UNIT # 01: GRAPHICAL REPRESENTATION OF FUNCTIONS

Graph of Quadratic Functions by Factorization Method:

Suppose after factorizing $y = f(x) = ax^2 + bx + c$, we obtain $f(x) = a(x-h)(x-k)$, where $h \leq k$.
Also h, k are x-intercepts.

Case-I:

After factorization if $h < k$ then $f(x) = a(x-h)(x-k)$.

Case-II:

After factorization if $h < k$ then $f(x) = a(x-h)^2$.

Note:

To find value of "a" use given point on the graph.

EXERCISE # 1.1

Q.1: (i) $\boldsymbol{y = 3(x+1)(x-1)}$

Solution:

x-intercept:

put $y = 0$,

$$0 = 3(x+1)(x-1)$$
$$0 = (x+1)(x-1)$$
$$0 = (x+1) \qquad 0 = (x-1)$$
$$-1 = x \qquad 1 = x$$

y-intercept:

put $x = 0$,

$$y = 3(0+1)(0-1)$$
$$y = 3(1)(-1)$$
$$y = -3$$

Vertex:

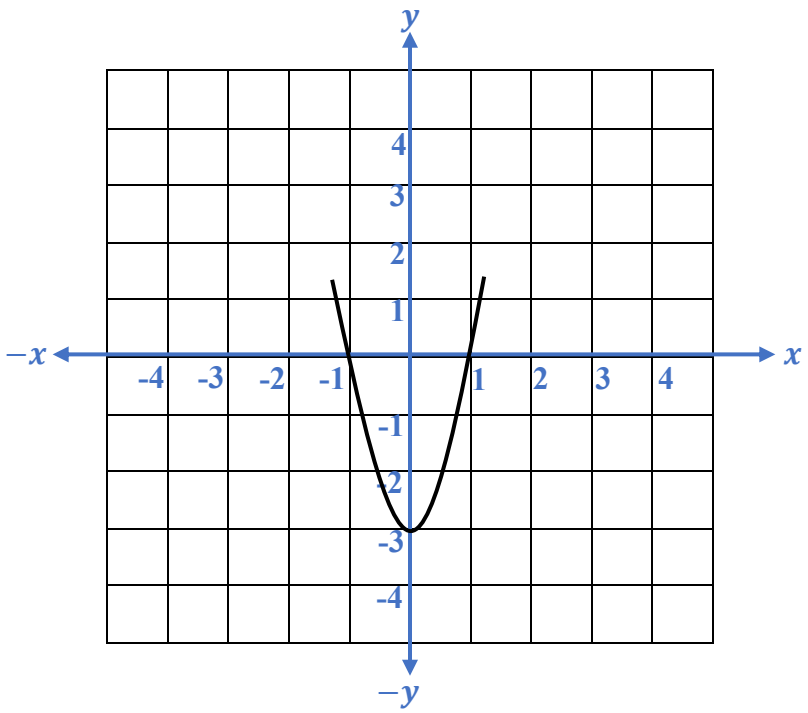
$$x = \frac{h+k}{2}$$
$$x = \frac{-1+1}{2}$$
$$x = \frac{0}{2} = 0$$

Put $x = 0$ in given equation,

$$y = 3(0+1)(0-1)$$
$$y = 3(1)(-1)$$
$$y = -3$$

So, $Vertext = (0, -3)$

Graph:



Prof: MUHAMMAD IRFAN DOGAR
MATHEMATICS INSTRUCTOR / TRAINER
(F.Sc. / I.Cs. / O/A-Level / Entry Test Expert)

Visit us at: MathCity.org
https://www.youtube.com/@mirfandogar
https://www.facebook.com/mirfandogar786
0300-1920009

Q.1: (ii) $y = -2(x-1)(x-2)$

Solution:

x-intercept:

put $y = 0$,

$$0 = -2(x-1)(x-2)$$
$$0 = (x-1)(x-2)$$
$$0 = (x-1) \qquad 0 = (x-2)$$
$$1 = x \qquad 2 = x$$

y-intercept:

put $x = 0$,

$$y = -2(0-1)(0-2)$$
$$y = -2(-1)(-2)$$
$$y = -5$$

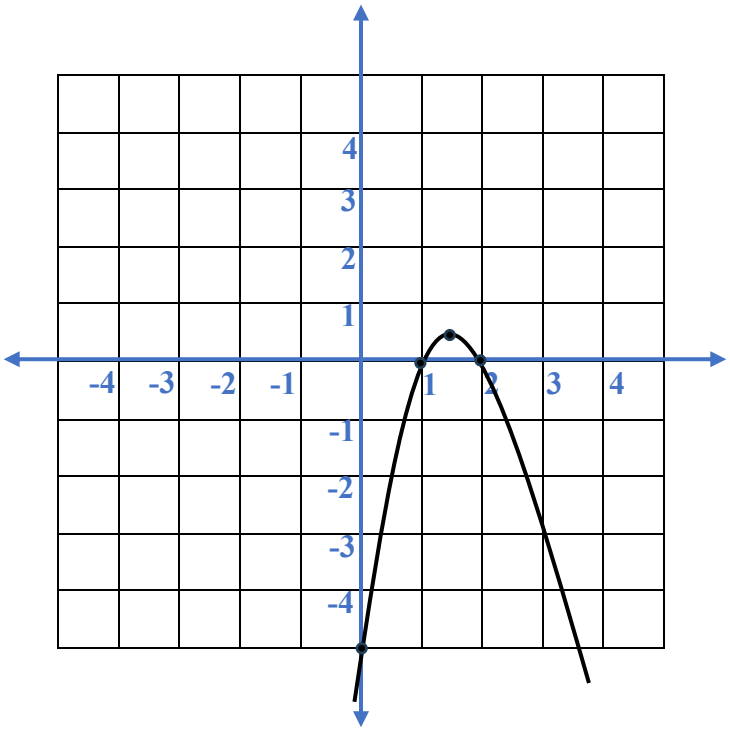
Vertex:

$$x = \frac{h+k}{2}$$
$$x = \frac{1+2}{2}$$
$$x = \frac{3}{2} = 1.5$$

Put $x = \frac{3}{2}$ in given equation,

$$y = -2\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-2\right)$$
$$y = -2\left(\frac{3-2}{2}\right)\left(\frac{3-4}{2}\right)$$
$$y = -2\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right) = \frac{1}{2} = 0.5$$
$$\rightarrow V\left(\frac{3}{2},\frac{1}{2}\right) = V(1.5,0.5)$$

Graph:



Prof: MUHAMMAD IRFAN DOGAR
MATHEMATICS INSTRUCTOR / TRAINER
(F.Sc. / I.Cs. / O/A-Level / Entry Test Expert)

Visit us at: MathCity.org
https://www.youtube.com/@mirfandogar
https://www.facebook.com/mirfandogar786
0300-1920009

Q.1: (iii) $y = (2x - 1)(2x + 1)$

Solution:

x-intercept:

put $y = 0$,

$$0 = (2x - 1)(2x + 1)$$

$0 = (2x - 1)$ $\quad$ $0 = (2x + 1)$

$\frac{1}{2} = x$ $\quad$ $-\frac{1}{2} = x$

y-intercept:

put $x = 0$,

$$y = (2(0) - 1)(2(0) + 1)$$
$$y = (-1)(1)$$
$$y = -1$$

Vertex:

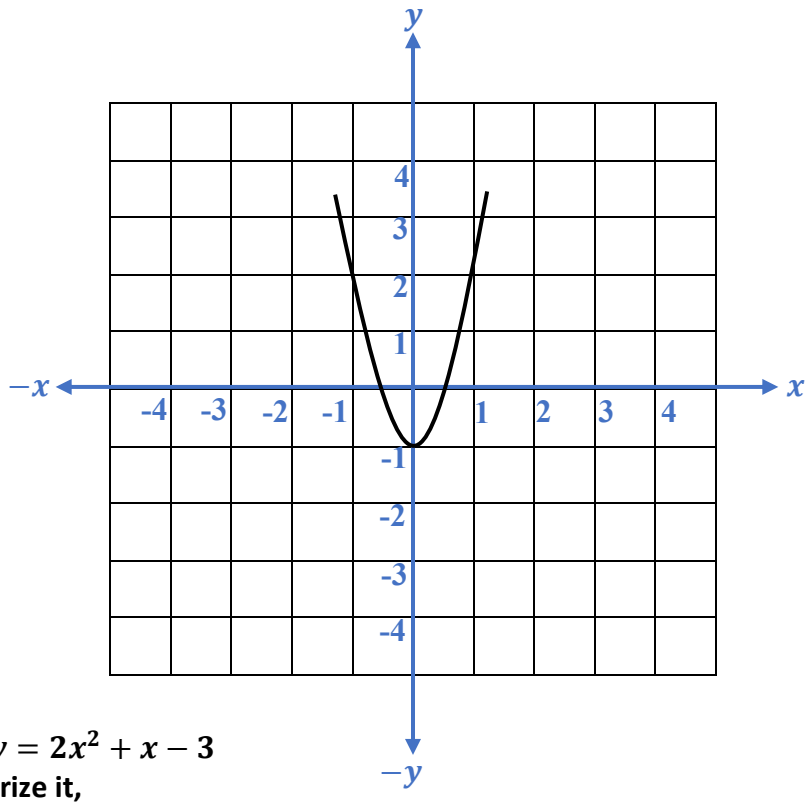
$$x = \frac{h + k}{2}$$
$$x = \frac{\frac{1}{2} + \left(-\frac{1}{2}\right)}{2}$$
$$x = \frac{0}{2} = 0$$

Put $x = 0$ in given equation,

$$y = (0 - 1)(0 + 1)$$
$$y = (-1)(1)$$
$$y = -1$$

➔ $V(0, -1)$

Graph:



Q.1: (iv) $y = 2x^2 + x - 3$

Solution: First we factorize it,

$$y = 2x^2 + 3x - 2x - 3$$
$$y = x(2x + 3) - 1(2x + 3)$$
$$y = (2x + 3)(x - 1)$$

x-intercept:

put $y = 0$,

y-intercept:

put $x = 0$,

$$0 = (2x+3)(x-1)$$

$$0 = (2x+3) \qquad 0 = (x-1)$$

$$\frac{-3}{2} = x \qquad 1 = x$$

$$y = (2(0)+3)(0-1)$$

$$y = (3)(-1)$$

$$y = -3$$

Vertex:

$$x = \frac{h+k}{2}$$

$$x = \frac{\frac{-3}{2}+1}{2}$$

$$x = \frac{\frac{-3+2}{2}}{2} = \frac{\frac{-1}{2}}{2} = -\frac{1}{4}$$

Put $x = 0$ in given equation,

$$y = \left(2\left(-\frac{1}{4}\right)+3\right)\left(-\frac{1}{4}-1\right)$$

$$y = \left(-\frac{1}{2}+3\right)\left(\frac{-1-4}{4}\right)$$

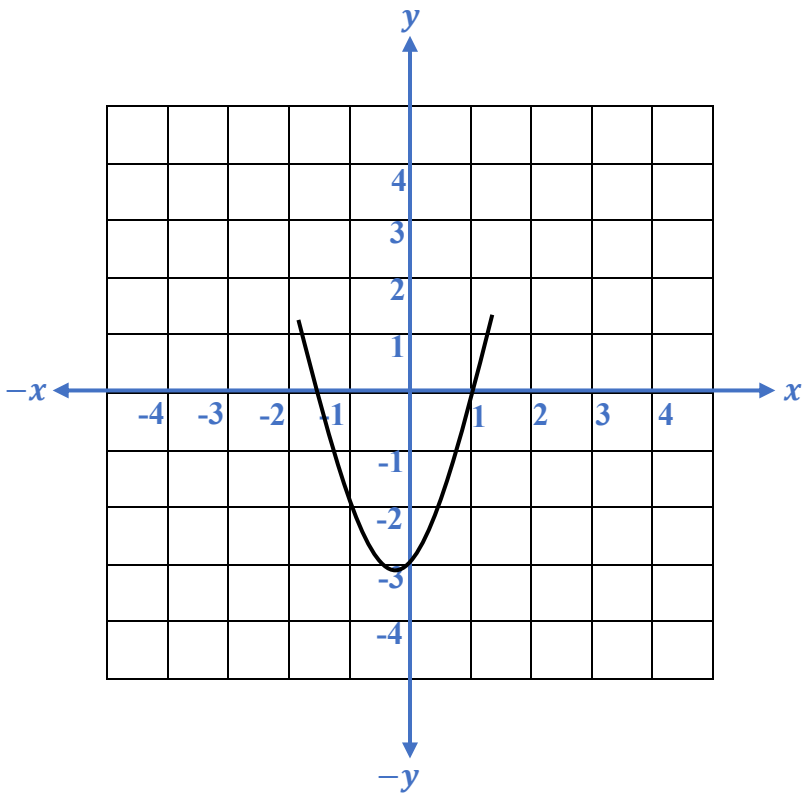
$$y = \left(\frac{-1+6}{2}\right)\left(\frac{-1-4}{4}\right)$$

$$y = \left(\frac{5}{2}\right)\left(\frac{-5}{4}\right) = \frac{-25}{8}$$

$$y = -3.125$$

➔ $V\left(-\frac{1}{4}, -\frac{25}{8}\right) = V(-0.25, -3.125)$

Graph:



Q.1: (v) $y = x^2 + 2x + 1$

Solution: First we factorize it,

$$y = x^2 + 2x + 1$$

$$y = x^2 + 2(x)(1) + (1)^2$$

Prof: MUHAMMAD IRFAN DOGAR
MATHEMATICS INSTRUCTOR / TRAINER
(F.Sc. / I.Cs. / O/A-Level / Entry Test Expert)

Visit us at: MathCity.org
https://www.youtube.com/@mirfandogar
https://www.facebook.com/mirfandogar786
0300-1920009

$$y = (x+1)^2$$

x-intercept:

put $y = 0$,

$$0 = (x+1)^2$$
$$0 = (x+1) \qquad 0 = (x+1)$$
$$-1 = x \qquad -1 = x$$

y-intercept:

put $x = 0$,

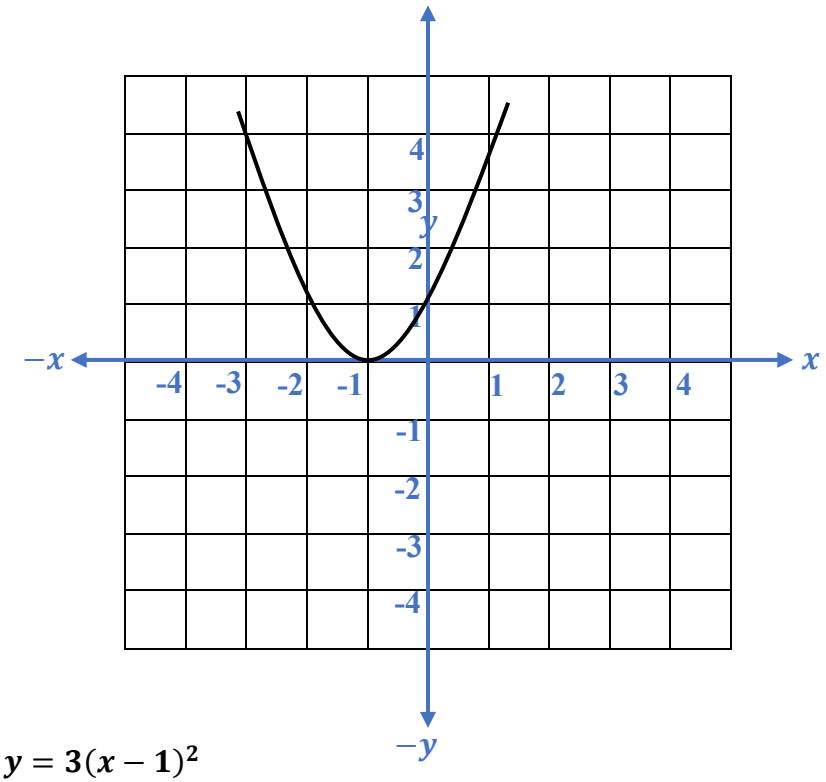
$$y = (0+1)^2$$
$$y = (1)^2$$
$$y = 1$$

Vertex:

Put $x = -1$ in given equation,

$$y = (-1)^2 + 2(-1) + 1$$
$$y = 1 - 2 + 1$$
$$y = 0$$
$$\rightarrow V(-1,0)$$

Graph:



Q.1: (vi) $y = 3(x-1)^2$

Solution:

x-intercept:

put $y = 0$,

$$0 = 3(x-1)^2$$
$$0 = (x-1)$$
$$1 = x$$

y-intercept:

put $x = 0$,

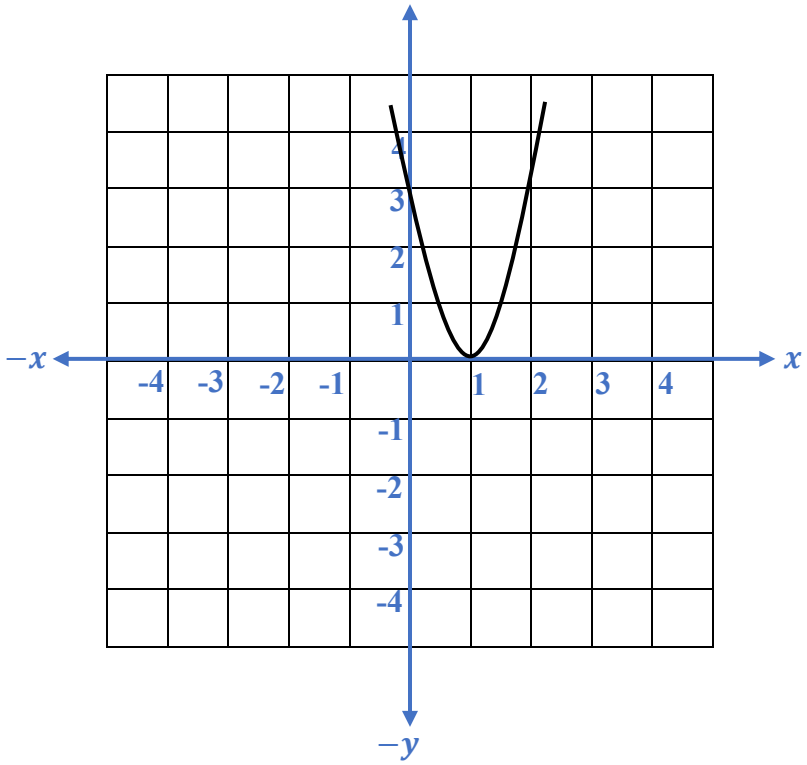
$$y = 3(0-1)^2$$
$$y = 3(-1)^2$$
$$y = 3$$

Vertex:

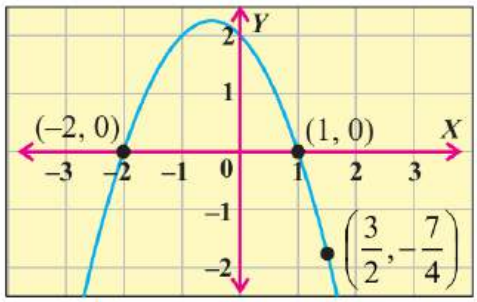
Put $x = 1$ in given equation,

$$y = 3(1-1)^2$$
$$y = 3(0)^2$$
$$y = 0$$
$$\rightarrow V(1,0)$$

Graph:



Q.2: Find the function whose graphs are shown in the following figures,
(i):



Solution: From figure it is clear that x-intercepts are, $h = -2$ & $k = 1$.
Therefore, the function can be written in the form,

$$f(x) = a(x-h)(x-k)$$
$$f(x) = a\big(x-(-2)\big)\big(x-(1)\big)$$
$$f(x) = a(x+2)(x-1)$$

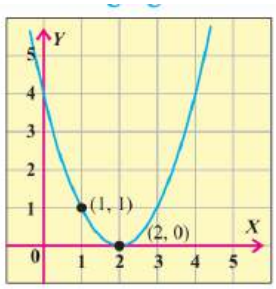
To find value of "a" we put point $(0,2)$,

$$2 = a(0+2)(0-1)$$
$$2 = a(2)(-1)$$
$$2 = a(-2)$$
$$-1 = a$$

Hence,

$$f(x) = -1(x+2)(x-1)$$
$$f(x) = -1(x^2 - x + 2x - 2)$$
$$f(x) = -1(x^2 + x - 2)$$
$$\boxed{f(x) = -x^2 - x + 2}$$

(ii)



Solution: From figure it is clear that x-intercept is, $h = 2$.
Therefore, the function can be written in the form,

$$f(x) = a(x-h)^2$$
$$f(x) = a(x-2)^2$$

To find value of "a" we put point $(1,1)$,

$$1 = a(1-2)^2$$
$$1 = a(-1)^2$$
$$1 = a(1)$$
$$1 = a$$

Hence,

$$f(x) = 1(x-2)^2$$
$$\boxed{f(x) = (x-2)^2}$$

Q.3: Find an equation of the graph whose x-intercepts are $x = -1$ and $x = -2$ and which passes through the point (-3,2).
Solution:
As given that x-intercepts are $h = -1$ and $k = -2$
Therefore, the function can be written in the form,

$$f(x) = a(x-h)(x-k)$$
$$f(x) = a\big(x-(-1)\big)\big(x-(-2)\big)$$
$$f(x) = a(x+1)(x+2)$$

To find value of "a" we put point $(-3,2)$,

$$2 = a(-3+1)(-3+2)$$
$$2 = a(-2)(-1)$$
$$2 = a(2)$$
$$1 = a$$

Hence,

$$f(x) = (1)(x+1)(x+2)$$
$$f(x) = x^2 + 2x + x + 2$$
$$\boxed{f(x) = x^2 + 3x + 2}$$

Q.4: Find an equation of the graph which touches x-axix at $x = -1$ and cuts y-axis at $y = -2$.
Solution: In this question there is only one x-intercept which is $h = -1$,
Therefore, the function $f(x)$ is,

$$f(x) = a(x-h)^2$$
$$f(x) = a\big(x-(-1)\big)^2$$
$$f(x) = a(x+1)^2$$

To find value of "a" we put y-intercept $(0,-2)$,

$$-2 = a(0+1)^2$$
$$-2 = a(1)^2$$

$$-2 = a$$

Hence,

$$f(x) = (-2)(x+1)^2$$
$$f(x) = (-2)(x^2 + 2x + 1)$$
$$\boxed{f(x) = -2x^2 - 4x - 2}$$

Q.5: Find an equation of the function whose graph cuts x-axis at $x = -1$ and $x = 1$, and whose vertex is at $(0, -2)$.

Solution:

As given that x-intercepts are $h = -1$ and $k = 1$

Therefore, the function can be written in the form,

$$f(x) = a(x-h)(x-k)$$
$$f(x) = a\big(x-(-1)\big)\big(x-(1)\big)$$
$$f(x) = a(x+1)(x-1)$$

To find value of "a" we put point $(0, -2)$,

$$-2 = a(0+1)(0-1)$$
$$-2 = a(1)(-1)$$
$$-2 = -a$$
$$2 = a$$

Hence,

$$f(x) = (2)(x+1)(x-1)$$
$$f(x) = (2)((x)^2 - (1)^2)$$
$$f(x) = (2)(x^2 - 1)$$
$$\boxed{f(x) = 2x^2 - 2}$$