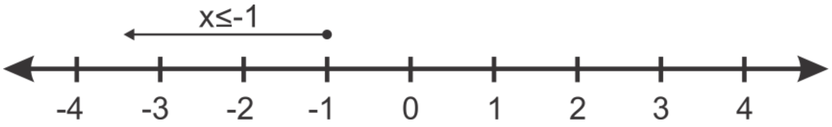
$5x - 15 \geq 16x - 4$

$-15 + 4 \geq 16x - 5x$

$-11 \geq 11x$

$-\frac{{}^{1}\cancel{11}}{\cancel{11}} \geq x$

$-1 \geq x$ OR $x \leq -1$



Question # 3: Find the solution region of the following linear inequalities:

(i) $\mathbf{3x - 4y \leq 12}$; $\mathbf{3x + 2y \geq 3}$

(a) Associated Equations

$3x - 4y = 12$ ______(A)	$3x + 2y = 3$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$3x - 4(0) = 12$	$3x + 2(0) = 3$
$3x = 12$	$3x = 3$
$x = \frac{12}{3}$	$x = \frac{3}{3}$
$x = 4$	$x = 1$
P_1 (4,0)	P_3 (1,0)

(c) y – Intercept

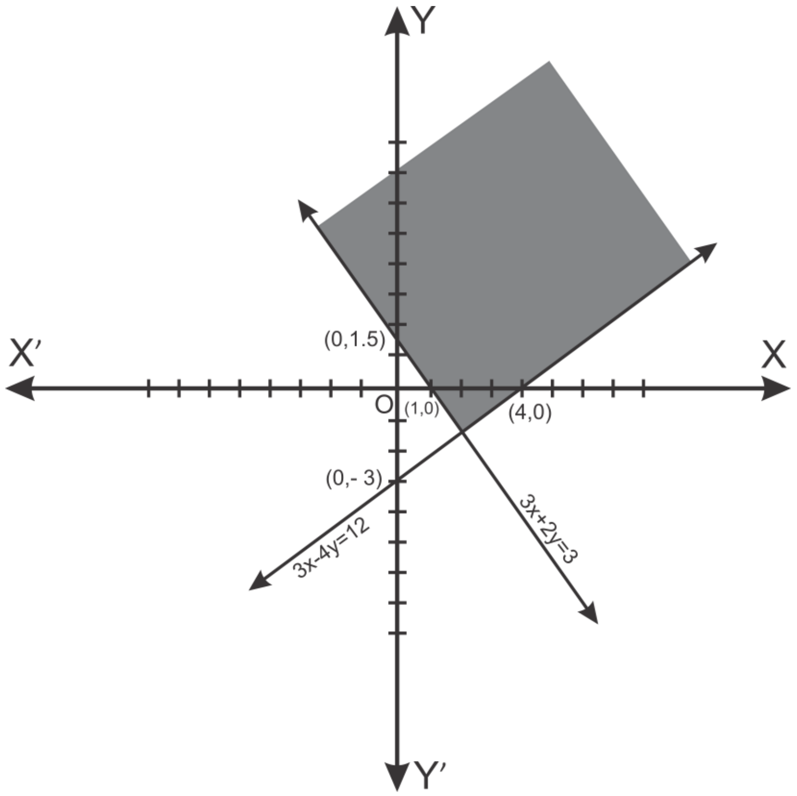
put $x = 0$ in equations (A) and (B)

$3(0) - 4y = 12$	$3(0) + 2y = 3$
$-4y = 12$	$2y = 3$
$y = \frac{12}{-4}$	$y = \frac{3}{2}$
$y = -3$	P_4 (0,1.5)
P_2 $(0, -3)$	

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$3(0) - 4(0) \leq 12$	$3(0) + 2(0) \geq 3$
$0 \leq 12$ (True)	$0 \geq 3$ (False)
Solution region lies towards the origin	Solution region lies away from the origin



(ii) $\mathbf{2x + y \leq 4 \quad ; \quad x + 2y \leq 6}$

(a) **Associated Equations**

$2x + y = 4$ ______(A)

$x + 2y = 6$ _____(B)

(b) **x – Intercept**

put $y = 0$ in equations (A) and (B)

$2x + 0 = 4$
$2x = 4$
$x = \frac{4}{2}$
$x = 2$
P_1 (2,0)

$x + 2(0) = 6$
$x + 0 = 6$
$x = 6$
P_3 (6,0)

(c) **y – Intercept**

put $x = 0$ in equations (A) and (B)

$2(0) + y = 4$
$y = 4$
P_2 (0,4)

$0 + 2y = 6$
$2y = 6$
$y = \frac{6}{2}$
$y = 3$
P_4 (0,3)

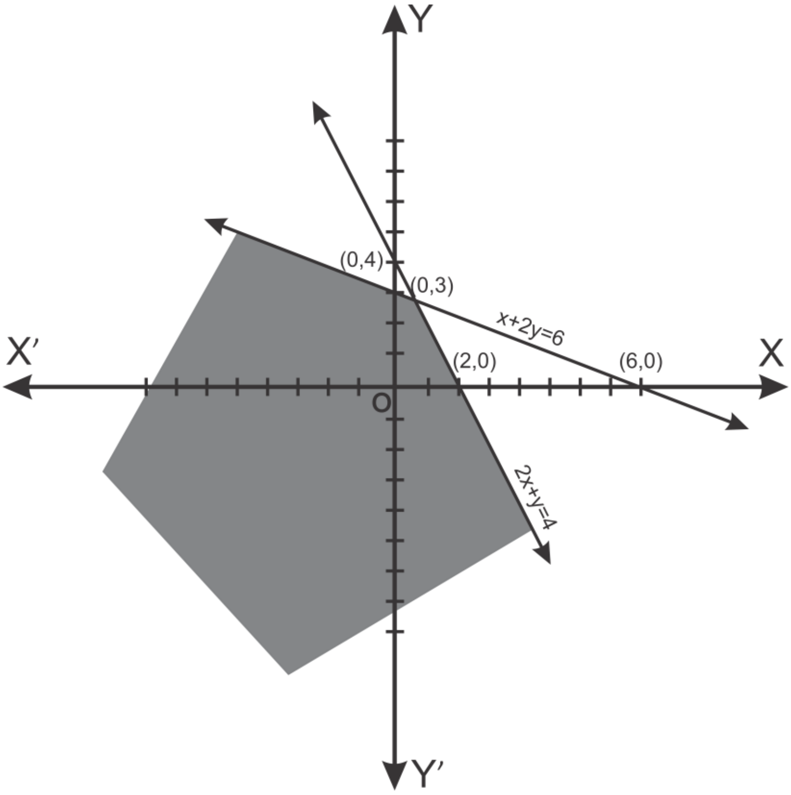
(d) **Test Point (0,0)**

put $x = 0, y = 0$ in given inequalities

$2(0) + 0 \leq 4$
$0 \leq 4$ (True)

$0 + 2(0) \leq 6$
$0 \leq 6$ (True)

Solution regions lie towards the origin



Question # 4: Find the maximum value of $g(x,y) = x + 4y$ subject to constraints:

$$\boldsymbol{x + y \leq 4} \quad ; \quad \boldsymbol{x \geq 0; y \geq 0}$$

(a) Associated Equations

$x + y = 4$ ______(A)

(b) x – Intercept

put $y = 0$ in equation (A)

$$x + 0 = 4$$

$$x = 4$$

$P_1\ (4,0)$

(c) y – Intercept

put $x = 0$ in equation (A)

$$0 + y = 4$$

$$y = 4$$

$P_2\ (0,4)$

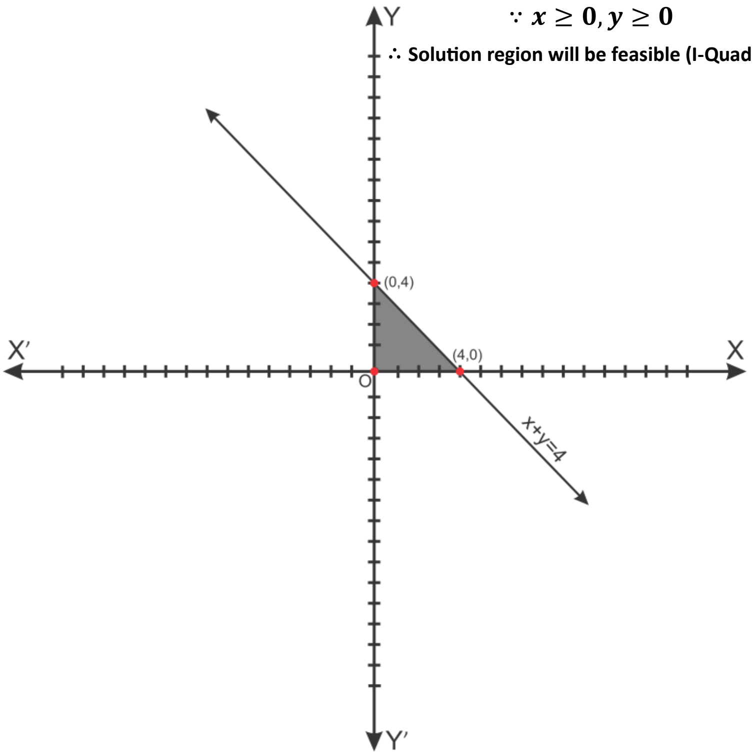
(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequality

$$0 + 0 \leq 4$$

$$0 \leq 4 \text{ (True)}$$

Solution Region lies towards the Origin



$\because \boldsymbol{x \geq 0, y \geq 0}$

∴ **Solution region will be feasible (I-Quadrant)**

(e) Corner Points

$$(0,0), (4,0), (0,4)$$

$$\because g(x,y) = x + 4y$$

put $x = 0, y = 0$

$$g(0,0) = 0 + 4(0) = 0 + 0 = 0$$

put $x = 4, y = 0$

$$g(4,0) = 4 + 4(0) = 4 + 0 = 4$$

put $x = 0, y = 4$

$$g(0,4) = 0 + 4(4) = 0 + 16 = 16$$

Hence, $g(x,y)$ is maximized at $(0,4)$

Question # 5: Find the maximum value of $f(x,y) = 3x + 5y$ subject to the constraints:

$$x + 3y \geq 3 \quad ; \quad x + y \geq 2 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$x + 3y = 3$ ______(A)

$x + y = 2$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$x + 3(0) = 3$

$x + 0 = 3$

$x = 3$

P_1 (3,0)

$x + 0 = 2$

$x = 2$

P_3 (2,0)

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$0 + 3y = 3$

$3y = 3$

$y = \frac{3}{3}$

$y = 1$

P_2 (0,1)

$0 + y = 2$

$y = 2$

P_4 (0,2)

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

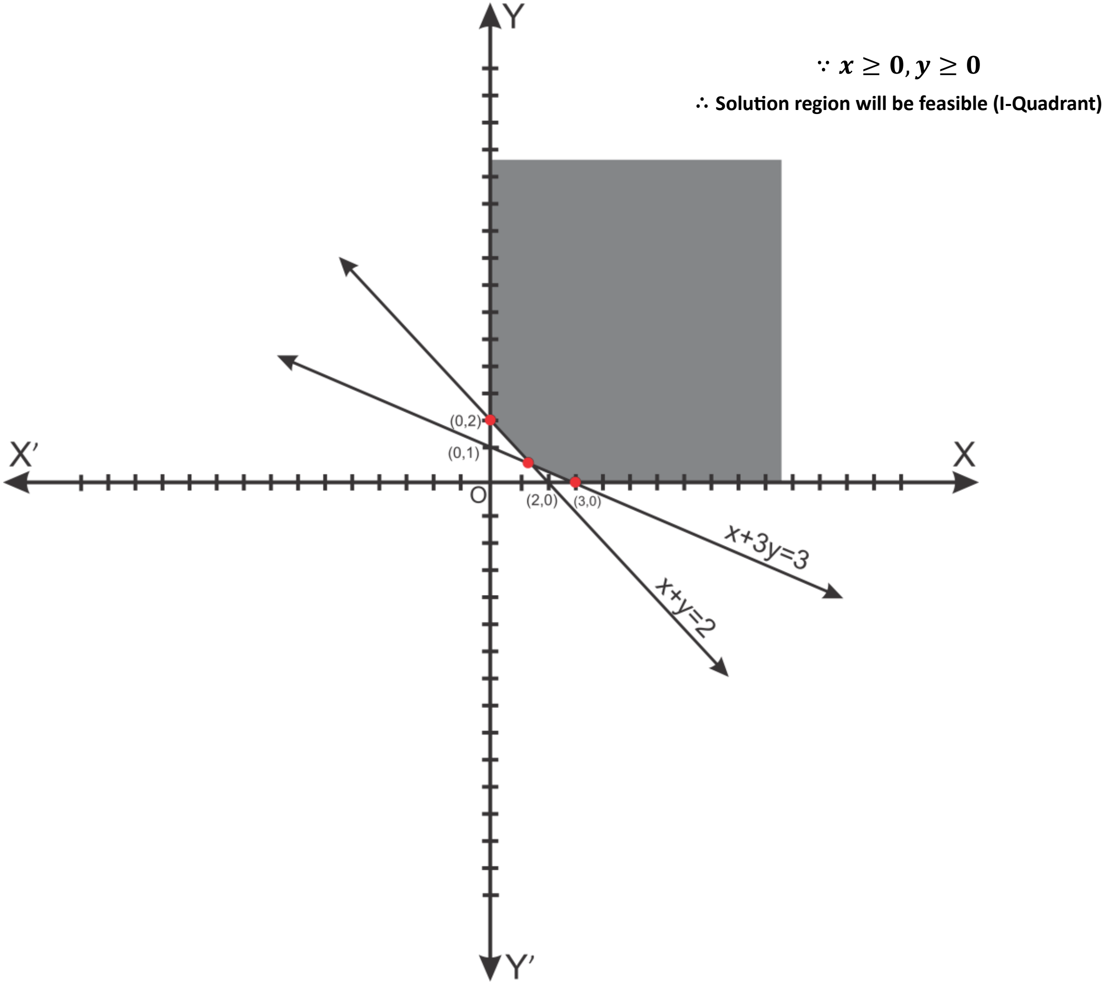
$0 + 3(0) \geq 3$

$0 \geq 3$ (True)

$0 + 0 \geq 2$

$0 \geq 2$ (True)

Solution Regions lie away from the Origin



$\because x \geq 0, y \geq 0$

∴ Solution region will be feasible (I-Quadrant)

(e) Point of Intersection

$$x + 3y = 3 \text{ \_\_\_\_\_\_ (A)}$$

$$x + y = 2 \text{ \_\_\_\_\_\_ (B)}$$

(A) − (B)

$$\cancel{x} + 3y = 3$$

$$\pm\cancel{x} \pm y = \pm 2$$

$$2y = 1$$

$$y = \frac{1}{2}$$

Put in equation (B)

$$x + \frac{1}{2} = 2$$

$$x = 2 - \frac{1}{2}$$

$$x = \frac{3}{2}$$

(f) Corner Points

$$(3,0), (0,2), \left(\frac{3}{2}, \frac{1}{2}\right)$$

$$\because f(x, y) = 3x + 5y$$

put $x = 3, y = 0$

$$f(0,0) = 3(3) + 5(0) = 9 + 0 = 9$$

put $x = 0, y = 2$

$$f(5,0) = 3(0) + 5(2) = 0 + 10 = 10$$

put $x = \frac{3}{2}, y = \frac{1}{2}$

$$f(0,7) = 3\left(\frac{3}{2}\right) + 5\left(\frac{1}{2}\right) = \frac{9}{2} + \frac{5}{2} = \frac{14}{2} = 7$$

Hence, $f(x, y)$ is minimized at $\left(\frac{3}{2}, \frac{1}{2}\right)$