

Chapter # 1

Real Numbers

Exercise # 1.1

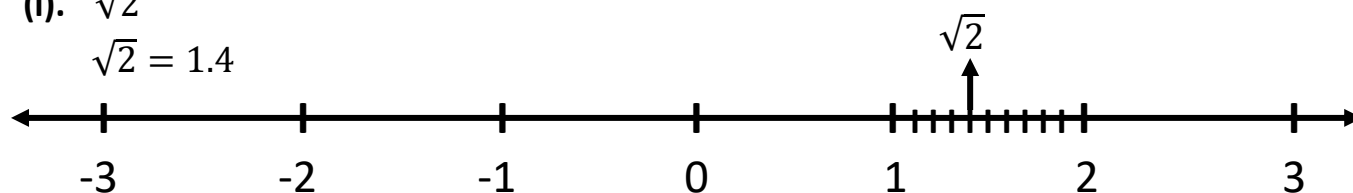
Question # 1: Identify each of the following as a rational or irrational number.

- | | | | |
|---------------------------------|--|-------------------------------------|---|
| (i) 2.353535
Rational | (ii) $0.\bar{6}$
Rational | (iii) 2.236067.....
Irrational | (iv) $\sqrt{7}$
Irrational |
| (v) e
Irrational | (vi) π
Irrational | (vii) $5 + \sqrt{11}$
Irrational | (viii) $\sqrt{3} + \sqrt{13}$
Irrational |
| (ix) $\frac{15}{4}$
Rational | (x) $(2 - \sqrt{2})(2 + \sqrt{2})$
Rational | | |

Question # 2: Represent the following numbers on number line.

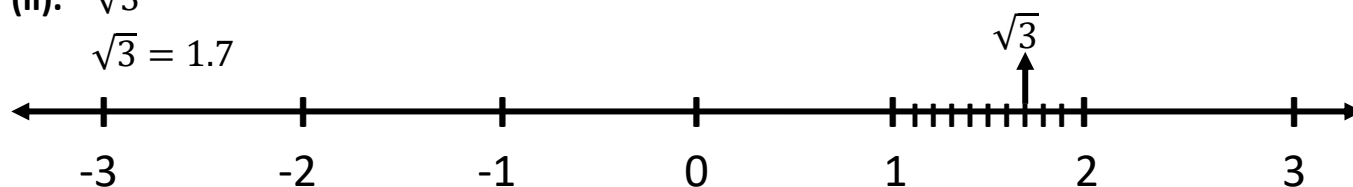
(i). $\sqrt{2}$

$\sqrt{2} = 1.4$



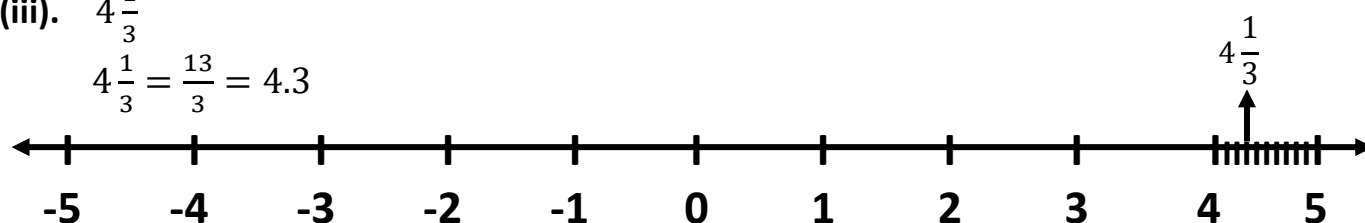
(ii). $\sqrt{3}$

$\sqrt{3} = 1.7$



(iii). $4\frac{1}{3}$

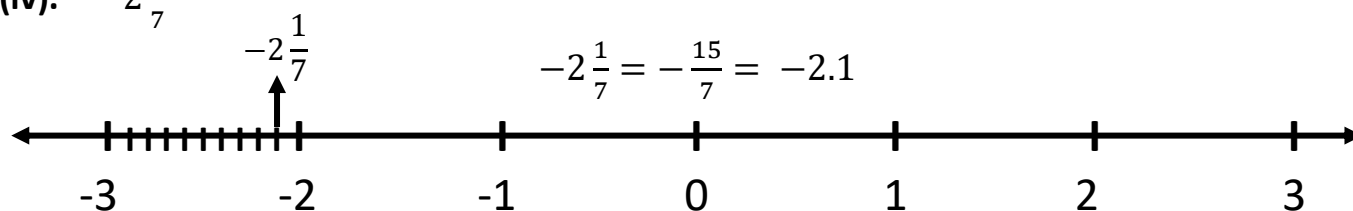
$4\frac{1}{3} = \frac{13}{3} = 4.3$



(iv). $-2\frac{1}{7}$

$-2\frac{1}{7}$

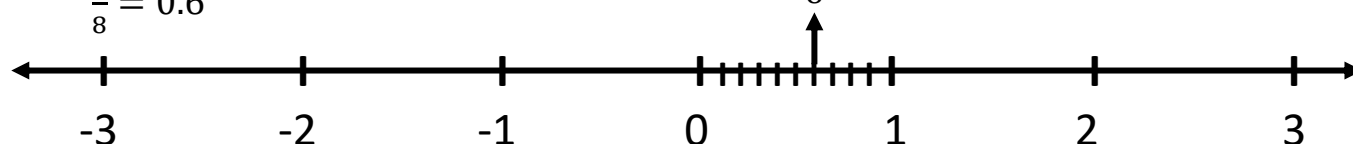
$-2\frac{1}{7} = -\frac{15}{7} = -2.1$

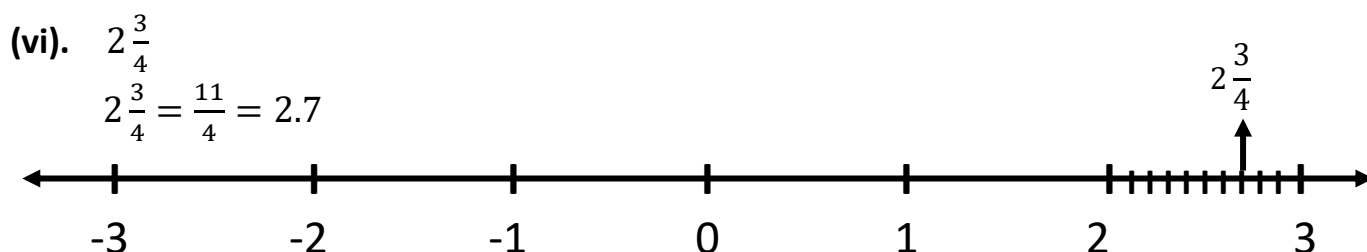


(v). $\frac{5}{8}$

$\frac{5}{8} = 0.6$

$\frac{5}{8}$





Question # 3: Express the following as rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$:

(i). $0.\bar{4}$

Let,

$$x = 0.444 \dots \text{ (A)}$$

Multiply by '10' on both sides,

$$10x = 4.444 \dots \text{ (B)}$$

Subtract eq (A) from eq (B)

$$10x - x = 4.444 \dots - 0.444 \dots$$

$$9x = 4$$

$$x = \frac{4}{9} \text{ (Answer)}$$

(ii). $0.\bar{37}$

Let,

$$x = 0.373737 \dots \text{ (A)}$$

Multiply by '100' on both sides,

$$100x = 37.373737 \dots \text{ (B)}$$

Subtract eq (A) from eq (B)

$$100x - x = 37.373737 \dots - 0.373737 \dots$$

$$99x = 37$$

$$x = \frac{37}{99} \text{ (Answer)}$$

(iii). $0.\bar{21}$

Let,

$$x = 0.212121 \dots \text{ (A)}$$

Multiply by '100' on both sides,

$$100x = 21.212121 \dots \text{ (B)}$$

Subtract eq (A) from eq (B)

$$100x - x = 21.212121 \dots - 0.212121 \dots$$

$$99x = 21$$

$$x = \frac{21}{99} \text{ (Answer)}$$

Question # 4: Name the property used in the following:

(i) $(a + 4) + b = a + (4 + b)$

Associative property w.r.t addition

(iii) $x - x = 0$

Additive inverse

(v) $16 + 0 = 16$

Additive identity

(vii) $4 \times (5 \times 8) = (4 \times 5) \times 8$

Associative property w.r.t multiplication

(ii) $\sqrt{2} + \sqrt{3} = \sqrt{3} + \sqrt{2}$

Commutative property w.r.t addition

(iv) $a(b + c) = ab + ac$

Left distributive property

(vi) $100 \times 1 = 100$

Multiplicative identity

(viii) $ab = ba$

Commutative property w.r.t multiplication

Question # 5: Name the property used in the following:

(i) $-3 < -1 \Rightarrow 0 < 2$

Additive property

(iii) If $a < b$ then $a + c < b + c$

Additive property

(ii) If $a < b$ then $\frac{1}{a} > \frac{1}{b}$

Reciprocal property

(iv) If $ac < bc$ and $c > 0$ then $a < b$

Cancellation property

(v) If $ac < bc$ and $c < 0$ then $a > b$

Cancellation property

(vi) Either $a > b$ or $a = b$ or $a < b$

Trichotomy property

Question # 6: Insert two rational numbers between:(i). $\frac{1}{3}$ and $\frac{1}{4}$

$$1^{\text{st}} \text{ rational number} = \left(\frac{1}{3} + \frac{1}{4} \right) \div 2$$

$$= \left(\frac{4+3}{12} \right) \times \frac{1}{2}$$

$$= \frac{7}{12} \times \frac{1}{2}$$

$$= \frac{7}{24}$$

$$2^{\text{nd}} \text{ rational number} = \left(\frac{1}{3} + \frac{7}{24} \right) \div 2$$

$$= \left(\frac{8+7}{24} \right) \times \frac{1}{2}$$

$$= \frac{15}{24} \times \frac{1}{2}$$

$$= \frac{15}{48}$$

3	3,24
2	1,8
2	1,4
2	1,2
	1,1

(iii). $\frac{3}{5}$ and $\frac{4}{5}$

$$1^{\text{st}} \text{ rational number} = \left(\frac{3}{5} + \frac{4}{5} \right) \div 2$$

$$= \left(\frac{3+4}{5} \right) \times \frac{1}{2}$$

$$= \frac{7}{5} \times \frac{1}{2}$$

$$= \frac{7}{10}$$

$$2^{\text{nd}} \text{ rational number} = \left(\frac{3}{5} + \frac{7}{10} \right) \div 2$$

$$= \left(\frac{6+7}{10} \right) \times \frac{1}{2}$$

$$= \frac{13}{10} \times \frac{1}{2}$$

$$= \frac{13}{20}$$

2	5, 10
5	5, 5
	1, 1

(ii). 3 and 4

$$1^{\text{st}} \text{ rational number} = (3 + 4) \div 2$$

$$= (7) \times \frac{1}{2}$$

$$= \frac{7}{2}$$

$$2^{\text{nd}} \text{ rational number} = \left(3 + \frac{7}{2} \right) \div 2$$

$$= \left(\frac{6+7}{2} \right) \times \frac{1}{2}$$

$$= \frac{13}{2} \times \frac{1}{2}$$

$$= \frac{13}{4}$$

Chapter # 1

Real Numbers

Exercise # 1.2

Question # 1: Rationalize the denominator of the following:

$$\begin{aligned}
 \text{(i). } & \frac{13}{4+\sqrt{3}} \\
 &= \frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} \quad \because a^2 - b^2 = (a+b)(a-b) \\
 &= \frac{13(4-\sqrt{3})}{(4)^2 - (\sqrt{3})^2} \\
 &= \frac{13(4-\sqrt{3})}{16-3} \\
 &= \frac{13(4-\sqrt{3})}{13} \\
 &= 4 - \sqrt{3} \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii). } & \frac{\sqrt{2}-1}{\sqrt{5}} \\
 &= \frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\
 &= \frac{(\sqrt{2} \times \sqrt{5} - \sqrt{5})}{(\sqrt{5})^2} \\
 &= \frac{(\sqrt{10} - \sqrt{5})}{5} \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v). } & \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \\
 &= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} \\
 &= \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2} \quad \because (a-b)^2 = a^2 + b^2 - 2ab \\
 &\quad \quad \quad a^2 - b^2 = (a+b)(a-b) \\
 &= \frac{(\sqrt{3})^2 + (\sqrt{2})^2 - 2(\sqrt{3})(\sqrt{2})}{3-2} \\
 &= \frac{3+2-2\sqrt{3} \times 2}{1} \\
 &= 5 - 2\sqrt{6} \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii). } & \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \\
 &= \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
 &= \frac{(\sqrt{2} \times \sqrt{3} + \sqrt{5} \times \sqrt{3})}{(\sqrt{3})^2} \\
 &= \frac{(\sqrt{6} + \sqrt{15})}{3} \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv). } & \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \\
 &= \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}} \\
 &= \frac{(6-4\sqrt{2})^2}{(6)^2 - (4\sqrt{2})^2} \quad \because (a-b)^2 = a^2 + b^2 - 2ab \\
 &\quad \quad \quad a^2 - b^2 = (a+b)(a-b) \\
 &= \frac{(6)^2 + (4\sqrt{2})^2 - 2(6)(4\sqrt{2})}{36 - (16 \times 2)} \\
 &= \frac{36 + (16 \times 2) - 48\sqrt{2}}{36 - 32} \\
 &= \frac{36 + 32 - 48\sqrt{2}}{4} \\
 &= \frac{68 - 48\sqrt{2}}{4} \\
 &= \frac{4(17 - 12\sqrt{2})}{4} \\
 &= 17 - 12\sqrt{2} \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi). } & \frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \\
 &= \frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} \\
 &= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{(\sqrt{7})^2 - (\sqrt{5})^2} \quad \because a^2 - b^2 = (a+b)(a-b) \\
 &= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{7-5} \\
 &= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{2} \\
 &= 2\sqrt{3}(\sqrt{7}-\sqrt{5}) \quad \text{(Answer)}
 \end{aligned}$$

Question # 2: Simplify the following:

$$\begin{aligned}
 \text{(i). } & \left(\frac{81}{16}\right)^{-\frac{3}{4}} \\
 &= \left(\frac{16}{81}\right)^{\frac{3}{4}} \\
 &= \frac{2^{4 \times \frac{3}{4}}}{3^{4 \times \frac{3}{4}}} \\
 &= \frac{2^3}{3^3} \\
 &= \frac{8}{27} \quad (\text{Answer})
 \end{aligned}$$

$$\begin{array}{r|l}
 2 & 16 \\
 \hline
 2 & 8 \\
 2 & 4 \\
 2 & 2 \\
 & 1 \\
 \hline
 3 & 81 \\
 3 & 27 \\
 3 & 9 \\
 3 & 3 \\
 & 1
 \end{array}$$

$$\begin{aligned}
 \text{(ii). } & \left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^3 \times \frac{16}{27} \\
 &= \left(\frac{4}{3}\right)^2 \div \frac{4^3}{3^{2 \times 3}} \times \frac{4^2}{3^3} \\
 &= \frac{4^2}{3^2} \times \frac{3^6}{4^3} \times \frac{4^2}{3^3} \\
 &= 4^{2+2-3} \times 3^{6-2-3} \\
 &= 4 \times 3 \\
 &= 12 \quad (\text{Answer})
 \end{aligned}$$

$$\begin{array}{r|l}
 3 & 81 \\
 3 & 27 \\
 3 & 9 \\
 3 & 3 \\
 & 1
 \end{array}$$

$$\begin{aligned}
 \text{(iii). } & (0.027)^{-\frac{1}{3}} \\
 &= \left(\frac{27}{1000}\right)^{-\frac{1}{3}} \\
 &= \left(\frac{1000}{27}\right)^{\frac{1}{3}} \\
 &= \left(\frac{10^3}{3^3}\right)^{\frac{1}{3}} \\
 &= \frac{10^{3 \times \frac{1}{3}}}{3^{3 \times \frac{1}{3}}} \\
 &= \frac{10}{3} \quad (\text{Answer})
 \end{aligned}$$

$$\begin{array}{r|l}
 3 & 27 \\
 3 & 9 \\
 3 & 3 \\
 & 1
 \end{array}$$

$$\begin{aligned}
 \text{(iv). } & \sqrt[7]{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} z^7}} \\
 &= (x^{14} y^{21-14} z^{35-7})^{\frac{1}{7}} \\
 &= (x^{14} y^7 z^{28})^{\frac{1}{7}} \\
 &= x^{14 \times \frac{1}{7}} y^{7 \times \frac{1}{7}} z^{28 \times \frac{1}{7}} \\
 &= x^2 y z^4 \quad (\text{Answer})
 \end{aligned}$$

$$\begin{aligned}
 \text{(v). } & \frac{5 \cdot (25)^{n+1} - 25 \cdot (5)^{2n}}{5 \cdot (5)^{2n+3} - (25)^{n+1}} \\
 &= \frac{5 \cdot 5^{2(n+1)} - 5^2 \cdot 5^{2n}}{5 \cdot 5^{2n+3} - 5^{2(n+1)}} \\
 &= \frac{5 \cdot 5^{2n+2} - 5^{2n+2}}{5 \cdot 5^{2n+2} \cdot 5 - 5^{2n+2}} \\
 &= \frac{5^{2n+2} (5 - 1)}{5^{2n+2} (5 \cdot 5 - 1)} \\
 &= \frac{4}{25-1} \\
 &= \frac{4}{24} \\
 &= \frac{1}{6} \quad (\text{Answer})
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi). } & \frac{(16)^{x+1} + 20(4^{2x})}{(2)^{x-3} \times 8^{x+2}} \\
 &= \frac{(2^4)^{x+1} + 20(2^{2 \times 2x})}{2^{x-3} \times 2^{3(x+2)}} \\
 &= \frac{2^{4x+4} + 20(2^{4x})}{2^{x-3} \times 2^{3x+6}} \\
 &= \frac{2^{4x} \cdot 2^4 + 20(2^{4x})}{2^{x-3+3x+6}} \\
 &= \frac{2^{4x}(2^4 + 20)}{2^{4x+3}} \\
 &= \frac{2^{4x}(16+20)}{2^{4x} \cdot 2^3} \\
 &= \frac{36}{8} \\
 &= \frac{9}{2} \quad (\text{Answer})
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii). } & (64)^{\frac{-2}{3}} \div (9)^{\frac{-3}{2}} \\
 &= (4^3)^{\frac{-2}{3}} \div (3^2)^{\frac{-3}{2}} \\
 &= 4^{-2} \div 3^{-3} \\
 &= \frac{4^{-2}}{3^{-3}} \\
 &= \frac{3^3}{4^2} \\
 &= \frac{27}{16} \quad (\text{Answer})
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii). } & \frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}} \\
 &= \frac{3^n \times 3^{2(n+1)}}{3^{n-1} \times 3^{2(n-1)}} \\
 &= \frac{3^{2n+2}}{3^{-1} \times 3^{2n-2}}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{3^{2n} \times 3^2 \times 3^1}{3^{2n} \times 3^{-2}} \\
&= 3^{2+1+2} \\
&= 3^5 = 243 \quad (\text{Answer})
\end{aligned}$$

$$\begin{aligned}
\text{(ix). } & \frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 4 \times 5^n} \\
&= \frac{5^n \times 5^3 - 6 \times 5^n \times 5^1}{5^n(9 - 2^2)} \\
&= \frac{5^n(5^3 - 6 \times 5)}{5^n(9 - 4)} \\
&= \frac{125 - 30}{5} \\
&= \frac{95}{5} \\
&= 19 \quad (\text{Answer})
\end{aligned}$$

Question # 3: If $x = 3 + \sqrt{8}$ then find the value of:

$$\begin{aligned}
\frac{x}{1} &= \frac{3 + \sqrt{8}}{1} \\
\frac{1}{x} &= \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}} \\
&= \frac{3 - \sqrt{8}}{3^2 - (\sqrt{8})^2} \\
&= \frac{3 - \sqrt{8}}{9 - 8} \\
&= \frac{3 - \sqrt{8}}{1} \\
\frac{1}{x} &= 3 - \sqrt{8}
\end{aligned}$$

$$\begin{aligned}
\text{(i) } x + \frac{1}{x} &= 3 + \sqrt{8} + 3 - \sqrt{8} \\
&= 6 \quad (\text{Answer})
\end{aligned}$$

$$\begin{aligned}
\text{(ii) } x - \frac{1}{x} &= 3 + \sqrt{8} - (3 - \sqrt{8}) \\
&= \cancel{3} + \sqrt{8} - \cancel{3} + \sqrt{8} \\
&= 2\sqrt{8} \quad (\text{Answer})
\end{aligned}$$

$$\begin{aligned}
\text{(iii) } x^2 + \frac{1}{x^2} & \\
\because x + \frac{1}{x} &= 6
\end{aligned}$$

Taking square on both sides

$$\begin{aligned}
\left(x + \frac{1}{x}\right)^2 &= 6^2 \\
(x)^2 + \left(\frac{1}{x}\right)^2 + 2(x)\left(\frac{1}{x}\right) &= 36 \\
x^2 + \frac{1}{x^2} + 2 &= 36 \\
x^2 + \frac{1}{x^2} &= 36 - 2 \\
x^2 + \frac{1}{x^2} &= 34 \quad (\text{Answer})
\end{aligned}$$

$$\begin{aligned}
\text{(iv) } x^2 - \frac{1}{x^2} & \\
x^2 - \frac{1}{x^2} &= \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) \\
&= (6)(2\sqrt{8}) \\
&= 12\sqrt{8} \quad (\text{Answer})
\end{aligned}$$

$$\begin{aligned}
\text{(v) } x^4 + \frac{1}{x^4} & \\
\because x^2 + \frac{1}{x^2} &= 34 \\
\text{Taking square on both sides} & \\
\left(x^2 + \frac{1}{x^2}\right)^2 &= (34)^2 \\
(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(x^2)\left(\frac{1}{x^2}\right) &= 1156 \\
x^4 + \frac{1}{x^4} + 2 &= 1156 \\
x^4 + \frac{1}{x^4} &= 1156 - 2 \\
x^4 + \frac{1}{x^4} &= 1154 \quad (\text{Answer})
\end{aligned}$$

$$\begin{aligned}
\text{(vi) } \left(x - \frac{1}{x}\right)^2 & \\
&= (2\sqrt{8})^2 \\
&= 4 \times 8 \\
&= 32 \quad (\text{Answer})
\end{aligned}$$

Question # 4: Find the rational number

P and Q such that: $\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p + q\sqrt{2}$

$$\begin{aligned}
\frac{8-3\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} &= p + q\sqrt{2} \\
\frac{8(4-3\sqrt{2}) - 3\sqrt{2}(4-3\sqrt{2})}{4^2 - (3\sqrt{2})^2} &= p + q\sqrt{2} \\
\frac{32 - 24\sqrt{2} - 12\sqrt{2} + (3\sqrt{2})^2}{16 - (9 \times 2)} &= p + q\sqrt{2} \\
\frac{32 - 36\sqrt{2} + (9 \times 2)}{16 - 18} &= p + q\sqrt{2} \\
\frac{32 - 36\sqrt{2} + 18}{-2} &= p + q\sqrt{2}
\end{aligned}$$

$$\frac{50-36\sqrt{2}}{-2} = p + q\sqrt{2}$$

$$\frac{50}{-2} - \frac{36\sqrt{2}}{-2} = p + q\sqrt{2}$$

$$-25 + 18\sqrt{2} = p + q\sqrt{2}$$

By comparing we get,

$$p = -25 \text{ and } q = 18$$

Question # 5: Simplify the following:

(i). $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

3	243
3	81
3	27
3	9
3	3
	1

2	16
2	8
2	4
2	2
	1

$$= \frac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}}}$$

$$= \frac{5^3 \times 2^3}{2^5 \times 2^4}$$

$$= \frac{125 \times 27}{32 \times 16}$$

$$= \frac{3375}{512} \quad (\text{Answer})$$

(ii). $\frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$

$$= \frac{54 \times (3^3)^{\frac{2x}{3}}}{3^{2(x+1)} + 216(3^{2x} \times 3^{-1})}$$

$$= \frac{54 \times 3^{2x}}{3^{2x+2} + \frac{216(3^{2x})}{3}}$$

$$= \frac{54 \times 3^{2x}}{3^{2x} \times 3^2 + 72(3^{2x})}$$

$$= \frac{54 \times 3^{2x}}{3^{2x}(9+72)}$$

$$= \frac{54}{81}$$

$$= \frac{2}{3} \quad (\text{Answer})$$

(iii). $\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$= \sqrt{\frac{(2^3 \times 3^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{-3}{2}}}}$$

$$= \sqrt{\frac{(2^3)^{\frac{2}{3}} \times (3^3)^{\frac{2}{3}} \times 5^1}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{2^2 \times 3^2 \times 5}{(25)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{2^2 \times 3^2 \times 5}{(5^2)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{2^2 \times 3^2 \times 5}{5^3}}$$

$$= \sqrt{\frac{2^2 \times 3^2}{5^{3-1}}}$$

$$= \sqrt{\frac{2^2 \times 3^2}{5^2}}$$

$$= \frac{2 \times 3}{5}$$

$$= \frac{6}{5} \quad (\text{Answer})$$

(iv). $\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right)\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

$$\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$= \left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right)\left(\left(a^{\frac{1}{3}}\right)^2 - a^{\frac{1}{3}}b^{\frac{2}{3}} + \left(b^{\frac{2}{3}}\right)^2\right)$$

$$= \left(a^{\frac{1}{3}}\right)^3 + \left(b^{\frac{2}{3}}\right)^3$$

$$= a + b^2 \quad (\text{Answer})$$

Chapter # 1

Real Numbers

Exercise # 1.3

Question # 1: The sum of three consecutive integers is forty-two, find the three integers.

Let, three consecutive integers are: x , $x + 1$, $x + 2$

According to question:

$$x + x + 1 + x + 2 = 42$$

$$3x + 3 = 42$$

$$3x = 42 - 3$$

$$3x = 39$$

$$x = \frac{39}{3}$$

$$x = 13$$

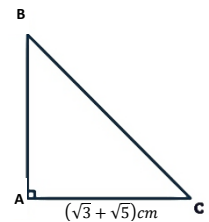
Also,

$$x + 1 = 13 + 1 = 14$$

$$x + 2 = 13 + 2 = 15$$

$$13, 14, 15 \quad (\text{Answer})$$

Question # 2: The diagram shows right angled ΔABC in which the length of \overline{AC} is $(\sqrt{3} + \sqrt{5})\text{cm}$. The area of ΔABC is $(1 + \sqrt{15})\text{cm}^2$. Find the length of \overline{AB} in the form of $(a\sqrt{3} + b\sqrt{5})\text{cm}$, where a and b are integers.



$$\overline{AC} = (\sqrt{3} + \sqrt{5})\text{cm}$$

$$\text{Area} = (1 + \sqrt{15})\text{cm}^2$$

$$\overline{AB} = ?$$

$$\text{Area} = \frac{\overline{AB} \times \overline{AC}}{2}$$

$$(1 + \sqrt{15}) = \frac{\overline{AB} \times (\sqrt{3} + \sqrt{5})}{2}$$

$$\overline{AB} = \frac{2(1 + \sqrt{15})}{(\sqrt{3} + \sqrt{5})}$$

$$\overline{AB} = \frac{2(1 + \sqrt{15})}{(\sqrt{3} + \sqrt{5})} \times \frac{(\sqrt{3} - \sqrt{5})}{(\sqrt{3} - \sqrt{5})}$$

$$= \frac{2(\sqrt{3} - \sqrt{5} + \sqrt{15} \times 3 - \sqrt{15} \times 5)}{(\sqrt{3})^2 - (\sqrt{5})^2} \quad \because a^2 - b^2 = (a + b)(a - b)$$

$$= \frac{2(\sqrt{3} - \sqrt{5} + \sqrt{45} - \sqrt{75})}{3 - 5}$$

$$= \frac{2(\sqrt{3} - \sqrt{5} + \sqrt{9 \times 3} - \sqrt{25 \times 3})}{-2}$$

$$= -(\sqrt{3} - \sqrt{5} + 3\sqrt{3} - 5\sqrt{3})$$

$$= -\sqrt{3} + \sqrt{5} - 3\sqrt{3} + 5\sqrt{3}$$

$$\overline{AB} = 4\sqrt{3} - 2\sqrt{5}$$

(Answer)

Question # 3: A rectangle has sides of length $2 + \sqrt{18} \text{ m}$ and $\left(5 - \frac{4}{\sqrt{2}}\right) \text{ m}$. Express the area of rectangle in the form $a + b\sqrt{2}$, where a and b are integers.

$\begin{aligned} \text{length} = l &= 2 + \sqrt{18} \text{ m} \\ l &= 2 + \sqrt{9 \times 2} \text{ m} \\ l &= 2 + 3\sqrt{2} \text{ m} \\ \text{breadth} = b &= 5 - \frac{4}{\sqrt{2}} \text{ m} \\ b &= 5 - \frac{2 \times 2}{\sqrt{2}} \text{ m} \\ b &= 5 - \frac{2\sqrt{2} \times \sqrt{2}}{\sqrt{2}} \text{ m} \quad \because 2 = \sqrt{2} \times \sqrt{2} \\ b &= 5 - 2\sqrt{2} \text{ m} \\ \text{Area of rectangle} &= l \times b \\ &= (2 + 3\sqrt{2}) \times (5 - 2\sqrt{2}) \end{aligned}$	$\begin{aligned} &= 2(5 - 2\sqrt{2}) + 3\sqrt{2}(5 - 2\sqrt{2}) \\ &= 10 - 4\sqrt{2} + 15\sqrt{2} - 6(\sqrt{2})^2 \\ &= 10 + 11\sqrt{2} - (6 \times 2) \\ &= 10 + 11\sqrt{2} - 12 \\ \text{Area} &= (11\sqrt{2} - 2) \text{ m}^2 \\ &\quad \text{(Answer)} \end{aligned}$
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Question # 4: Find two numbers whose sum is 68 and difference is 22.

Let, two numbers are: x, y

According to question:

$\begin{aligned} x + y &= 68 \quad \text{---(A)} \\ 22 + y + y &= 68 \\ 2y &= 68 - 22 \\ 2y &= 46 \\ y &= \frac{46}{2} \\ y &= 23 \\ \text{put in equation (B)} \\ \text{using equation (B),} \\ x &= 22 + 23 \\ x &= 45 \\ 23, 45 &\quad \text{(Answer)} \end{aligned}$	$\begin{aligned} x - y &= 22 \\ x &= 22 + y \quad \text{---(B)} \\ \text{put in equation (A)} \end{aligned}$
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Question # 5: The weather in Lahore was usually warm during the summer of 2024. The TV news reported temperature as high as 48°C . By using the formula, $(^\circ\text{F} = \frac{9}{5}^\circ\text{C} + 32)$ find the temperature as Fahrenheit scale.

Temperature in $^\circ\text{C} = 48^\circ\text{C}$

$$\begin{aligned} \therefore ^\circ\text{F} &= \frac{9}{5}^\circ\text{C} + 32 \\ &= \frac{9}{5} \times 48 + 32 \\ &= 86.4 + 32 \\ ^\circ\text{F} &= 118.4^\circ \quad \text{(Answer)} \end{aligned}$$

Question # 6: The sum of the ages of the father and son is 72 years. Six years ago, the father's age was 2 times the age of the son. What was son's age six years ago?

Let,

age of son = x and age of father = y

According to question,

$$x + y = 72 \text{ (A)}$$

Before '6' years ago the ages of both were:

$$2(x - 6) = y - 6$$

$$2x - 12 = y - 6$$

$$2x - 12 + 6 = y$$

$$2x - 6 = y$$

put the value of 'y' in equation (A)

$$x + 2x - 6 = 72$$

$$3x = 72 + 6$$

$$3x = 78$$

$$x = \frac{78}{3}$$

$$x = 26 \text{ years}$$

Before six years ago,

$$\text{Age of son} = 26 - 6 = 20 \text{ years}$$

(Answer)

Question # 7: Mirha bought a toy for Rs. 1500 and sold for Rs. 1520. What was her profit percentage?

$$CP = 1500 \text{ Rs}$$

$$SP = 1520 \text{ Rs}$$

$$Profit = SP - CP$$

$$= 1520 - 1500$$

$$= 20 \text{ Rs}$$

$$\% Profit = \frac{Profit}{CP} \times 100\%$$

$$= \frac{20}{1500} \times 100\%$$

$$= 0.0133 \times 100\%$$

$$= 1.33\% \quad (\text{Answer})$$

Question # 8: The annual income of Tayyab is Rs. 9,60,000 while the exempted amount is Rs. 1,30,000. How much tax would he have to pay at the rate of 0.75%?

$$\text{Annual Income} = 9,60,000 \text{ Rs}$$

$$\text{Exempted Amount} = 1,30,000 \text{ Rs}$$

$$\text{Taxable Income} = 9,60,000 - 1,30,000$$

$$= 8,30,000 \text{ Rs}$$

$$\text{Tax rate} = 0.075\%$$

$$\text{Tax amount} = 0.075\% \times 8,30,000$$

$$= \frac{0.075}{100} \times 8,30,000$$

$$= 6225 \text{ Rs} \quad (\text{Answer})$$

Question # 9: Find the compound markup on Rs. 3,75,000 for one year at the rate of 14% compounded annually.

Principal Amount = 3,75,000 Rs

Time = 1 year

Rate = 14%

Compound Markup = ?

$$\begin{aligned}\text{Profit/Compound Markup} &= \frac{\text{Principal Amount} \times \text{time} \times \text{rate}}{100} \\ &= \frac{3,75,000 \times 1 \times 14}{100} \\ &= 52500 \text{ Rs} \quad (\text{Answer})\end{aligned}$$

Chapter # 1

Real Numbers

Review Exercise # 1

Question # 1: Four options are given against each statement. Encircle the correct option.

#	Answer	#	Answer
i	C	vi	B
ii	D	vii	A
iii	D	viii	B
iv	D	ix	D
v	A	x	D

Question # 2: If $a = \frac{3}{2}$, $b = \frac{5}{3}$ and $c = \frac{7}{5}$, then verify that:

(i) $a(b + c) = ab + ac$

$$\begin{aligned} \frac{3}{2} \left(\frac{5}{3} + \frac{7}{5} \right) &= \left(\frac{3}{2} \right) \left(\frac{5}{3} \right) + \left(\frac{3}{2} \right) \left(\frac{7}{5} \right) \\ \frac{3}{2} \left(\frac{25+21}{15} \right) &= \frac{15}{6} + \frac{21}{10} \\ \frac{3}{2} \left(\frac{46}{15} \right) &= \frac{75+63}{30} \end{aligned}$$

2	6,10
3	3,5
5	1,5
	1,1

$$\frac{138}{30} = \frac{138}{30}$$

Hence Proved

(ii) $(a + b)c = ac + bc$

$$\begin{aligned} \left(\frac{3}{2} + \frac{5}{3} \right) \frac{7}{5} &= \left(\frac{3}{2} \right) \left(\frac{7}{5} \right) + \left(\frac{5}{3} \right) \left(\frac{7}{5} \right) \\ \left(\frac{9+10}{6} \right) \frac{7}{5} &= \frac{21}{10} + \frac{35}{15} \\ \left(\frac{19}{6} \right) \frac{7}{5} &= \frac{63+70}{30} \end{aligned}$$

2	10,15
3	5,15
5	5,5
	1,1

$$\frac{133}{30} = \frac{133}{30}$$

Hence Proved

Question # 3: If $a = \frac{4}{3}$, $b = \frac{5}{2}$, $c = \frac{7}{4}$, then verify the associative property of real numbers w.r.t addition and multiplication.

Addition $a + (b + c) = (a + b) + c$

$$\begin{aligned} \frac{4}{3} + \left(\frac{5}{2} + \frac{7}{4} \right) &= \left(\frac{4}{3} + \frac{5}{2} \right) + \frac{7}{4} \\ \frac{4}{3} + \left(\frac{10+7}{4} \right) &= \left(\frac{8+15}{6} \right) + \frac{7}{4} \\ \frac{4}{3} + \frac{17}{4} &= \frac{23}{6} + \frac{7}{4} \\ \frac{16+51}{12} &= \frac{46+21}{12} \end{aligned}$$

2	4,6
2	2,3
3	1,3
	1,1

$$\frac{67}{12} = \frac{67}{12}$$

Hence Proved

Multiplication $a(bc) = (ab)c$

$$\begin{aligned} \frac{4}{3} \left(\frac{5}{2} \times \frac{7}{4} \right) &= \left(\frac{4}{3} \times \frac{5}{2} \right) \frac{7}{4} \\ \frac{4}{3} \left(\frac{35}{8} \right) &= \left(\frac{20}{6} \right) \frac{7}{4} \\ \frac{140}{24} &= \frac{140}{24} \end{aligned}$$

Hence Proved

Question # 4: Is 0 a rational number? Explain.

Ans: Yes, zero (0) is a rational number. It satisfies the definition of rational numbers.

e.g. $\frac{0}{2}$, $\frac{0}{-9}$ both are rational numbers.

Question # 5: State trichotomy property of real numbers.

Ans: For $a, b \in \mathcal{R}$, either $a = b$ **or** $a > b$ **or** $a < b$

Question # 6: Find two rational numbers between 4 and 5.

$$\begin{aligned} 1^{\text{st}} \text{ rational number} &= (4 + 5) \div 2 \\ &= (9) \times \frac{1}{2} \\ &= \frac{9}{2} \end{aligned}$$

$$\begin{aligned} 2^{\text{nd}} \text{ rational number} &= \left(4 + \frac{9}{2}\right) \div 2 \\ &= \left(\frac{8+9}{2}\right) \times \frac{1}{2} \\ &= \frac{17}{2} \times \frac{1}{2} \\ &= \frac{17}{4} \end{aligned}$$

Question # 7: Simplify the following:

$$\begin{aligned} \text{(i). } & \sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}} \\ &= \left(\frac{x^{15}y^{35}}{z^{20}}\right)^{\frac{1}{5}} \\ &= \frac{x^{15 \times \frac{1}{5}} y^{35 \times \frac{1}{5}}}{z^{20 \times \frac{1}{5}}} \\ &= \frac{x^3 y^7}{z^4} \quad (\text{Answer}) \end{aligned}$$

$$\begin{aligned} \text{(ii). } & \sqrt[3]{(27)^{2x}} \\ &= (27)^{\frac{2x}{3}} \\ &= (3^3)^{\frac{2x}{3}} \\ &= (3)^{2x} \\ &= 9^{2x} \quad (\text{Answer}) \end{aligned}$$

3	27
3	9
3	3
	1

$$\begin{aligned} \text{(iii). } & \frac{6(3)^{n+2}}{3^{n+1} - 3^n} \\ &= \frac{6 \times 3^n \times 3^2}{3^n \times 3^1 - 3^n} \\ &= \frac{6 \times 3^n \times 9}{3^n(3-1)} \\ &= \frac{54}{2} \\ &= 27 \quad (\text{Answer}) \end{aligned}$$

Question # 8: The sum of three consecutive odd integers is 51. Find the three integers.

Let, three consecutive odd integers are: $x, x + 2, x + 4$

According to question:

$$x + x + 2 + x + 4 = 51$$

$$3x + 6 = 51$$

$$3x = 51 - 6$$

$$3x = 45$$

$$x = \frac{45}{3}$$

$$x = 15$$

Also,

$$x + 2 = 15 + 2 = 17$$

$$x + 4 = 15 + 4 = 19$$

$$15, 17, 19 \quad (\text{Answer})$$

Question # 9: Abdullah picked up 96 balls and placed them into two buckets. One bucket has twenty-eight more balls than the other bucket. How many balls were in each bucket?

Let,

$$\text{Balls in 1st bucket} = x$$

$$\text{Balls in 2nd bucket} = x + 28$$

$$\text{Total balls} = 96$$

According to question:

$$x + x + 28 = 96$$

$$2x = 96 - 28$$

$$2x = 68$$

$$x = \frac{68}{2}$$

$$\text{Balls in 1st Bucket} = x = 34$$

$$\begin{aligned} \text{Balls in 2nd Bucket} &= x + 28 \\ &= 34 + 28 = 62 \end{aligned}$$

Question # 10: Salma invested Rs. 3,50,000 in a bank, which paid simple profit at the rate of $7\frac{1}{4}\%$ per annum. After 2 years, the rate was increased to 8 % per annum. Find the amount she had at the end of 7 years.

For 2 years:

$$\text{Principal Amount} = 3,50,000 \text{ Rs}$$

$$\text{Rate} = 7\frac{1}{4}\% = 7.25\%$$

$$\text{Time} = 2 \text{ years}$$

$$\begin{aligned} \text{Profit} = P_1 &= \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100} \\ &= \frac{3,50,000 \times 7.25 \times 2}{100} \\ &= 50750 \text{ Rs} \end{aligned}$$

For Next 5 years:

$$\text{Principal Amount} = 3,50,000 \text{ Rs}$$

$$\text{Rate} = 8\%$$

$$\text{Time} = 5 \text{ years}$$

$$\begin{aligned} \text{Profit} = P_2 &= \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100} \\ &= \frac{3,50,000 \times 8 \times 5}{100} \\ &= 1,40,000 \text{ Rs} \end{aligned}$$

At end of 7 years:

$$\begin{aligned} \text{Total Amount} &= \text{Principal Amount} + P_1 + P_2 \\ &= 3,50,000 + 50,750 + 1,40,000 \\ &= 5,40,750 \text{ Rs} \end{aligned}$$