Important limits in Exercise 2.3

① $\lim_{x\to\infty} \frac{\text{number}}{x} = 0 \Rightarrow \frac{\text{number}}{\infty} = 0$ $\qquad \lim_{x\to a} f(x) = L$

② $\lim_{x\to 0} \frac{\text{number}}{x} = \infty$

③ $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$ $\qquad e \approx 2.718281\ldots$

④ $\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$

⑤ $\lim_{x\to 0} \frac{a^x-1}{x} = \ln a$, where $a>0$, $a\neq 1$.

⑥ $\lim_{x\to 0} \frac{(1+x)^n-1}{x} = n$, where $n\in\mathbb{Q}$.

⑦ $\lim_{x\to 0} \frac{\sin x}{x} = 1$

L-Hospital Rule.

If $\lim_{x\to a} \frac{f(x)}{g(x)}$ is indeterminate form $\left(\frac{0}{0}, \frac{\infty}{\infty}, 0^\infty, \infty^0, 0^0, 1^\infty\right)$ then we use L-Hospital Rule, i.e.,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

① $(k)' = 0$, $(x^n)' = nx^{n-1}$, $(x)' = 1$

$$\left(f(x)^n\right)' = n\, f(x)^{n-1} \cdot f'(x).$$

$a>0, a\neq 1$ $\quad (a^x)' = a^x \ln a$, $(e^x)' = 1.$

$(\sin ax)' = a\cos ax$

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0332-6297570
Mukhtar Abbas.

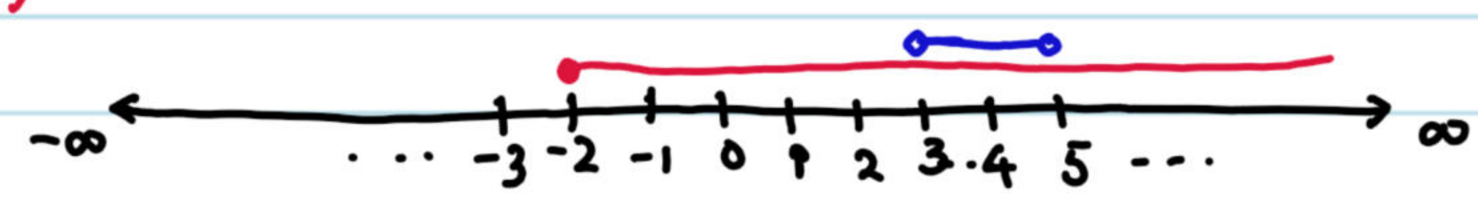
Exercise 2.3

Sindh Board.

Q # 01 Find the following:

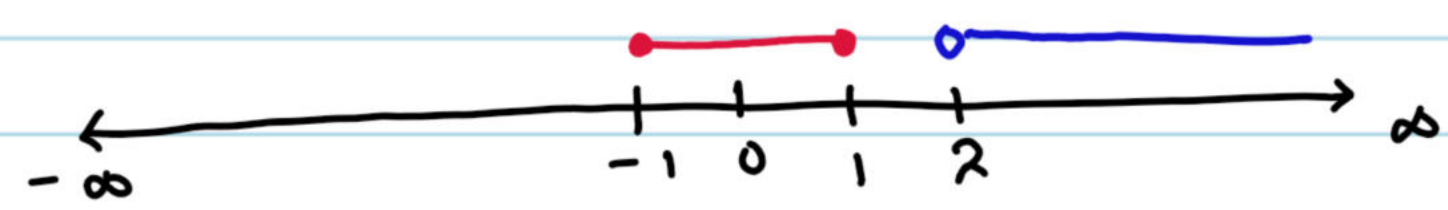
(i) $[2,\infty) \cup (3,5)$

$= [2,\infty)$



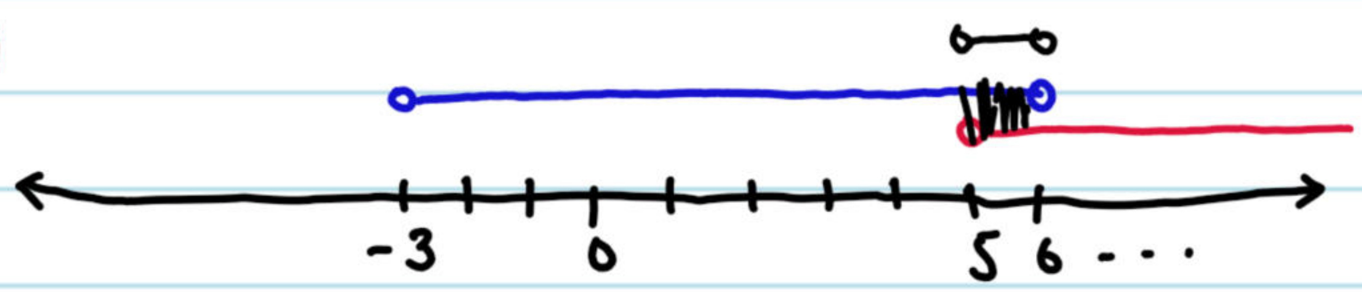
(ii) $[-1,1] - (2,\infty)$

$= [-1,1]$



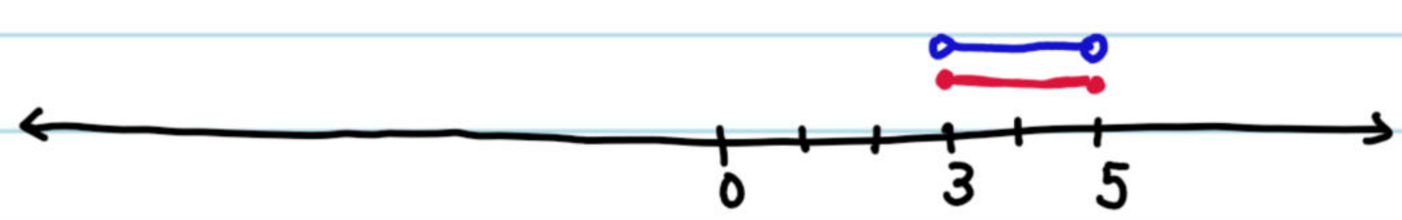
(iii) $(5,\infty) \cap (-3,6)$

$= (5,6)$



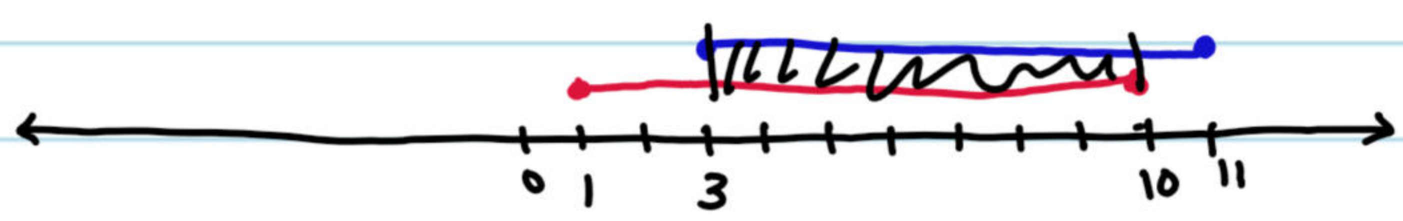
(iv) $[3,5] - (3,5)$

$= \{3,5\}$



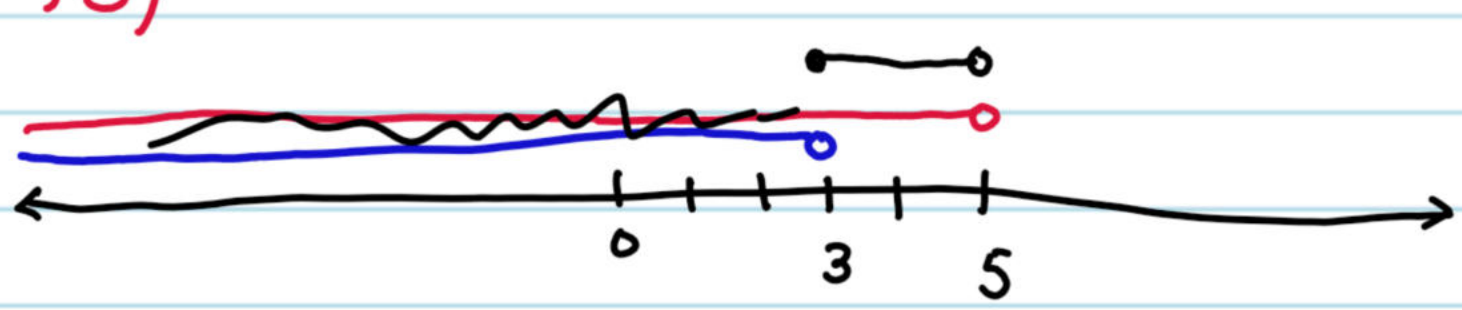
(v) $[1,10] \cap [3,11]$

$= [3,10]$



(vi) $(-\infty,5) - (-\infty,3)$

$= [3,5)$



Q # 02 Find the nth term and limit of the following sequences.

(i) $\frac{1}{2^0}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \ldots$

$$a_n = \frac{1}{2^{n-1}}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{2^{n-1}} = \frac{1}{2^{\infty-1}} = \frac{1}{2^{\infty}}$$

$$= \frac{1}{\infty} = 0$$ Ans.

(ii) $\frac{1 \cdot 2}{3 \cdot 4}, \frac{3 \cdot 4}{5 \cdot 6}, \frac{5 \cdot 6}{7 \cdot 8}, \ldots$

$1, 3, 5, \ldots$

$b_1 = 1$

$d = 2$

$b_n = b_1 + (n-1)d$

$= 1 + (n-1)2$

$= 1 + 2n - 2$

$= 2n - 1$

$$a_n = \frac{(2n-1)\cdot(2n)}{(2n+1)\cdot(2n+2)}$$

$$a_n = \frac{4n^2 - 2n}{4n^2 + 4n + 2n + 2}$$

$$a_n = \frac{4n^2 - 2n}{4n^2 + 6n + 2}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{4n^2 - 2n}{4n^2 + 6n + 2}$$

$$= \lim_{n\to\infty} \frac{\left(\frac{4n^2 - 2n}{n^2}\right)}{\left(\frac{4n^2 + 6n + 2}{n^2}\right)}$$

$$= \lim_{n\to\infty} \frac{\left(4 - \frac{2}{n}\right)}{\left(4 + \frac{6}{n} + \frac{2}{n^2}\right)}$$

$$= \frac{4 - 0}{4 + 0 + 0} = \frac{4}{4} = 1$$

Ans.

Q # 03 Find the limit of the following sequences whose nth terms are

(i) $a_n = \dfrac{1+5n}{7n}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1+5n}{7n}$$

$$= \lim_{n\to\infty} \frac{\left(\frac{1+5n}{n}\right)}{\left(\frac{7n}{n}\right)} = \lim_{n\to\infty} \frac{\left(\frac{1}{n}+5\right)}{7}$$

$$= \frac{0+5}{7} = \frac{5}{7}$$

(ii) $a_n = \dfrac{(3n-1)(n^4-n)}{(n^2+5)(n^3-7)} = \dfrac{3n^5-3n^2-n^4+n}{n^5-7n^2+5n^3-35}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\left(\frac{3n^5-3n^2-n^4+n}{n^5}\right)}{\left(\frac{n^5-7n^2+5n^3-35}{n^5}\right)}$$

$$= \lim_{n\to\infty} \frac{\left(3-\frac{3}{n^3}-\frac{1}{n}+\frac{1}{n^4}\right)}{\left(1-\frac{7}{n^3}+\frac{5}{n^2}-\frac{35}{n^5}\right)} = \frac{3-0-0+0}{1-0+0-0}$$

$$= \frac{3}{1} = 3 \quad \text{Ans}$$

(iii) $a_n = \dfrac{(n+1)!}{n!-(n+1)!}$

$7! = 7\cdot 6!$

$(n+1)! = (n+1)\,n!$

$$a_n = \frac{(n+1)\,n!}{n!-(n+1)\,n!}$$

$$a_n = \frac{(n+1)\cancel{n!}}{\cancel{n!}[1-(n+1)]}$$

$$a_n = \frac{n+1}{\cancel{1}-n-\cancel{1}} = \frac{n+1}{n} = 1+\frac{1}{n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(1+\frac{1}{n}\right) = 1+0 = 1 \quad \text{Ans}$$

Q # 04 Find the limit of the function

$$y = \frac{5x}{x+1} \quad \text{for} \quad x \to \infty.$$

$$\lim_{x \to \infty} y = \lim_{x \to \infty} \frac{5x}{x+1}$$

$$= \lim_{x \to \infty} \frac{\left(\frac{5\cancel{x}}{\cancel{x}}\right)}{\left(\frac{x+1}{x}\right)} = \lim_{x \to \infty} \frac{5}{\left(1 + \frac{1}{x}\right)}$$

$$= \frac{5}{1+0} = \frac{5}{1} = 5 \quad \text{Ans.}$$

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0332-6297570

AKHTAR ABBAS.