Chapter # 5

Linear Equations and Inequalities

Exercise # 5.2

Question # 1: Maximize $f(x,y) = 2x + 5y$; subject to the constraints:

$2y - x \leq 8$; $x - y \leq 4$; $x \geq 0; y \geq 0$

$\because x \geq 0, y \geq 0$

$\therefore$ Solution region will be feasible (I-Quadrant)

(a) Associated Equations

$2y - x = 8$ ______(A) $\quad\|\quad$ $x - y = 4$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$2(0) - x = 8$
$-x = 8$
$x = -8$
$P_1\ (-8,0)$

$x - 0 = 4$
$x = 4$
$P_3\ (4,0)$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$2y - 0 = 8$
$2y = 8$
$y = \frac{8}{2}$
$y = 4$
$P_2\ (0,4)$

$0 - y = 4$
$-y = 4$
$y = -4$
$P_4\ (0,-4)$

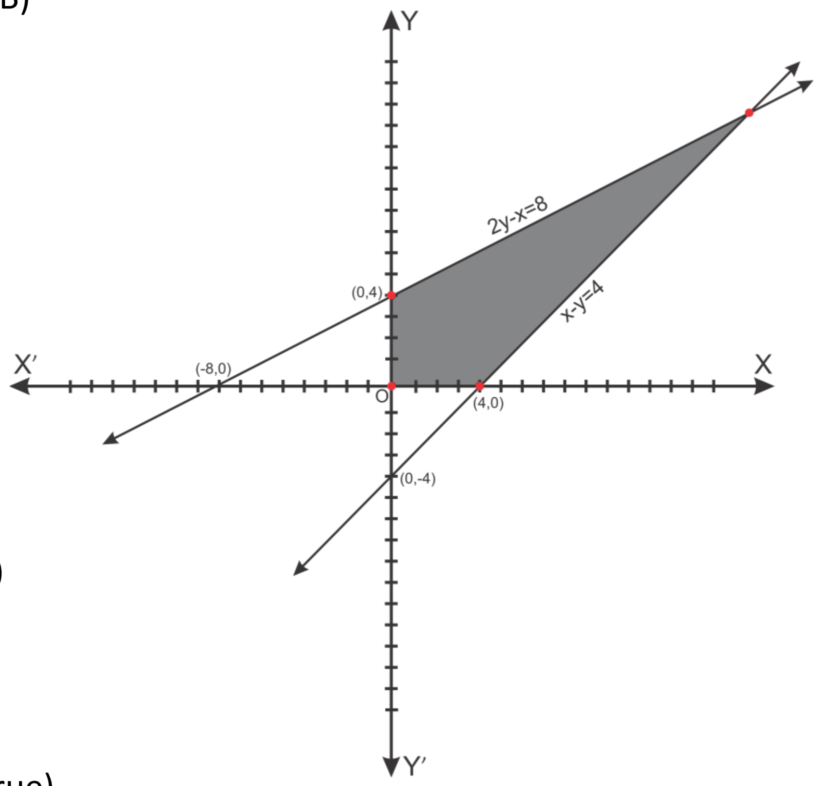
(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$2(0) - 0 \leq 8$
$0 \leq 8$ (True)

$0 - 0 \leq 4$
$0 \leq 4$ (True)

Solution Regions lie towards the Origin



(e) Point of Intersection

$2y - x = 8$ ______ (A)

$x - y = 4$ ______ (B)

(A) + (B)

$-x + 2y = 8$

$x - y = 4$

$y = 12$

Put in equation (B)

$x - 12 = 4$

$x = 4 + 12$

$x = 16$

(f) Corner Points

$(0,0), (0,4), (4,0), (16,12)$

$\because f(x, y) = 2x + 5y$

put $x = 0, y = 0$

$f(0,0) = 2(0) + 5(0) = 0 + 0 = 0$

put $x = 0, y = 4$

$f(0,4) = 2(0) + 5(4) = 0 + 20 = 20$

put $x = 4, y = 0$

$f(4,0) = 2(4) + 5(0) = 8 + 0 = 8$

put $x = 16, y = 12$

$f(16,12) = 2(16) + 5(12) = 32 + 60 = 92$

Hence, $f(x, y)$ is maximized at $(16,12)$

Question # 2: Maximize $f(x, y) = x + 3y$; subject to the constraints:

$\mathbf{2x + 5y \leq 30}$; $\mathbf{5x + 4y \leq 20}$; $\mathbf{x \geq 0; y \geq 0}$

(a) Associated Equations

$2x + 5y = 30$ ______(A)

$5x + 4y = 20$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$2x + 5(0) = 30$

$2x = 30$

$x = \frac{30}{2}$

$x = 15$

$P_1\ (15,0)$

$5x + 4(0) = 20$

$5x = 20$

$x = \frac{20}{5}$

$x = 4$

$P_3\ (4,0)$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$$2(0) + 5y = 30$$
$$5y = 30$$
$$y = \frac{30}{5}$$
$$y = 6$$
P_2 (0,6)

$$5(0) + 4y = 20$$
$$4y = 20$$
$$y = \frac{20}{4}$$
$$y = 5$$
P_4 (0,5)

(d) Test Point (0,0)

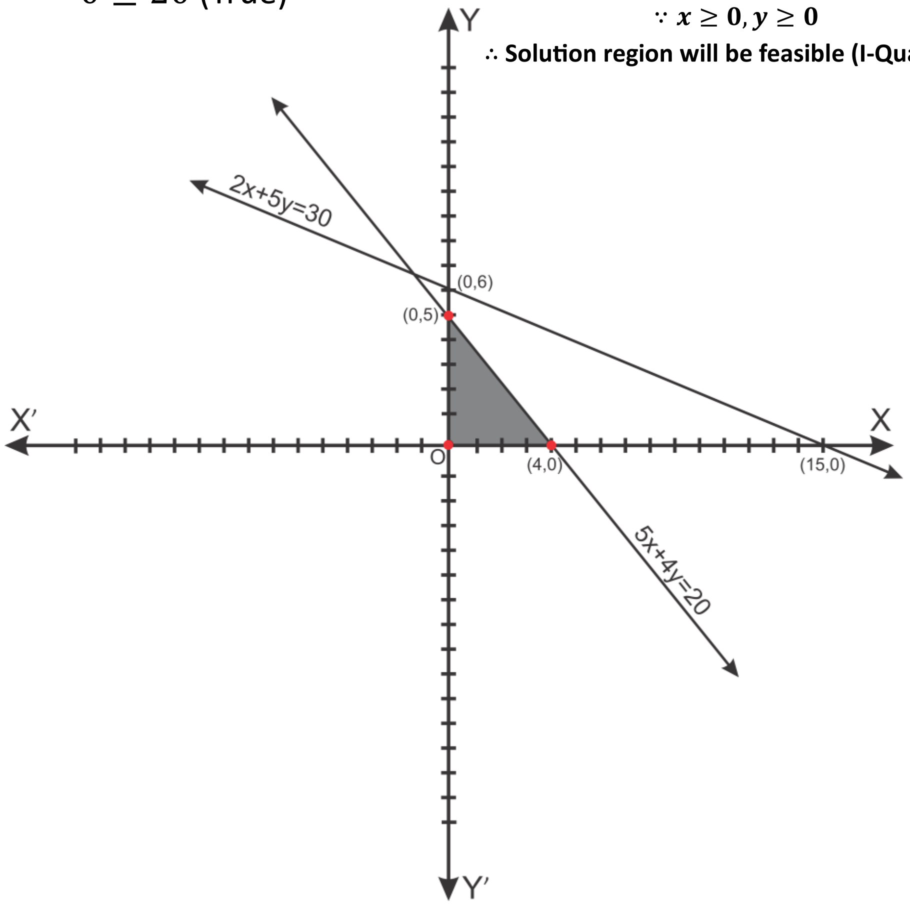
put $x = 0, y = 0$ in given inequalities

$$2(0) + 5(0) \leq 30$$
$0 \leq 30$ (True)

$$5(0) + 4(0) \leq 20$$
$0 \leq 20$ (True)

Solution Regions lie towards the Origin

$\because x \geq 0, y \geq 0$
$\therefore$ **Solution region will be feasible (I-Quadrant)**



(e) Corner Points

(0,0), (4,0), (0,5)

$\because f(x, y) = x + 3y$

put $x = 0, y = 0$

$$f(0,0) = 0 + 3(0) = 0 + 0 = 0$$

put $x = 4, y = 0$

$$f(4,0) = 4 + 3(0) = 4 + 0 = 4$$

put $x = 0, y = 5$

$$f(0,5) = 0 + 3(5) = 0 + 15 = 15$$

Hence, $f(x, y)$ is maximized at (0,5)

Question # 3: Maximize $z = 2x + 3y$; subject to the constraints:

$$\mathbf{2x + y \leq 4 \quad ; \quad 4x - y \leq 2 \quad ; \quad x \geq 0; y \geq 0}$$

(a) Associated Equations

$2x + y = 4$ ______(A) ‖ $4x - y = 2$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$$2x + 0 = 4$$
$$2x = 4$$
$$x = \frac{4}{2}$$
$$x = 2$$
$P_1\,(2,0)$

‖

$$4x - 0 = 2$$
$$4x = 2$$
$$x = \frac{2}{4}$$
$$x = \frac{1}{2}$$
$P_3\,(\frac{1}{2}, 0)$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$$2(0) + y = 4$$
$$0 + y = 4$$
$$y = 4$$
$P_2\,(0,4)$

‖

$$4(0) - y = 2$$
$$-y = 2$$
$$y = -2$$
$P_4\,(0, -2)$

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$$2(0) + 0 \leq 4$$
$0 \leq 4$ (True)

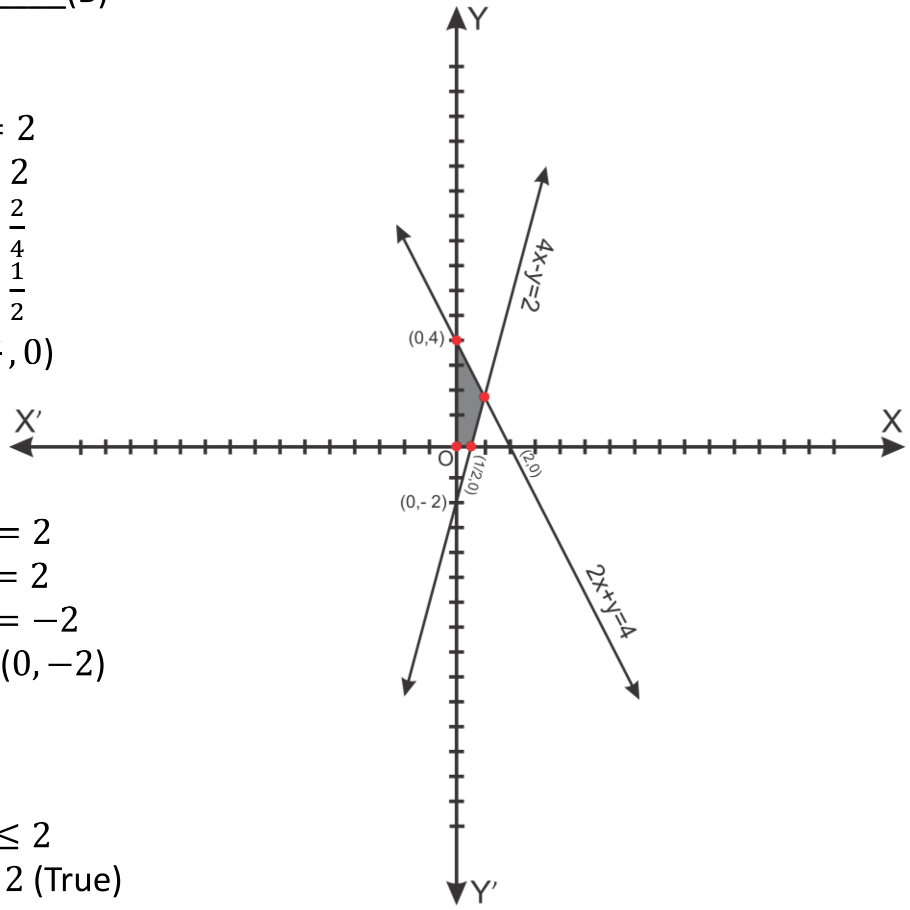
‖

$$4(0) - 0 \leq 2$$
$0 \leq 2$ (True)

Solution Regions lie towards the Origin

$\because x \geq 0, y \geq 0$

$\therefore$ **Solution region will be feasible (I-Quadrant)**



(e) Point of Intersection

$2x + y = 4$ _____ (A)

$4x - y = 2$ _____ (B)

(A) + (B)

$2x + y = 4$

$4x - y = 2$

$6x = 6$

$x = \frac{6}{6}$

$x = 1$

Put in equation (A)

$2(1) + y = 4$

$2 + y = 4$

$y = 4 - 2$

$y = 2$

(f) Corner Points

$(0,0), (0,4), (1,0), (1,2)$

$\because z = 2x + 3y$

put $x = 0, y = 0$

$z = 2(0) + 3(0) = 0 + 0 = 0$

put $x = 0, y = 4$

$z = 2(0) + 3(4) = 0 + 12 = 12$

put $x = 1, y = 0$

$z = 2(1) + 3(0) = 2 + 0 = 2$

put $x = 16, y = 12$

$z = 2(1) + 3(2) = 2 + 6 = 8$

Hence, z is maximized at $(0,4)$

Question # 4: Maximize $z = 2x + y$; subject to the constraints:

$$x + y \geq 3 \quad ; \quad 7x + 5y \leq 35 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$x + y = 3$ ______(A)

$7x + 5y = 35$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$x + 0 = 3$

$x = 3$

$P_1\ (3,0)$

$7x + 5(0) = 35$

$7x = 35$

$x = \frac{35}{7}$

$x = 5$

$P_3\ (5,0)$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$0 + y = 3$

$y = 4$

$P_2\ (0,4)$

$7(0) + 5y = 35$

$5y = 35$

$y = \frac{35}{5}$

$y = 7$

$P_4\ (0,7)$

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$0 + 0 \geq 3$

$0 \geq 4$ (False)

Solution region lies away from the origin

$7(0) + 5(0) \leq 35$

$0 \leq 35$ (True)

Solution region lies towards the origin

(e) Corner Points

$(3,0), (0,3), (5,0), (0,7)$

$\because z = 2x + y$

put $x = 3, y = 0$

$z = 2(3) + 0 = 6 + 0 = 6$

put $x = 0, y = 3$

$z = 2(0) + 3 = 0 + 3 = 3$

put $x = 5, y = 0$

$z = 2(5) + 0 = 10 + 0 = 10$

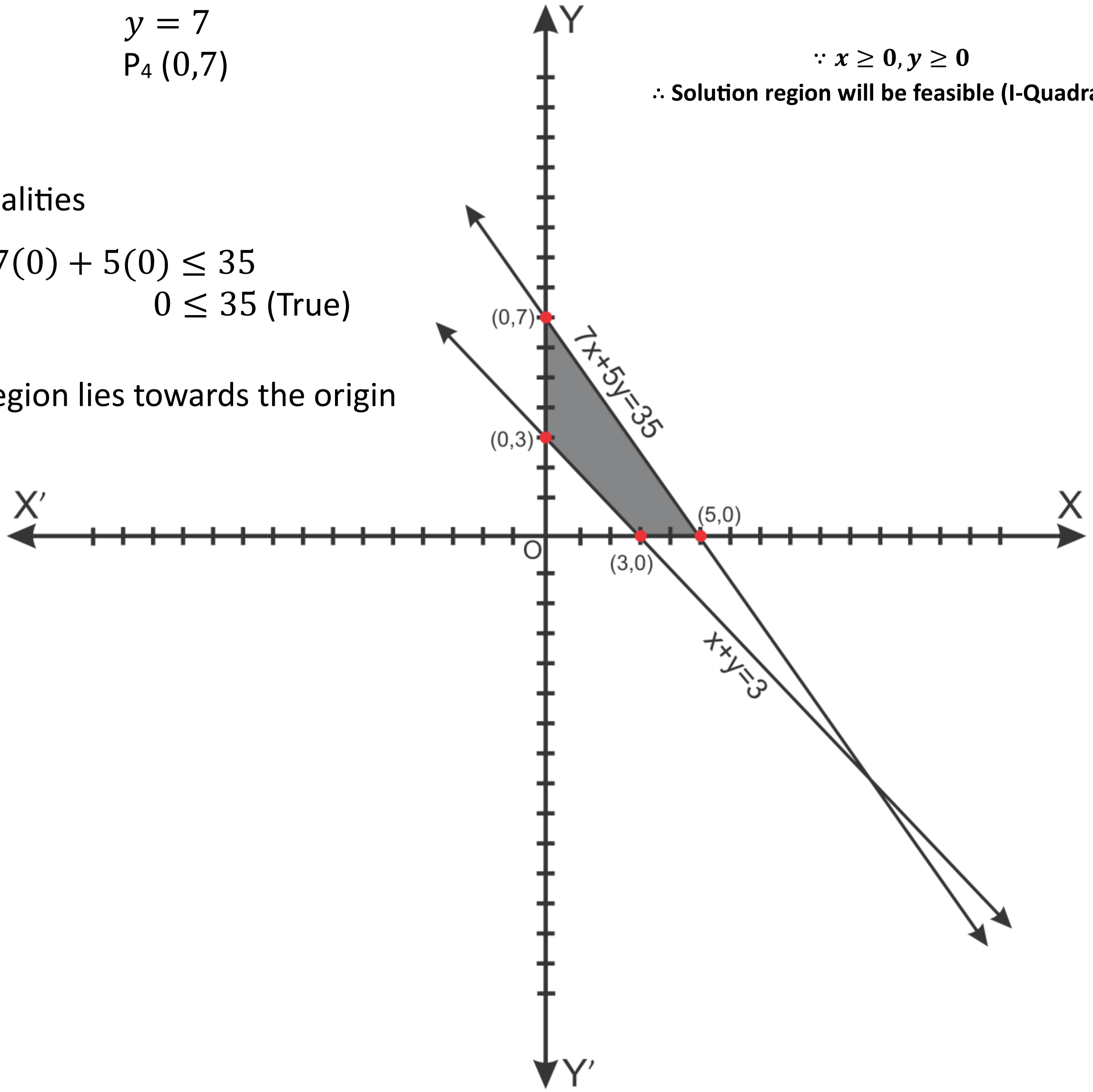
put $x = 0, y = 7$

$z = 2(0) + 7 = 0 + 7 = 7$

Hence, z is minimized at $(0,3)$

$\because \boldsymbol{x \geq 0, y \geq 0}$

∴ Solution region will be feasible (I-Quadrant)



Question # 5: Maximize the function defined as: $f(x, y) = 2x + 3y$; subject to the constraints:

$$2x + y \leq 10 \quad ; \quad x + 2y \leq 14 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$2x + y = 10$ ______(A)

$x + 2y = 14$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$$2x + 0 = 10$$
$$2x = 10$$
$$x = \frac{10}{2}$$
$$x = 5$$
P1 (5,0)

$$x + 2(0) = 14$$
$$x + 0 = 14$$
$$x = 14$$
P3 (5,0)

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$$2(0) + y = 10$$
$$0 + y = 10$$
$$y = 10$$
P2 (0,10)

$$0 + 2y = 14$$
$$2y = 14$$
$$y = \frac{14}{2}$$
$$y = 7$$
P4 (0,7)

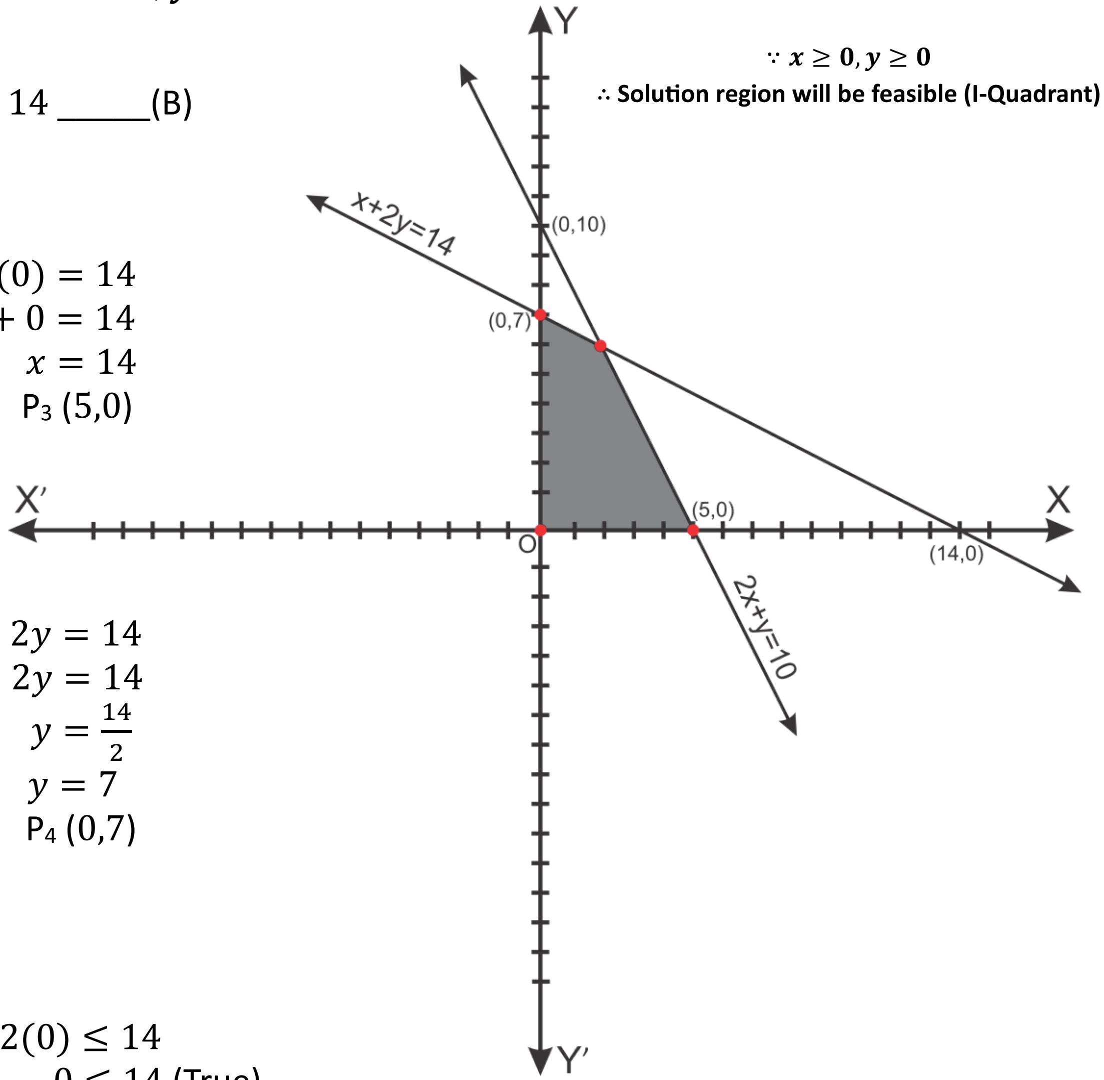
(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$$2(0) + 0 \leq 10$$
$0 \leq 10$ (True)

$$0 + 2(0) \leq 14$$
$0 \leq 14$ (True)

Solution Regions lie towards the origin



(e) Point of Intersection

$2x + y = 10 \;;\; x + 2y = 14$ ______ (D)

Multiply by '2'

$4x + 2y = 20$ ________ (C)

(C) − (D)

$$4x + 2y = 20$$
$$\pm x \pm 2y = \pm 14$$
$$3x = 6$$
$$x = \frac{6}{3}$$
$$x = 2$$

Put in equation (D)

$$2 + 2y = 14$$
$$2y = 14 - 2$$
$$2y = 12$$
$$y = \frac{12}{2}$$
$$y = 6$$

(f) Corner Points

$(0,0), (5,0), (0,7), (2,6)$

$\because f(x, y) = 2x + 3y$

put $x = 0, y = 0$

$f(0,0) = 2(0) + 3(0) = 0 + 0 = 0$

put $x = 5, y = 0$

$f(5,0) = 2(5) + 3(0) = 10 + 0 = 10$

put $x = 0, y = 7$

$f(0,7) = 2(0) + 3(7) = 0 + 21 = 21$

put $x = 2, y = 6$

$f(2,6) = 2(2) + 3(6) = 4 + 18 = 22$

Hence, z is maximized at $(2,6)$

Question # 6: Find minimum and maximum values of $z = 3x + y$; subject to the constraints:

$\mathbf{3x + 5y \geq 15}$; $\mathbf{x + 3y \leq 9}$; $\mathbf{x \geq 0; y \geq 0}$

(a) Associated Equations

$3x + 5y = 15$ ______(A)

$x + 3y = 9$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$$3x + 5(0) = 15$$
$$3x = 15$$
$$x = \frac{15}{3}$$
$$x = 5$$

$P_1\,(5,0)$

$$x + 3(0) = 9$$
$$x + 0 = 9$$
$$x = 9$$

$P_3\,(9,0)$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$3(0) + 5y = 15$	$0 + 3y = 9$
$5y = 15$	$3y = 9$
$y = \frac{15}{5}$	$y = \frac{9}{3}$
$y = 3$	$y = 3$
$P_2\ (0,3)$	$P_4\ (0,3)$

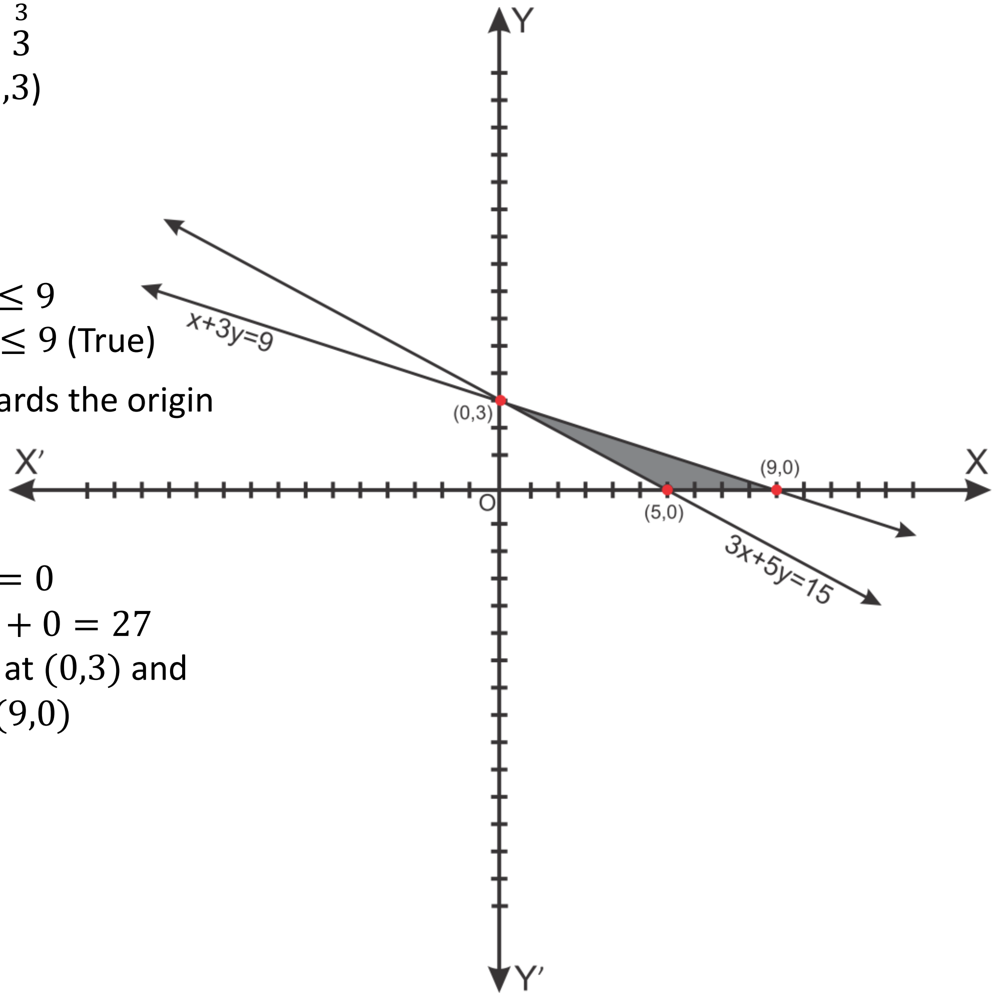
(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$3(0) + 5(0) \geq 15$	$0 + 3(0) \leq 9$
$0 \geq 15$ (False)	$0 \leq 9$ (True)
Solution region lies away from the origin	Solution region lies towards the origin

$\because x \geq 0, y \geq 0$

∴ Solution region will be feasible (I-Quadrant)



(e) (Croner Points)

$(0,3), (5,0), (9,0)$

$\because z = 3x + y$

put $x = 0, y = 3$

$z = 3(0) + 3 = 0 + 3 = 3$

put $x = 5, y = 0$

$z = 3(5) + 0 = 15 + 0 = 15$

put $x = 9, y = 0$

$z = 3(9) + 0 = 27 + 0 = 27$

Hence, z is minimized at $(0,3)$ and maximized at $(9,0)$