Unit 2 Logarithms

EXERCISE 2.1

1. Express the following numbers in scientific notation:

(i) 2000000 (ii) 48900 (iii) 0.0042
(iv) 0.0000009 (v) 73×10^3 (vi) 0.65×10^2

Solution

(i) 2×10^6 (ii) 4.89×10^4 (iii) 4.2×10^{-3} (iv) 9×10^{-7} (v) 7.3×10^4

(vi) 6.5×10^1

2. Express the following numbers in ordinary notation:

(i) 8.04×10^2 (ii) 3×10^5 (iii) 1.5×10^{-2}
(iv) 1.77×10^7 (v) 5.5×10^{-6} (vi) 4×10^{-5}

Solution

(i) 804 (ii) 300000 (iii) 0.015 (iv) 17700000 (v) 0.0000055 (vi) 0.00004

3. The speed of light is approximately 3×10^8 metres per second. Express it in standard form.
4. The circumference of the Earth at the equator is about 40075000 metres. Express this number in scientific notation.
5. The diameter of Mars is 6.7779×10^3 km. Express this number in standard form.
6. The diameter of Earth is about 1.2756×10^4 km. Express this number in standard form.

Solution

3. 300,000,000 m/sec **4.** 4.0075×10^7 m

5. 6779 km **6.** 12756 km

EXERCISE 2.2

1. Express each of the following in logarithmic form:

(i) $10^3 = 1000$ (ii) $2^8 = 256$ (iii) $3^{-3} = \frac{1}{27}$

(iv) $20^2 = 400$ (v) $16^{-\frac{1}{4}} = \frac{1}{2}$ (vi) $11^2 = 121$

(vii) $p = q^r$ (viii) $(32)^{\frac{-1}{5}} = \frac{1}{2}$

Solution: $\mathbf{\log_b(x) = y \Leftrightarrow b^y = x \quad ; b > 0, x > 0, b \neq 1}$

(i) $\log_{10} 1000 = 3$ (ii) $\log_2 256 = 8$ (iii) $\log_3 \frac{1}{27} = -3$ (iv) $\log_{20} 400 = 2$

(v) $\log_{16} \frac{1}{2} = -\frac{1}{4}$ (vi) $\log_{11} 121 = 2$ (vii) $\log_q p = r$ (viii) $\log_{32} \frac{1}{2} = -\frac{1}{5}$

2. Express each of the following in exponential form:

(i) $\log_5 125 = 3$ (ii) $\log_2 16 = 4$ (iii) $\log_{23} 1 = 0$

(iv) $\log_5 5 = 1$ (v) $\log_2 \frac{1}{8} = -3$ (vi) $\frac{1}{2} = \log_9 3$

(vii) $5 = \log_{10} 100000$ (viii) $\log_4 \frac{1}{16} = -2$

Solution: $\mathbf{\log_b(x) = y \Leftrightarrow b^y = x \quad ; b > 0, x > 0, b \neq 1}$

(i) $5^3 = 125$ (ii) $2^4 = 16$ (iii) $23^0 = 1$ (iv) $5^1 = 5$

(v) $2^{-3} = \frac{1}{8}$ (vi) $9^{\frac{1}{2}} = 3$ (vii) $10^5 = 100000$ (viii) $4^{-2} = \frac{1}{16}$

3. Find the value of x in each of the following:

(i)	$\log_x 64 = 3$	(ii)	$\log_5 1 = x$	(iii)	$\log_x 8 = 1$
(iv)	$\log_{10} x = -3$	(v)	$\log_4 x = \frac{3}{2}$	(vi)	$\log_2 1024 = x$

Solution: $\mathbf{\log_b(x) = y \Leftrightarrow b^y = x \quad ; b > 0, x > 0, b \neq 1}$

i. $\log_x 64 = 3 \Rightarrow x^3 = 64 \Rightarrow x^3 = 4^3 \Rightarrow \mathbf{x = 4}$

ii. $\log_5 1 = x \Rightarrow 5^x = 1 \Rightarrow 5^x = 5^0 \Rightarrow \mathbf{x = 0}$

iii. $\log_x 8 = 1 \Rightarrow x^1 = 8 \Rightarrow \mathbf{x = 8}$

iv. $\log_{10} x = -3 \Rightarrow 10^{-3} = x \Rightarrow x = \frac{1}{10^3} \Rightarrow \mathbf{x = \frac{1}{1000}}$

v. $\log_4 x = \frac{3}{2} \Rightarrow 4^{\frac{3}{2}} = x \Rightarrow x = (2^2)^{\frac{3}{2}} \Rightarrow x = 2^3 \Rightarrow \mathbf{x = 8}$

vi. $\log_2 1024 = x \Rightarrow 2^x = 1024 \Rightarrow 2^x = 2^{10} \Rightarrow \mathbf{x = 10}$

EXERCISE 2.3

1. Find characteristic of the following numbers:

(i)	5287	(ii)	59.28	(iii)	0.0567
(iv)	234.7	(v)	0.000049	(vi)	145000

Solution

(i) 3 (ii) 1 (iii) -2 (iv) 2 (v) -5 (vi) 5

2. Find logarithm of the following numbers:

(i)	43	(ii)	579	(iii)	1.982
(iv)	0.0876	(v)	0.047	(vi)	0.000354

Solution

i. $\log 43 = 1.6335$ Characteristic $= 1$, Mantissa $= 0.6335$

ii. $\log 579 = 2.7627$ Characteristic $= 2$, Mantissa $= 0.7627$

iii. $\log 19.82 = 1.2971$ Characteristic $= 1$, Mantissa $= 0.2971$

iv. $\log 0.0876 = -2 + 0.9425 = -1.0575$ Characteristic $= -2$, Mantissa $= 0.9425$

v. $\log 0.047 = -2 + 0.6721 = -1.3279$ Characteristic $= -2$, Mantissa $= 0.6721$

vi. $\log 0.000354 = -4 + 0.5490 = -3.4518$ Characteristic $= -4$, Mantissa $= 0.5490$

3. If $\log 3.177 = 0.5019$, then find:

(i)	log 3177	(ii)	log 31.77	(iii)	log 0.03177

Solution

i. $\log 3177 = 3.5019$ Characteristic $= 3$, Mantissa $= 0.5019$

ii. $\log 31.77 = 1.5019$ Characteristic $= 1$, Mantissa $= 0.5019$

iii. $\log 0.03177 = -2 + 0.5019 = -1.4981$ Characteristic $= -2$, Mantissa $= 0.5019$

4. Find the value of x.

(i) $\log x = 0.0065$ (ii) $\log x = 1.192$ (iii) $\log x = -3.434$

(iv) $\log x = -1.5726$ (v) $\log x = 4.3561$ (vi) $\log x = -2.0184$

Solution

i. $\log x = 0.0065 \Rightarrow x = \text{antilog}(0.0065) \Rightarrow x = 1.015$

ii. $\log x = 1.192 \Rightarrow x = \text{antilog}(1.192) \Rightarrow x = 15.56$

iii. $\log x = -3.434 \Rightarrow \log x = -4 + 4 - 3.434 \Rightarrow x = \text{antilog}(\bar{4}.566)$

$\Rightarrow x = 0.0003681$

iv. $\log x = -1.5726 \Rightarrow \log x = -2 + 2 - 1.5726 \Rightarrow x = \text{antilog}(\bar{2}.4274)$

$\Rightarrow x = 0.02675$

v. $\log x = 4.3561 \Rightarrow x = \text{antilog}(4.3561) \Rightarrow x = 2270$

vi. $\log x = -2.0184 \Rightarrow \log x = -3 + 3 - 2.0184 \Rightarrow x = \text{antilog}(\bar{3}.9816)$

$\Rightarrow x = 0.009585$

EXERCISE 2.4

1. Without using calculator, evaluate the following:

(i) $\log_2 18 - \log_2 9$ (ii) $\log_2 64 + \log_2 2$ (iii) $\frac{1}{3}\log_3 8 - \log_3 18$

(iv) $2\log 2 + \log 25$ (v) $\frac{1}{3}\log_4 64 + 2\log_5 25$ (vi) $\log_3 12 + \log_3 0.25$

Solution

i. $\log_2 18 - \log_2 9 = \log_2(2 \times 9) - \log_2 9 = \log_2 2 + \log_2 9 - \log_2 9$

$= \log_2 2 = \mathbf{1}$

ii. $\log_2 64 + \log_2 2 = \log_2(2 \times 2 \times 2 \times 2 \times 2 \times 2) + \log_2 2$

$= \log_2(2^6) + \log_2 2 = 6\log_2 2 + \log_2 2 = 7\log_2 2 = 7(1) = \mathbf{7}$

iii. $\frac{1}{3}\log_3 8 - \log_3 18 = \frac{1}{3}\log_3(2 \times 2 \times 2) - \log_3(2 \times 3 \times 3)$

$= \frac{1}{3}\log_3(2^3) - \log_3(2 \times 3^2) = \frac{3}{3}\log_3 2 - \log_3 2 - 2\log_3 3$

$= \log_3 2 - \log_3 2 - 2\log_3 3 = -2(1) = \mathbf{-2}$

iv. $2\log 2 + \log 25 = 2\log 2 + \log(5^2) = 2\log 2 + 2\log 5 = 2(\log 2 + \log 5)$

$= 2\log(2 \times 5) = 2\log 10 = 2(1) = \mathbf{2}$

v. $\frac{1}{3}\log_4 64 + 2\log_5 25 = \frac{1}{3}\log_4(4^3) + 2\log_5(5^2) = \frac{3}{3}\log_4 4 + 2 \times 2\log_5 5$

$= \log_4 4 + 4\log_5 5 = (1) + 4(1) = 1 + 4 = \mathbf{5}$

vi. $\log_3 12 + \log_3 0.25 = \log_3 12 + \log_3 \frac{25}{100} = \log_3 12 + \log_3 \frac{1}{4} = \log_3 \frac{12}{4}$

$= \log_3 3 = \mathbf{1}$

2. Write the following as a single logarithm:

(i) $\frac{1}{2}\log 25 + 2\log 3$ (ii) $\log 9 - \log\frac{1}{3}$

(iii) $\log_5 b^2 . \log_a 5^3$ (iv) $2\log_3 x + \log_3 y$

(v) $4\log_5 x - \log_5 y + \log_5 z$ (vi) $2\ln a + 3\ln b - 4\ln c$

Solution

i. $\frac{1}{2}\log 25 + 2\log 3 = \frac{1}{2}\log(5^2) + \log(3^2) = \log 5 + \log 9 = \log(5 \times 9) = \mathbf{\log 45}$

ii. $\log 9 - \log\frac{1}{3} = \log\left(\frac{9}{\frac{1}{3}}\right) = \log(9 \times 3) = \mathbf{\log 27}$

iii. $\log_5 b^2 . \log_a 5^3 = 2\log_5 b \times 3\log_a 5 = 2\frac{\log_a b}{\cancel{\log_a} 5} \times 3\frac{\log_a 5}{\cancel{\log_a} a} = 6\frac{\log_a b}{(1)} = \mathbf{6\log_a b}$

vi. $2\log_3 x + \log_3 y = \log_3(x^2) + \log_3 y = \mathbf{\log_3 x^2 y}$

vi. $2\ln a + 3\ln b - 4\ln c = \ln a^2 + \ln b^3 - \ln c^4 = \mathbf{\ln\frac{a^2 b^3}{c^4}}$

3. Expand the following using laws of logarithms:

(i) $\log\left(\frac{11}{5}\right)$ (ii) $\log_5\sqrt{8a^6}$ (iii) $\ln\left(\frac{a^2 b}{c}\right)$

(iv) $\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}$ (v) $\ln\sqrt[3]{16x^3}$ (vi) $\log_2\left(\frac{1-a}{b}\right)^5$

Solution

i. $\log\left(\frac{11}{5}\right) = \mathbf{\log 11 - \log 5}$

ii. $\log_5\sqrt{8a^6} = \log_5(2^3 \times a^6)^{\frac{1}{2}} = \log_5\left(2^{\frac{3}{2}} \times a^3\right) = \mathbf{\frac{3}{2}\log_5 2 + 3\log_5 a}$

iii. $\ln\left(\frac{a^2 b}{c}\right) = \ln a^2 + \ln b - \ln c = \mathbf{2\ln a + \ln b - \ln c}$

iv. $\ln\left(\frac{xy}{z}\right)^{\frac{1}{9}} = \frac{\mathbf{1}}{\mathbf{9}}\ln\left(\frac{xy}{z}\right) = \frac{\mathbf{1}}{\mathbf{9}}[\mathbf{\ln x + \ln y - \ln z}]$

v. $\ln\sqrt[3]{16x^3} = \ln(2^4 \times x^3)^{\frac{1}{3}} = \ln\left(2^{\frac{4}{3}} \times x\right) = \mathbf{\frac{4}{3}\ln 2 + \ln x}$

vi. $\log_2\left(\frac{1-a}{b}\right)^5 = \mathbf{5}\log_2\left(\frac{1-a}{b}\right) = \mathbf{5[\log_2(1-a) - \log_2 b]}$

4. Find the value of x in the following equations:

(i) $\log 2 + \log x = 1$ (ii) $\log_2 x + \log_2 8 = 5$

(iii) $(81)^x = (243)^{x+2}$ (iv) $\left(\frac{1}{27}\right)^{x-6} = 27$

(v) $\log(5x - 10) = 2$ (vi) $\log_2(x+1) - \log_2(x-4) = 2$

Solution

i. $\log 2 + \log x = 1 \Rightarrow \log 2x = \log 10 \Rightarrow 2x = 10 \Rightarrow \mathbf{x = 5}$

ii. $\log_2 x + \log_2 8 = 5 \Rightarrow \log_2 x + \log_2 8 = 5\log_2 2 \Rightarrow \log_2 8x = \log_2 2^5 \Rightarrow 8x = 32 \Rightarrow \mathbf{x = 4}$

iii. $(81)^x = (243)^{x+2} \Rightarrow (3^4)^x = (3^5)^{x+2} \Rightarrow 3^{4x} = 3^{5x+10} \Rightarrow 5x + 10 = 4x \Rightarrow \mathbf{x = -10}$

iv. $\left(\frac{1}{27}\right)^{x-6} = 27 \Rightarrow (3^{-3})^{x-6} = 3^3 \Rightarrow 3^{-3x+18} = 3^3 \Rightarrow -3x + 18 = 3 \Rightarrow \mathbf{x = 5}$

v. $\log(5x - 10) = 2 \Rightarrow \log(5x - 10) = 2\log 10 \Rightarrow \log(5x - 10) = \log 10^2$

$\Rightarrow 5x - 10 = 100 \Rightarrow 5x = 110 \Rightarrow \mathbf{x = 22}$

vi. $\log_2(x+1) - \log_2(x-4) = 2 \Rightarrow \log_2\left(\frac{x+1}{x-4}\right) = 2\log_2 2$

$\Rightarrow \log_2\left(\frac{x+1}{x-4}\right) = \log_2 2^2 \Rightarrow \frac{x+1}{x-4} = 4 \Rightarrow x + 1 = 4x - 16$

$\Rightarrow 3x = 17 \Rightarrow x = \frac{17}{3} \Rightarrow \mathbf{x = 5\frac{2}{3}}$

5. Find the values of the following with the help of logarithm table:

(i) $\frac{3.68 \times 4.21}{5.234}$ (ii) $4.67 \times 2.11 \times 2.397$

(iii) $\frac{(20.46)^2 \times (2.4122)}{754.3}$ (iv) $\frac{\sqrt[3]{9.364} \times 21.64}{3.21}$

5(i). $\mathbf{\log\left(\frac{3.68\times4.21}{5.234}\right)}$ =???

Solution

Let $x = \frac{3.68\times4.21}{5.234}$

$\log x = \log\left(\frac{3.68\times4.21}{5.234}\right)$ taking logarithm on both sides

$\log x = \log(3.68) + \log(4.21) - \log(5.234)$

$\log x = 0.5658 + 0.6243 - 0.7188$

$\log x = 0.4713$

$x = \text{antilog}(0.4713)$

$\Rightarrow \mathbf{\log\left(\frac{3.68\times4.21}{5.234}\right) = 2.960}$

5(ii). $\mathbf{log(4.67 \times 2.11 \times 2.397)}$ =???

Solution

Let $x = 4.67 \times 2.11 \times 2.397$

$\log x = \log(4.67 \times 2.11 \times 2.397)$ taking logarithm on both sides

$\log x = \log(4.67) + \log(2.11) + \log(2.397)$

$\log x = 0.6693 + 0.3243 + 0.3797$

$\log x = 1.3733$

$x = \text{antilog}(1.3733)$

$\Rightarrow \mathbf{log(4.67 \times 2.11 \times 2.397) = 23.62}$

5(iii). $\mathbf{log\left[\frac{(20.46)^2 \times (2.4122)}{754.3}\right]}$ =???

Solution

Let $x = \frac{(20.46)^2 \times (2.4122)}{754.3}$

$\log x = \log\left[\frac{(20.46)^2 \times (2.4122)}{754.3}\right]$ taking logarithm on both sides

$\log x = 2\log(20.46) + \log(2.4122) - \log(754.3)$

$\log x = 2(1.3109) + 0.3824 - 2.8776$

$\log x = 0.1266$

$x = \text{antilog}(0.1266)$

$\Rightarrow \mathbf{log\left[\frac{(20.46)^2 \times (2.4122)}{754.3}\right] = 1.339}$

5(iv). $\mathbf{log\left[\frac{\sqrt[3]{9.364} \times (21.64)}{3.21}\right]}$ =???

Solution

Let $x = \frac{\sqrt[3]{9.364} \times (21.64)}{3.21}$

$\log x = \log\left[\frac{\sqrt[3]{9.364} \times (21.64)}{3.21}\right]$ taking logarithm on both sides

$\log x = \frac{1}{3}\log(9.364) + \log(21.64) - \log(3.21)$

$\log x = \frac{1}{3}(0.9715) + 1.3353 - 0.5065$

$\log x = 1.1526$

$x = \text{antilog}(1.1526)$

$\Rightarrow \mathbf{log\left[\frac{\sqrt[3]{9.364} \times (21.64)}{3.21}\right] = 14.21}$

6. The formula to measure the magnitude of earthquakes is given by $M = \log_{10}\left(\frac{A}{A_o}\right)$. If amplitude ($A$) is 10,000 and reference amplitude (A_o) is 10. What is the magnitude of the earthquake?

Solution

$$\mathbf{M = \log_{10}\left[\frac{A}{A_0}\right] = \log_{10}\left[\frac{10000}{10}\right]} = ???$$

$$M = \log_{10}\left[\frac{10000}{10}\right] \Rightarrow M = \log_{10}[1000] \Rightarrow M = \log_{10}[10^3] \Rightarrow \mathrm{M} = 3\log_{10}(10)$$

$$\Rightarrow \mathbf{M = \log_{10}\left[\frac{A}{A_0}\right] = \log_{10}\left[\frac{10000}{10}\right] = 3} \ \textbf{rector scale}$$

7. Abdullah invested Rs. 100,000 in a saving scheme and gains interest at the rate of 5% per annum so that the total value of this investment after t years is Rs y. This is modelled by an equation $y = 100{,}000\,(1.05)^t$, $t \geq 0$. Find after how many years the investment will be double.

Solution

Initial investment = Rs. 100000

Interest rate = 5% per annum

Total value after t years = y

The equation modeling this situation is:

$$\mathrm{y} = 100000 \times (1.05)^{\mathrm{t}}$$

We want to find years when the investment will be double, i.e., y = 2,00,000

$$2{,}00{,}000 = 1{,}00{,}000 \times (1.05)^{\mathrm{t}}$$

$$2 = (1.05)^t \Rightarrow \log 2 = \log(1.05)^{\mathrm{t}} \Rightarrow \log 2 = \mathrm{t} \times \log(1.05)$$

$$\Rightarrow \mathrm{t} = \frac{\log 2}{\log(1.05)} \Rightarrow t = \frac{0.3010}{0.0212} \Rightarrow \mathbf{t \approx 14.21} \ \textbf{years}$$

8. Huria is hiking up a mountain where the temperature (T) decreases by 3% (or a factor of 0.97) for every 100 metres gained in altitude. The initial temperature (T_i) at sea level is 20°C. Using the formula $T = T_i \times 0.97^{\frac{h}{100}}$, calculate the temperature at an altitude (h) of 500 metres.

Solution

First method	**Second method**
$\mathrm{T} = \mathrm{T_i} \times (0.97)^{\frac{h}{100}}$	$\mathrm{T} = \mathrm{T_i} \times (0.97)^{\frac{h}{100}}$
$\mathrm{T} = 20 \times (0.97)^{\frac{500}{100}}$	$\mathrm{T} = 20 \times (0.97)^{\frac{500}{100}}$
$\mathrm{T} = 20 \times (0.97)^5$	$\log\mathrm{T} = \log(20 \times (0.97)^5)$
$\mathrm{T} = 20 \times 0.859$	$\log\mathrm{T} = \log 20 + 5\log(0.97) = 1.3011 + 5(-0.0133)$
$\mathbf{T \approx 17.17°C}$	$\log\mathrm{T} = 1.2346 \Rightarrow \mathrm{T} = \mathrm{antilog}(1.2346)$
	$\mathbf{T \approx 17.17°C}$

REVIEW EXERCISE 2

1. Four options are given against each statement. Encircle the correct option.

(i) The standard form of 5.2×10^6 is:

(a) 52,000 (b) 520,000 (c) ✓ 5,200,000 (d) 52,000,000

(ii) Scientific notation of 0.00034 is:

(a) 3.4×10^3 (b) ✓ 3.4×10^{-4} (c) 3.4×10^4 (d) 3.4×10^{-3}

(iii) The base of common logarithm is:

(a) 2 (b) ✓ 10 (c) 5 (d) e

(iv) $\log_2 2^3 =$ __________.

(a) 1 (b) 2 (c) 5 (d) ✓ 3

(v) log 100 = __________.

(a) ✓ 2 (b) 3 (c) 10 (d) 1

(vi) If log 2 = 0.3010, then log 200 is:

(a) 1.3010 (b) 0.6010 (c) ✓ 2.3010 (d) 2.6010

(vii) log (0) = __________.

(a) positive (b) negative (c) zero (d) ✓ undefined

(viii) log 10,000 =

(a) 2 (b) 3 (c) ✓ 4 (d) 5

(ix) log 5 + log 3 = __________.

(a) log 0 (b) log 2 (c) $\log\left(\frac{5}{3}\right)$ (d) ✓ log 15

(x) $3^4 = 81$ in logarithmic form is:

(a) $\log_3 4 = 81$ (b) $\log_4 3 = 81$

(c) ✓ $\log_3 81 = 4$ (d) $\log_4 81 = 3$

2. Express the following numbers in scientific notation:

(i) 0.000567 (ii) 734 (iii) 0.33×10^3

Solution

(i) 5.67×10^{-4} (ii) 7.34×10^2 (iii) 3.3×10^2

3. Express the following numbers in ordinary notation:

(i) 2.6×10^3 (ii) 8.794×10^{-4} (iii) 6×10^{-6}

Solution

(i) 2600 (ii) 0.0008794 (iii) 0.000006

4. Express each of the following in logarithmic form:

(i) $3^7 = 2187$ (ii) $a^b = c$ (iii) $(12)^2 = 144$

Solution

(i) $\log_3 2187 = 7$ (ii) $\log_a c = b$ (iii) $\log_{12} 144 = 2$

5. Express each of the following in exponential form:

(i) $\log_4 8 = x$ (ii) $\log_9 729 = 3$ (iii) $\log_4 1024 = 5$

Solution

(i) $4^x = 8$ (ii) $9^3 = 729$ (iii) $4^5 = 1024$

6. Find value of x in the following:

(i) $\log_9 x = 0.5$ (ii) $\left(\frac{1}{9}\right)^{3x} = 27$ (iii) $\left(\frac{1}{32}\right)^{2x} = 64$

Solution

i. $\log_9 x = 0.5 \Rightarrow x = 9^{0.5} \Rightarrow x = (3^2)^{\frac{1}{2}} \Rightarrow \mathbf{x = 3}$

ii. $\left(\frac{1}{9}\right)^{3x} = 27 \Rightarrow \left(\frac{1}{3^2}\right)^{3x} = 3^3 \Rightarrow (3^{-2})^{3x} = 3^3 \Rightarrow 3^{-6x} = 3^3$

$\Rightarrow -6x = 3 \Rightarrow x = -\frac{3}{6} \Rightarrow \mathbf{x = -\frac{1}{2}}$

iii. $\left(\frac{1}{32}\right)^{2x} = 64 \Rightarrow \left(\frac{1}{2^5}\right)^{2x} = 2^6 \Rightarrow (2^{-5})^{2x} = 2^6 \Rightarrow 2^{-10x} = 2^6$

$\Rightarrow -10x = 6 \Rightarrow x = -\frac{6}{10} \Rightarrow \mathbf{x = -\frac{3}{5}}$

7. Write the following as a single logarithm:

(i) $7 \log x - 3\log y^2$ (ii) $3 \log 4 - \log 32$

(iii) $\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$

Solution

i. $7\log x - 3\log y^2 = \log x^7 - \log y^6 = \mathbf{\log \frac{x^7}{y^6}}$

ii. $3\log 4 - \log 32 = \log 4^3 - \log 32 = \log \frac{4^3}{32} = \log \frac{64}{32} = \mathbf{\log 2}$

iii. $\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3 = \frac{1}{3}[\log_5(8 \times 27)] - \log_5 3$

$= \frac{1}{3}[\log_5(216)] - \log_5 3 = \log_5(6^3)^{\frac{1}{3}} - \log_5 3$

$= \log_5 6 - \log_5 3 = \log_5 \frac{6}{3} = \mathbf{\log_5 2}$

8. Expand the following using laws of logarithms:

(i) $\log(x y z^6)$ (ii) $\log_3 \sqrt[6]{m^5 n^3}$ (iii) $\log \sqrt{8x^3}$

Solution

i. $\log(xyz^6) = \log x + \log y + \log z^6 = \mathbf{\log x + \log y + 6\log z}$

ii. $\log_3 \sqrt[6]{m^5 n^3} = \log_3(m^5 n^3)^{\frac{1}{6}} = \frac{1}{6}[\log_3 m^5 + \log_3 n^3] = \mathbf{\frac{1}{6}[5\log_3 m + 3\log_3 n]}$

iii. $\log\sqrt{8x^3} = \log(8x^3)^{\frac{1}{2}} = \log(2^3 x^3)^{\frac{1}{2}} = \log(2x)^{\frac{3}{2}} = \mathbf{\frac{3}{2}[\log 2 + \log x]}$

9. Find the values of the following with the help of logarithm table:

(i) $\sqrt[3]{68.24}$ (ii) 319.8×3.543 (iii) $\frac{36.12 \times 750.9}{113.2 \times 9.98}$

9(i). $\mathbf{\log[\sqrt[3]{68.24}]}$ =???

Solution

Let $x = \sqrt[3]{68.24} = (68.24)^{\frac{1}{3}}$

$\log x = \log(68.24)^{\frac{1}{3}}$ taking logarithm on both sides

$\log x = \frac{1}{3}\log(68.24) = \frac{1}{3}(1.8340)$

$\log x = 0.6113$

$x = \text{antilog}(0.6113)$

$\Rightarrow \mathbf{\log[\sqrt[3]{68.24}] = 4.086}$

9(ii). $\mathbf{log(319.8 \times 3.543)} = ???$

Solution

Let $x = 319.8 \times 3.543$

$\log x = \log(319.8 \times 3.543)$ taking logarithm on both sides

$\log x = \log(319.8) + \log(3.543)$

$\log x = 2.5049 + 0.5494$

$\log x = 3.0543$

$x = \text{antilog}(3.0543)$

$\Rightarrow \mathbf{log(319.8 \times 3.543) = 1133}$

9(iii). $\mathbf{log\left(\frac{36.12\times 750.9}{113.2\times 9.98}\right)} = ???$

Solution

Let $x = \frac{36.12\times 750.9}{113.2\times 9.98}$

$\log x = \log\left(\frac{36.12\times 750.9}{113.2\times 9.98}\right)$ taking logarithm on both sides

$\log x = \log(36.12) + \log(750.9) - \log(113.2) - \log(9.98)$

$\log x = 1.5578 + 2.8756 - 2.0539 - 0.9991$

$\log x = 1.3804$

$x = \text{antilog}(1.3804)$

$\Rightarrow \mathbf{log\left(\frac{36.12\times 750.9}{113.2\times 9.98}\right) = 24.01}$

10. In the year 2016, the population of a city was 22 millions and was growing at a rate of 2.5% per year. The function $p(t) = 22(1.025)^t$ gives the population in millions, t years after 2016. Use the model to determine in which year the population will reach 35 millions. Round the answer to the nearest year.

Solution

$P(t) = 22 \times (1.025)^t$

$35 = 22 \times (1.025)^t$ when P(t) = 35

$1.591 = (1.025)^t$ dividing by 22

$\log 1.591 = t \times \log 1.025$ taking logarithm on both sides

$0.2014 = t \times 0.0107 \Rightarrow t = \frac{0.2014}{0.0107}$

$t = 18.81 \approx 19$ years

Since t represents years after 2016, add 19 to 2016:

Year ≈ 2016 + 19 ≈ 2035