Very Imp for Long Question

Exc 8.3

ADDITION and SUBTRACTION FORMULAS FOR INVERSE TRIG FN

i) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$

ii) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$

iii) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{(1-x^2)(1-y^2)}\right]$

iv) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left[xy + \sqrt{(1-x^2)(1-y^2)}\right]$

v) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$

vi) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right)$

vii) $\cot^{-1}x + \cot^{-1}y = \cot^{-1}\left(\dfrac{xy-1}{x+y}\right)$

viii) $\cot^{-1}x - \cot^{-1}y = \cot^{-1}\left(\dfrac{xy+1}{y-x}\right)$

ix) $\sec^{-1}x + \sec^{-1}y = \cos^{-1}\left(\dfrac{1-\sqrt{(x^2-1)(y^2-1)}}{xy}\right)$

x) $\sec^{-1}x - \sec^{-1}y = \cos^{-1}\left(\dfrac{1+\sqrt{(x^2-1)(y^2-1)}}{xy}\right)$

xi) $\csc^{-1}x \pm \csc^{-1}y = \sin^{-1}\left[\dfrac{\sqrt{y^2-1} \pm \sqrt{x^2-1}}{xy}\right]$

Q#1 Establish the Identities

i) $\sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) = \frac{\pi}{2}$

Sol As L.H.S

$$\sin^{-1}A + \sin^{-1}B = \sin^{-1}\left[A\sqrt{1-B^2} + B\sqrt{1-A^2}\right]$$

$$= \sin^{-1}\left[\frac{1}{3}\sqrt{1-\left(\frac{2\sqrt{2}}{3}\right)^2} + \frac{2\sqrt{2}}{3}\sqrt{1-\left(\frac{1}{3}\right)^2}\right]$$

$$= \sin^{-1}\left[\frac{1}{3}\sqrt{1-\frac{8}{9}} + \frac{2\sqrt{2}}{3}\sqrt{1-\frac{1}{9}}\right]$$

$$= \sin^{-1}\left[\frac{1}{3}\sqrt{\frac{1}{9}} + \frac{2\sqrt{2}}{3}\sqrt{\frac{8}{9}}\right]$$

$$= \sin^{-1}\left[\frac{1}{3}\cdot\frac{1}{3} + \frac{2\sqrt{2}}{3}\cdot\frac{2\sqrt{2}}{3}\right]$$

$$= \sin^{-1}\left[\frac{1}{9} + \frac{8}{9}\right] \Rightarrow \sin^{-1}\left[\frac{9}{9}\right]$$

$$= \sin^{-1}(1) = \frac{\pi}{2} \quad \text{R.H.S}$$

ii) $\cos^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$

Sol As

$$\cos^{-1}A + \cos^{-1}B = \cos^{-1}\left[AB - \sqrt{(1-A^2)(1-B^2)}\right]$$

$$= \cos^{-1}\left[\frac{3}{5}\cdot\frac{4}{5} - \sqrt{\left(1-\left(\frac{3}{5}\right)^2\right)\left(1-\left(\frac{4}{5}\right)^2\right)}\right]$$

$$= \cos^{-1}\left[\frac{12}{25} - \sqrt{\left(1-\frac{9}{25}\right)\left(1-\frac{16}{25}\right)}\right]$$

$$= \cos^{-1}\left[\frac{12}{25} - \sqrt{\left(\frac{16}{25}\right)\left(\frac{9}{25}\right)}\right]$$

$$= \cos^{-1}\left[\frac{12}{25} - \frac{12}{25}\right]$$

$$= \cos^{-1}(0) = \frac{\pi}{2} \qquad \text{R.H.S}$$

(iii) $\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$

Sol

$$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\left(\frac{3}{4}\right)\left(\frac{1}{7}\right)}\right)$$

$$= \tan^{-1}\left(\frac{\frac{21+4}{\cancel{28}}}{\frac{28-3}{\cancel{28}}}\right)$$

$$= \tan^{-1}\left(\frac{25}{25}\right) \Rightarrow \tan^{-1}(1) = \frac{\pi}{4}$$

(iv) $\cot^{-1}(2+\sqrt{3}) + \cot^{-1}(2-\sqrt{3}) = \frac{\pi}{2}$

Sol $\cot^{-1}A + \cot^{-1}B = \cot^{-1}\left(\frac{AB-1}{A+B}\right)$

$= \cot^{-1}\left(\frac{(2+\sqrt{3})(2-\sqrt{3})-1}{2+\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}}}\right)$

$= \cot^{-1}\left(\frac{[(2)^2-(\sqrt{3})^2]-1}{4}\right]$

$= \cot^{-1}\left(\frac{4-3-1}{4}\right) = \cot^{-1}(0) = \frac{\pi}{2}$

$\cot^{-1}(0) = \frac{\pi}{2} - \tan^{-1}(0)$

$= \frac{\pi}{2}$ Ans.

(V) $\operatorname{cosec}^{-1}(\sqrt{2}) + \operatorname{cosec}^{-1}(\sqrt{2}) = \frac{\pi}{2}$

$\operatorname{cosec}^{-1}A + \operatorname{cosec}^{-1}B = \sin^{-1}\left[\frac{\sqrt{B^2-1}+\sqrt{A^2-1}}{AB}\right]$

$= \sin^{-1}\left[\frac{\sqrt{2-1}+\sqrt{2-1}}{\sqrt{2}\sqrt{2}}\right]$

$= \sin^{-1}\left[\frac{2}{2}\right]$

$= \sin^{-1}[1] = \frac{\pi}{2}$

vi) $\sec^{-1}(2) + \sec^{-1}(2) = \frac{2\pi}{3}$

Sol

$$\sec^{-1}A + \sec^{-1}B = \cos^{-1}\left(\frac{1 - \sqrt{(A^2-1)(B^2-1)}}{AB}\right)$$

$$= \cos^{-1}\left(\frac{1 - \sqrt{(4-1)(4-1)}}{2 \cdot 2}\right)$$

$$= \cos^{-1}\left(\frac{1-3}{4}\right) = \cos^{-1}\left(\frac{-2}{4}\right)$$

$$= \cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$$

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vii) $\sin^{-1}(x) + \sin^{-1}(-x) = 0$

$$\sin^{-1}A + \sin^{-1}B = \sin^{-1}\left[A\sqrt{1-B^2} + B\sqrt{1-A^2}\right]$$

$$= \sin^{-1}\left[x\sqrt{1-x^2} + (-x)\sqrt{1-x^2}\right]$$

$$= \sin^{-1}\left[x\sqrt{1-x^2} - x\sqrt{1-x^2}\right]$$

$$= \sin^{-1}[0]$$

$$= 0$$

(viii) $\cos^{-1}x + \cos^{-1}(-x) = \pi$

Sol

$$\cos^{-1}A + \cos^{-1}B = \cos^{-1}\left[AB - \sqrt{(1-A^2)(1-B^2)}\right]$$

$$= \cos^{-1}\left[x(-x) - \sqrt{(1-x^2)(1-x^2)}\right]$$

$$= \cos^{-1}\left[-x^2 - \sqrt{(1-x^2)^{\cancel{2}}}\right]$$

$$= \cos^{-1}\left[-\cancel{x^2} - 1 + \cancel{x^2}\right]$$

$$= \cos^{-1}[-1] \qquad = \pi$$

(ix) $\tan^{-1}\left(\frac{\alpha}{\beta}\right) - \tan^{-1}\left(\frac{\alpha-\beta}{\alpha+\beta}\right) = \frac{\pi}{4}$

$$\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$$

$$= \tan^{-1}\left[\frac{\frac{\alpha}{\beta} - \frac{\alpha-\beta}{\alpha+\beta}}{1 + \frac{\alpha}{\beta}\cdot\frac{\alpha-\beta}{\alpha+\beta}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{\alpha(\alpha+\beta) - \beta(\alpha-\beta)}{\cancel{\beta(\alpha+\beta)}}}{\frac{\beta(\alpha+\beta) + \alpha(\alpha-\beta)}{\cancel{\beta(\alpha+\beta)}}}\right]$$

$$= \tan^{-1}\left[\frac{\alpha^2 + \cancel{\alpha\beta} - \cancel{\alpha\beta} + \beta^2}{\cancel{\alpha\beta} + \beta^2 + \alpha^2 - \cancel{\alpha\beta}}\right]$$

$$= \tan^{-1}\left[\frac{\alpha^2 + \beta^2}{\alpha^2 + \beta^2}\right] \Rightarrow \tan^{-1}(1) = \frac{\pi}{4}$$

(X) $\cot^{-1}(1) - \cot^{-1}(\sqrt{3}) = \frac{\pi}{12}$

Sol: $\cot^{-1}A - \cot^{-1}B = \cot^{-1}\left(\frac{AB+1}{B-A}\right)$

$$= \cot^{-1}\left(\frac{(1)(\sqrt{3})+1}{\sqrt{3}-1}\right)$$

$$= \cot^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$$

$$= \cot^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$$

$$= \cot^{-1}\left(\frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2-(1)^2}\right)$$

$$= \cot^{-1}\left(\frac{(\sqrt{3})^2+(1)^2+2(\sqrt{3})(1)}{3-1}\right)$$

$$= \cot^{-1}\left(\frac{3+1+2\sqrt{3}}{2}\right)$$

$$= \cot^{-1}\left(\frac{4+2\sqrt{3}}{2}\right)$$

$$= \cot^{-1}\left(2+\sqrt{3}\right)$$

$$= \frac{\pi}{2} - \tan^{-1}(2+\sqrt{3})$$

$$= \frac{\pi}{2} - \frac{5\pi}{12}$$

$$= \frac{\pi}{12} \quad \text{Ans.}$$

xi) $\operatorname{Cosec}^{-1}(\sqrt{2}) - \operatorname{Cosec}^{-1}(2) = \frac{\pi}{12}$

Sol $\operatorname{Cosec}^{-1} A - \operatorname{Cosec}^{-1} B = \operatorname{Sin}^{-1}\left[\frac{\sqrt{B^2-1} - \sqrt{A^2-1}}{AB}\right]$

$= \operatorname{Sin}^{-1}\left[\frac{\sqrt{4-1} - \sqrt{2-1}}{2\sqrt{2}}\right]$

$= \operatorname{Sin}^{-1}\left[\frac{\sqrt{3} - \sqrt{1}}{2\sqrt{2}}\right] \Rightarrow \operatorname{Sin}^{-1}\left[\frac{\sqrt{3}-1}{2\sqrt{2}}\right]$

$= \frac{\pi}{12}$

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xii) $\operatorname{Cos}^{-1}\left(\frac{1}{2}\right) - \operatorname{Cos}^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

Sol

$\operatorname{Cos}^{-1} A - \operatorname{Cos}^{-1} B = \operatorname{Cos}^{-1}\left[AB + \sqrt{(1-A^2)(1-B^2)}\right]$

$= \operatorname{Cos}^{-1}\left[\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \sqrt{\left(1-\frac{1}{4}\right)\left(1-\frac{3}{4}\right)}\right]$

$= \operatorname{Cos}^{-1}\left[\frac{\sqrt{3}}{4} + \sqrt{\frac{3}{4}\cdot\frac{1}{4}}\right]$

$= \operatorname{Cos}^{-1}\left[\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}\right] = \operatorname{Cos}^{-1}\left[\frac{2\sqrt{3}}{4}\right]$

$= \operatorname{Cos}^{-1}\left[\frac{\sqrt{3}}{2}\right] = \frac{\pi}{6}$ R.H.S

Q#2 Find

$\sec^{-1}\left(\frac{x+1}{x}\right) + \sec^{-1}\left(\frac{x-1}{x}\right)$ confirm the result is 0 if $x=0$

Sol $\sec^{-1}A + \sec^{-1}B = \cos^{-1}\left(\frac{1 - \sqrt{(A^2-1)(B^2-1)}}{AB}\right)$

$$= \cos^{-1}\left[\frac{1 - \sqrt{\left(\left(\frac{x+1}{x}\right)^2 - 1\right)\left(\left(\frac{x-1}{x}\right)^2 - 1\right)}}{\frac{x+1}{x} \cdot \frac{x-1}{x}}\right]$$

$$= \cos^{-1}\left[\frac{1 - \sqrt{\frac{\cancel{x^2}+1+2x-\cancel{x^2}}{x^2} \cdot \frac{\cancel{x^2}+1-2x-\cancel{x^2}}{x^2}}}{\frac{x^2-1}{x^2}}\right]$$

$$= \cos^{-1}\left[\frac{1 - \sqrt{\frac{(1)^2-(2x)^2}{x^4}}}{\frac{x^2-1}{x^2}}\right]$$

$$= \cos^{-1}\left[\frac{\frac{1}{1} - \frac{\sqrt{1-4x^2}}{x^2}}{\frac{x^2-1}{x^2}}\right]$$

$$= \cos^{-1}\left[\frac{\frac{x^2-\sqrt{1-4x^2}}{\cancel{x^2}}}{\frac{x^2-1}{\cancel{x^2}}}\right]$$

$$= \cos^{-1}\left[\frac{x^2-\sqrt{1-4x^2}}{x^2-1}\right]$$

If $x = 0$

$$= \cos^{-1}\left[\frac{0 - \sqrt{1-0}}{-1}\right]$$

$$= \cos^{-1}(1) = 0 \quad \text{Ans}$$

Apply inverse Cosecant Addition Formula:

$\operatorname{cosec}^{-1}\left(\frac{2x+1}{x}\right) + \operatorname{cosec}^{-1}\left(\frac{x+1}{x}\right)$ result is 0 If $x=0$

Sol: $\operatorname{cosec}^{-1}A + \operatorname{cosec}^{-1}B = \sin^{-1}\left[\frac{\sqrt{B^2-1} + \sqrt{A^2-1}}{AB}\right]$

$$= \sin^{-1}\left[\frac{\sqrt{\left(\frac{x+1}{x}\right)^2 - 1} + \sqrt{\left(\frac{2x+1}{x}\right)^2 - 1}}{\frac{2x+1}{x} \cdot \frac{x+1}{x}}\right]$$

$$= \sin^{-1}\left[\frac{\sqrt{\frac{\cancel{x^2}+2x+1-\cancel{x^2}}{x^2}} + \sqrt{\frac{4x^2+4x+1-x^2}{x^2}}}{\frac{(x+1)(2x+1)}{x^2}}\right]$$

$$= \sin^{-1}\left[\frac{\sqrt{\frac{2x+1}{x^2}} + \sqrt{\frac{3x^2+4x+1}{x^2}}}{\frac{2x^2+3x+1}{x^2}}\right]$$

$$= \sin^{-1}\left[\frac{\frac{\sqrt{2x+1}}{x} + \frac{\sqrt{3x^2+4x+1}}{x}}{\frac{2x^2+3x+1}{x^2}}\right]$$

$$= \sin^{-1}\left[\frac{\frac{\sqrt{2x+1} + \sqrt{3x^2+4x+1}}{x}}{\frac{2x^2+3x+1}{x^2}}\right]$$

$$= \sin^{-1}\left[\frac{x\left(\sqrt{2x+1} + \sqrt{3x^2+4x+1}\right)}{2x^2+3x+1}\right]$$

At $x = 0$

$$= \sin^{-1}\left[\frac{0\left(\sqrt{0+1} + \sqrt{0+0+1}\right)}{0+0+1}\right]$$

$$= \sin^{-1}(0)$$

$$= 0$$ Proved.

(iii) $\cot^{-1}(x-1) + \cot^{-1}(2x-1)$ and confirm that the result is $\frac{3\pi}{4}$ or $\left(-\frac{\pi}{4}\right)$ If $x = \pm 1$ wrong.

Sol

$$\cot^{-1}A + \cot^{-1}B = \cot^{-1}\left(\frac{AB-1}{A+B}\right)$$

$$= \cot^{-1}\left(\frac{(x-1)(2x-1) - 1}{x-1+2x-1}\right)$$

$$= \cot^{-1}\left(\frac{2x^2 - 3x + \cancel{1} - \cancel{1}}{3x - 2}\right)$$

$$= \cot^{-1}\left(\frac{x(2x-3)}{3x-2}\right)$$

At $x = 1$

$$= \cot^{-1}\left(\frac{1(-1)}{1}\right)$$

$$= \cot^{-1}(-1)$$

$$= \frac{\pi}{2} - \tan^{-1}(-1) = \frac{\pi}{2} - \left(-\frac{\pi}{4}\right)$$

$$= \frac{\pi}{2} + \frac{\pi}{4}$$

$$= \frac{2\pi + \pi}{4} = \frac{3\pi}{4}$$

At $x = -1$

$$\cot^{-1}\left(\frac{-1(-2-3)}{-3-2}\right) = \cot^{-1}\left(\frac{5}{-5}\right)$$

$$= \cot^{-1}(-1)$$

$$= \frac{\pi}{2} - \tan^{-1}(-1) = \frac{\pi}{2} - \left(-\frac{\pi}{4}\right)$$

$$= \frac{\pi}{2} + \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

Proved

Q# 3 Solve the following inverse Trigonometric eq algebraically.

(i) $\sin^{-1}(x-1) = \frac{\pi}{2}$

Sol

$$(x-1) = \sin\frac{\pi}{2}$$

$$x-1 = 1$$

$$x = 1+1$$

$$x = 2 \quad \text{Ans}$$

(ii) $\sin^{-1}x = \cos^{-1}\frac{5}{13}$

Sol

$$x = \sin\left[\cos^{-1}\frac{5}{13}\right]$$

Let $\theta = \cos^{-1}\frac{5}{13}$

$\cos\theta = \frac{5}{13}$

$B = 5 \qquad H = 13$

Using Phy Theorem

$$H^2 = P^2 + B^2$$

$$(13)^2 = P^2 + (5)^2$$

$$169 - 25 = P^2$$

$$P^2 = 144 \quad P = 12$$

$\sin\theta = \frac{12}{13}$

As $x = \sin\theta$

$x = \frac{12}{13}$ Ans:

(iii) $\tan^{-1} x = \sin^{-1} \frac{24}{25}$

Sol

$x = \tan\left[\sin^{-1} \frac{24}{25}\right]$

Let $\theta = \sin^{-1} \frac{24}{25}$

$\sin\theta = \frac{24}{25}$

$P = 24$ $H = 25$

Using phythagoras Th

$H^2 = P^2 + B^2$

$(25)^2 = (24)^2 + B^2$

$625 - 576 = B^2$

$B^2 = 49$

$B = 7$

$\tan\theta = \frac{P}{B} = \frac{24}{7}$

As

$x = \tan\theta$

$x = \frac{24}{7}$ Ans

(iv) $\cos^{-1}\left(x - \frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

Sol

$x - \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$x - \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

$$x = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \frac{\cancel{2}\sqrt{3}}{\cancel{2}}$$

$x = \sqrt{3}$ Ans.

(V) $\tan^{-1}\left(x + \frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

Sol

$$x + \frac{\sqrt{2}}{2} = \tan\frac{\pi}{4}$$

$$x + \frac{\sqrt{2}}{2} = 1$$

$$x = 1 - \frac{\sqrt{2}}{2}$$

$x = \frac{2-\sqrt{2}}{2}$ Ans

(Vi) $\cos^{-1}\left(x - \frac{1}{2}\right] = \frac{\pi}{3}$

Sol

$$x - \frac{1}{2} = \cos\frac{\pi}{3}$$

$$x - \frac{1}{2} = \frac{1}{2} \Rightarrow x = \frac{1}{2} + \frac{1}{2}$$

$x = \frac{2}{2}$ $x = 1$ Ans

(Vii) $6\cot^{-1}(2x) - 5\pi = 0$

Sol

$$6\cot^{-1}(2x) = 5\pi$$

$$\cot^{-1}(2x) = \frac{5\pi}{6}$$

$$2x = \cot\left(\frac{5\pi}{6}\right)$$

$$2x = \cot(150)$$

$$2x = \frac{1}{\tan 150} \Rightarrow 2x = \frac{1}{-1/\sqrt{3}}$$

$$2x = -\sqrt{3} \qquad \boxed{x = -\frac{\sqrt{3}}{2}}$$

viii) $9(\sin^{-1}x)^2 - \pi^2 = 0$

Sol

$$\sqrt{(\sin^{-1}x)^2} = \sqrt{\frac{\pi^2}{9}}$$

$$\sin^{-1}x = \pm\frac{\pi}{3}$$

$$x = \sin\left(\pm\frac{\pi}{3}\right) = \pm\frac{\sqrt{3}}{2} \quad \text{Ans.}$$

(ix) $\pi - 2\sin^{-1}x = 2\pi$

Sol

$$\pi - 2\pi = 2\sin^{-1}x$$

$$-\frac{\pi}{2} = \sin^{-1}x$$

$$x = \sin\left(-\frac{\pi}{2}\right)$$

$$x = -\sin\frac{\pi}{2}$$

$$x = -1 \quad \text{Ans}$$

(x) $4(\tan^{-1}x)^2 - 3\pi(\tan^{-1}x) - \pi^2 = 0$

Sol

Let $\tan^{-1}x = y$

$$4y^2 - 3\pi y - \pi^2 = 0$$

$a = 4 \quad b = -3\pi \quad c = -\pi^2$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-3\pi) \pm \sqrt{(-3\pi)^2 - 4(4)(-\pi^2)}}{2(4)}$$

$$y = \frac{3\pi \pm \sqrt{9\pi^2 + 16\pi^2}}{8}$$

$$y = \frac{3\pi \pm 5\pi}{8}$$

$y = \frac{3\pi + 5\pi}{8}$ | $y = \frac{3\pi - 5\pi}{8}$

$y = \frac{8\pi}{8}$ | $y = -\frac{2\pi}{8}$

$y = \pi$ | $y = -\frac{\pi}{4}$

$\tan^{-1}x = \pi$ | $\tan^{-1}x = -\frac{\pi}{4}$

$x = \tan(\pi)$ | $x = \tan\left(-\frac{\pi}{4}\right)$

$x = 0$ | $x = -\tan\frac{\pi}{4}$

$x = -1$

S.S $= \{0, -1\}$

(xi) $\tan^{-1}\left(\dfrac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\dfrac{\pi}{4}-\dfrac{x}{2}$

where $x \in \left(0, \dfrac{\pi}{2}\right)$

Sol

$1+\cos x = 2\cos^2\dfrac{x}{2}$ | $1-\cos x = 2\sin^2\dfrac{x}{2}$

$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$ | $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$$\tan^{-1}\left(\frac{\sqrt{2\cos^2 x/2}+\sqrt{2\sin^2 x/2}}{\sqrt{2\cos^2 x/2}-\sqrt{2\sin^2 x/2}}\right)=\frac{\pi}{4}-\frac{x}{2}$$

$$\tan^{-1}\left(\frac{\sqrt{2}\cos x/2+\sqrt{2}\sin x/2}{\sqrt{2}\cos x/2-\sqrt{2}\sin x/2}\right)=\frac{\pi}{4}-\frac{x}{2}$$

$$\tan^{-1}\left(\frac{\cancel{\sqrt{2}}(\cos x/2+\sin x/2)}{\cancel{\sqrt{2}}(\cos x/2-\sin x/2)}\right)=\frac{\pi}{4}-\frac{x}{2}$$

$$\tan^{-1}\left(\frac{\cos x/2+\sin x/2}{\cos x/2-\sin x/2}\right)=\frac{\pi}{4}-\frac{x}{2}$$

$$\frac{\cos x/2+\sin x/2}{\cos x/2-\sin x/2}=\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$\div$ ing by $\cos\dfrac{x}{2}$

$$\frac{1+\tan x/2}{1-\tan x/2}=\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\frac{\tan\frac{\pi}{4} + \tan\frac{x}{2}}{1 - \tan\frac{\pi}{4}\cdot\tan\frac{x}{2}} = \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\cancel{\tan}\left(\frac{\pi}{4} + \frac{x}{2}\right) = \cancel{\tan}\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\cancel{\frac{\pi}{4}} + \frac{x}{2} = \cancel{\frac{\pi}{4}} - \frac{x}{2} \Rightarrow \frac{x}{2} + \frac{x}{2} = 2x = 0$$

$$\boxed{x = 0} \text{ A}$$

Q#4 Find the values of x

(i) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1} x = \frac{\pi}{2}$

Sol

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1} x = \frac{\pi}{2}$$

$$\frac{\pi}{3} + \cos^{-1} x = \frac{\pi}{2}$$

$$\cos^{-1} x = \frac{\pi}{2} - \frac{\pi}{3}$$

$$\cos^{-1} x = \frac{3\pi - 2\pi}{6}$$

$$\cos^{-1} x = \frac{\pi}{6} \Rightarrow x = \cos\frac{\pi}{6} \Rightarrow x = \frac{\sqrt{3}}{2}$$

(ii) $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}(2x) = \frac{\pi}{6}$

Sol

$$\frac{\pi}{6} + \cos^{-1} 2x = \frac{\pi}{6}$$

$$\cos^{-1} 2x = \frac{\pi}{6} - \frac{\pi}{6}$$

$$\cos^{-1} 2x = 0$$

$$2x = \cos(0)$$

$$2x = 1 \qquad \boxed{x = \frac{1}{2}} \text{ Ans.}$$

(iii) $\sin^{-1} x + \cos^{-1} \frac{4}{5} = \pi$

Sol

$$\sin^{-1} x = \left(\pi - \cos^{-1} \frac{4}{5}\right)$$

$$x = \sin\left(\pi - \cos^{-1} \frac{4}{5}\right)$$

$\boxed{\sin(\pi - \theta) = \sin\theta}$ Allied Angle.

$$x = \sin\left[\cos^{-1} \frac{4}{5}\right]$$

Let $\theta = \cos^{-1} \frac{4}{5}$ $\qquad \cos\theta = \frac{4}{5}$

$B = 4 \quad H = 5$

Using Phy Theorem

$$H^2 = P^2 + B^2$$

$$25 = P^2 + 16$$

$$P^2 = 9$$

$$P = 3$$

$$\sin\theta = \frac{3}{5}$$

$$x = \sin\theta$$

$$\boxed{x = \frac{3}{5}} \text{ Ans.}$$

(iv) $\tan^{-1} x - \tan^{-1} \frac{1}{x} = \frac{\pi}{4}$

$$\tan^{-1} A - \tan^{-1} B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$$

$$\tan^{-1}\left(\frac{\frac{x}{1} - \frac{1}{x}}{1 + \cancel{x} \cdot \frac{1}{\cancel{x}}}\right) = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{\frac{x^2-1}{x}}{2}\right) = \frac{\pi}{4}$$

$$\frac{x^2-1}{2x} = \tan\left(\frac{\pi}{4}\right)$$

$$\frac{x^2-1}{2x} = 1 \Rightarrow x^2 - 1 = 2x$$

$$x^2 - 2x - 1 = 0$$

$a = 1$, $b = -2$, $c = -1$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+4}}{2} \Rightarrow x = \frac{2 \pm \sqrt{8}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$\boxed{x = 1 \pm \sqrt{2}}$$

Q#5 Prove that

(i) $\sin^{-1}\left(\frac{5}{13}\right) - \sin^{-1}\left(\frac{63}{65}\right) = \cos^{-1}\left(\frac{3}{5}\right)$

sol L.H.S

$\sin^{-1}\left(\frac{5}{13}\right) - \sin^{-1}\left(\frac{63}{65}\right)$

Let $\alpha = \sin^{-1}\frac{5}{13}$	$\beta = \sin^{-1}\frac{63}{65}$
$\sin\alpha = \frac{5}{13}$	$\sin\beta = \frac{63}{65}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$P = 5$ $H = 13$	$P = 63$ $H = 65$
Using Phy Theorem	Using Phy Theorem
$H^2 = P^2 + B^2$	$H^2 = P^2 + B^2$
$(13)^2 = (5)^2 + B^2$	$(65)^2 = (63)^2 + B^2$
$169 - 25 = B^2$	$4225 - 3969 = B^2$
$B^2 = 144$	$B^2 = 256$
$B = 12$	$B = 16$
$\cos\alpha = \frac{12}{13}$	$\cos\beta = \frac{16}{65}$

$\cos(\alpha-\beta) = \frac{12}{13}\cdot\frac{16}{65} + \frac{5}{13}\cdot\frac{63}{65}$

$$\cos(\alpha-\beta) = \frac{192}{845} + \frac{315}{845}$$

$$\cos(\alpha-\beta) = \frac{507}{845}$$

$$\alpha-\beta = \cos^{-1}\left(\frac{507}{845}\right)$$

$$\sin^{-1}\frac{5}{13} - \sin^{-1}\frac{63}{65} = \cos^{-1}\left(\frac{3}{5}\right)$$

(ii)

$$\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\frac{77}{36}$$

Sol L.H.S

$$\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5}$$

Let

$\alpha = \sin^{-1}\frac{8}{17}$	$\beta = \sin^{-1}\frac{3}{5}$
$\sin\alpha = \frac{8}{17}$	$\sin\beta = \frac{3}{5}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$P = 8$ $H = 17$	$P = 3$ $H = 5$
Using Phythagoras Theorem	Using Phythagoras Theorem
$H^2 = P^2 + B^2$	$H^2 = P^2 + B^2$
$(17)^2 = (8)^2 + B^2$	$(5)^2 = (3)^2 + B^2$

$289 - 64 = B^2$
$B^2 = 225$
$B = 15$

$\tan\alpha = \frac{8}{15}$

$25 - 9 = B^2$
$B^2 = 16$
$B = 4$

$\tan\beta = \frac{3}{4}$

$$\tan(\alpha+\beta) = \frac{\frac{8}{15} + \frac{3}{4}}{1 - \frac{8}{15} \cdot \frac{3}{4}}$$

$$\tan(\alpha+\beta) = \left(\frac{\frac{32+45}{60}}{\frac{60-24}{60}}\right)$$

$$\alpha+\beta = \tan^{-1}\left(\frac{77}{36}\right)$$

$$\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\left(\frac{77}{36}\right) \quad \text{proved.}$$

(iii) $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

Sol: L.H.S

$$\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right)$$

Let

$\alpha = \sin^{-1}\frac{3}{5}$	$\beta = \cos^{-1}\frac{12}{13}$
$\sin\alpha = \frac{3}{5}$	$\cos\beta = \frac{12}{13}$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$P = 3 \quad H = 5$	$B = 12 \quad H = 13$
Using Phythagores	Using Phy Theorem
$H^2 = P^2 + B^2$	$H^2 = P^2 + B^2$
$(5)^2 = (3)^2 + B^2$	$(13)^2 = P^2 + (12)^2$
$25 - 9 = B^2$	$169 - 144 = P^2$
$B = 4$	$P^2 = 25$
$\cos\alpha = \frac{4}{5}$	$P = 5$
	$\sin\beta = \frac{5}{13}$

$$\sin(\alpha+\beta) = \frac{3}{5}\cdot\frac{12}{13} + \frac{4}{5}\cdot\frac{5}{13}$$

$$\alpha+\beta = \sin^{-1}\left(\frac{36}{65} + \frac{20}{65}\right)$$

$$\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{12}{13} = \sin^{-1}\left(\frac{56}{65}\right)$$

vimp

(iv) $\text{Tan}^{-1}\left(\frac{1}{4}\right) + \text{Tan}^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\text{Tan}^{-1}\frac{4}{3}$

Sol L.H.S

$\text{Tan}^{-1}\frac{1}{4} + \text{Tan}^{-1}\frac{2}{9}$

Let $\alpha = \text{Tan}^{-1}\frac{1}{4}$ $\qquad$ $\beta = \text{Tan}^{-1}\frac{2}{9}$

$\text{Tan}\,\alpha = \frac{1}{4}$ $\qquad$ $\text{Tan}\,\beta = \frac{2}{9}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha+\beta) = \frac{1/4 + 2/9}{1 - \frac{1}{4}\cdot\frac{2}{9}}$$

$$\tan(\alpha+\beta) = \frac{\frac{9+8}{\cancel{36}}}{\frac{36-2}{\cancel{36}}}$$

$$\alpha+\beta = \tan^{-1}\left(\frac{\cancel{17}^{1}}{\cancel{34}_{2}}\right)$$

$$\text{Tan}^{-1}\frac{1}{4} + \text{Tan}^{-1}\frac{2}{9} = \text{Tan}^{-1}\left(\frac{1}{2}\right)$$

Now

Let $x = \text{Tan}^{-1}\frac{1}{2} \Rightarrow \text{Tan}\,x = \frac{1}{2}$

Use this Formula

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\tan 2x = \frac{2 \cdot \frac{1}{2}}{1 - \left(\frac{1}{2}\right)^2}$$

$$\tan 2x = \frac{1}{\frac{4}{1} - \frac{1}{4}} \Rightarrow \tan 2x = \frac{1}{\frac{3}{4}}$$

$$\tan 2x = \frac{4}{3} \Rightarrow 2x = \tan^{-1} \frac{4}{3}$$

$$x = \frac{1}{2} \tan^{-1} \frac{4}{3}$$

$$\tan^{-1}\left(\frac{1}{2}\right) = \frac{1}{2} \tan^{-1} \frac{4}{3}$$

$$\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \frac{1}{2} \tan^{-1} \frac{4}{3}$$ Proved.

(v) $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$

Let $\alpha = \cos^{-1} \frac{4}{5}$	$\beta = \cos^{-1} \frac{12}{13}$
$\cos\alpha = \frac{4}{5}$	$\cos\beta = \frac{12}{13}$

$$\cos(\alpha+\beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$B = 4 \quad H = 5$	$B = 12 \quad H = 13$
Using Phy Theorem	Using Phy Theorem
$H^2 = P^2 + B^2$	$H^2 = P^2 + B^2$

$(5)^2 = P^2 + (4)^2$	$(13)^2 = P^2 + (12)^2$
$25 - 16 = P^2$	$169 - 144 = P^2$
$P = 3$	$P = 5$
$\sin\alpha = \frac{3}{5}$	$\sin\beta = \frac{5}{13}$

$$\cos(\alpha+\beta) = \frac{4}{5} \cdot \frac{12}{13} - \frac{3}{5} \cdot \frac{5}{13}$$

$$= \frac{48}{65} - \frac{15}{65}$$

$$\alpha + \beta = \cos^{-1}\left(\frac{33}{65}\right)$$

$$\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\left(\frac{33}{65}\right) \quad \text{Proved.}$$

(vi) $2\,\mathrm{Tan}^{-1}\left(\frac{3}{4}\right) - \mathrm{Tan}^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$

Sol

$$\mathrm{Tan}^{-1}A + \mathrm{Tan}^{-1}B = \mathrm{Tan}^{-1}\left(\frac{A+B}{1-AB}\right)$$

replace B by A

$$\mathrm{Tan}^{-1}A + \mathrm{Tan}^{-1}A = \mathrm{Tan}^{-1}\left(\frac{A+A}{1-AA}\right)$$

$$\boxed{2\,\mathrm{Tan}^{-1}A = \mathrm{Tan}^{-1}\left(\frac{2A}{1-A^2}\right)}$$

$$2\ \mathrm{Tan}^{-1}\left(\frac{3}{4}\right) = \mathrm{Tan}^{-1}\left(\frac{2\cdot\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right)$$

$$= \mathrm{Tan}^{-1}\left(\frac{3/2}{7/16}\right) = \mathrm{Tan}^{-1}\left(\frac{3}{\cancel{2}_1}\times\frac{\cancel{16}^8}{7}\right)$$

$$2\ \mathrm{Tan}^{-1}\left(\frac{3}{4}\right) = \mathrm{Tan}^{-1}\left(\frac{24}{7}\right)$$

$$\mathrm{Tan}^{-1}\left(\frac{24}{7}\right) - \mathrm{Tan}^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$$

$$\mathrm{Tan}^{-1}A - \mathrm{Tan}^{-1}B = \mathrm{Tan}^{-1}\left(\frac{A-B}{1+AB}\right)$$

$$= \mathrm{Tan}^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\cdot\frac{17}{31}}\right)$$

$$= \mathrm{Tan}^{-1}\left(\frac{\frac{744-119}{\cancel{217}}}{\frac{217+408}{\cancel{217}}}\right)$$

$$= \mathrm{Tan}^{-1}\left(\frac{625}{625}\right)$$

$$= \mathrm{Tan}^{-1}(1)$$

$$= \frac{\pi}{4} \quad \text{R.H.S.}$$

viii) $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$

Sol.

$$\tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

L.H.S

$$=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{5}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{5}\right)}\right)+\tan^{-1}\left(\frac{1}{8}\right)$$

$$=\tan^{-1}\left(\frac{\frac{5+2}{\cancel{10}}}{\frac{10-1}{\cancel{10}}}\right)+\tan^{-1}\left(\frac{1}{8}\right)$$

$$=\tan^{-1}\left(\frac{7}{9}\right)+\tan^{-1}\left(\frac{1}{8}\right)$$

Again $\tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)$

$$=\tan^{-1}\left(\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{72}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{56+9}{\cancel{72}}}{\frac{72-7}{\cancel{72}}}\right)$$

$$=\tan^{-1}\left(\frac{65}{65}\right)$$

$$=\tan^{-1}(1)=\frac{\pi}{4}$$

(VIII) $\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right) = \frac{\pi}{2}$

Sol

$$\sin^{-1}A + \sin^{-1}B = \sin^{-1}\left[A\sqrt{1-B^2} + B\sqrt{1-A^2}\right]$$

L.H.S

$$= \sin^{-1}\left[\frac{4}{5}\sqrt{1-\frac{25}{169}} + \frac{5}{13}\sqrt{1-\frac{16}{25}}\right] + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left[\frac{4}{5}\cdot\frac{12}{13} + \frac{5}{13}\cdot\frac{3}{5}\right] + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left[\frac{48}{65} + \frac{15}{65}\right] + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\frac{63}{65} + \sin^{-1}\frac{16}{65}$$

Again use

$$\sin^{-1}A + \sin^{-1}B = \sin^{-1}\left[A\sqrt{1-B^2} + B\sqrt{1-A^2}\right]$$

$$\sin^{-1}\left[\frac{63}{65}\sqrt{1-\frac{256}{4225}} + \frac{16}{65}\sqrt{1-\frac{3969}{4225}}\right]$$

$$\sin^{-1}\left[\frac{63}{65}\cdot\frac{63}{65} + \frac{16}{65}\cdot\frac{16}{65}\right]$$

$$\sin^{-1}\left[\frac{4225}{4225}\right]$$

$$\sin^{-1}(1)$$

$$= \frac{\pi}{2}$$

(IX) $\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{8}{19}\right) = \frac{\pi}{4}$

Sol

$$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

L.H.S

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\cdot\frac{3}{5}}\right) - \tan^{-1}\left(\frac{8}{19}\right)$$

$$= \tan^{-1}\left(\frac{\frac{15+12}{\cancel{20}}}{\frac{20-9}{\cancel{20}}}\right) - \tan^{-1}\left(\frac{8}{19}\right)$$

$$= \tan^{-1}\left(\frac{27}{11}\right) - \tan^{-1}\left(\frac{8}{19}\right)$$

Again

$$\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$$

$$= \tan^{-1}\left(\frac{\frac{27}{11}-\frac{8}{19}}{1+\frac{27}{11}\cdot\frac{8}{19}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{513-88}{209}}{\frac{209+216}{209}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{425}{209}}{\frac{425}{209}}\right)$$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$

(x) $2\,\text{Tan}^{-1}\left(\frac{1}{5}\right) + \text{Sec}^{-1}\left(\frac{5\sqrt{2}}{7}\right) + 2\,\tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

Sol

$$= 2\left[\text{Tan}^{-1}\frac{1}{5} + \text{Tan}^{-1}\frac{1}{8}\right] + \text{Sec}^{-1}\left(\frac{5\sqrt{2}}{7}\right) =$$

$$= 2\left[\text{Tan}^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\cdot\frac{1}{8}}\right)\right] + \text{Cos}^{-1}\left(\frac{7}{5\sqrt{2}}\right)$$

$$= 2\left[\text{Tan}^{-1}\left(\frac{\frac{13}{\cancel{40}}}{\frac{39}{\cancel{40}}}\right)\right] + \text{Cos}^{-1}\left(\frac{7}{5\sqrt{2}}\right)$$

$$= 2\cdot\text{Tan}^{-1}\left(\frac{1}{3}\right) + \text{Cos}^{-1}\left(\frac{7}{5\sqrt{2}}\right)$$

$$2\,\text{Tan}^{-1}A = \text{Tan}^{-1}\left(\frac{2A}{1-A^2}\right)$$

$$= \text{Tan}^{-1}\left(\frac{2/3}{1-1/9}\right) + \text{Cos}^{-1}\left(\frac{7}{5\sqrt{2}}\right)$$

$$= \text{Tan}^{-1}\left(\frac{2/3}{8/9}\right) + \text{Cos}^{-1}\left(\frac{7}{5\sqrt{2}}\right)$$

$$= \text{Tan}^{-1}\left(\frac{\cancel{2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{9}^{3}}{\cancel{8}_{4}}\right) + \text{Cos}^{-1}\left(\frac{7}{5\sqrt{2}}\right)$$

$$= \text{Tan}^{-1}\left(\frac{3}{4}\right) + \text{Cos}^{-1}\left(\frac{7}{5\sqrt{2}}\right)$$

Let $\alpha = \cos^{-1}\left(\frac{7}{5\sqrt{2}}\right)$

$$\cos\alpha = \frac{7}{5\sqrt{2}}$$

$B = 7 \qquad H = 5\sqrt{2}$

Using Phythagoras

$$H^2 = P^2 + B^2$$

$$(5\sqrt{2})^2 - (7)^2 = P^2$$

$$50 - 49 = P^2$$

$$P = 1$$

$$\tan\alpha = \frac{P}{B} \Rightarrow \tan\alpha = \frac{1}{7}$$

$$\alpha = \tan^{-1}\left(\frac{1}{7}\right)$$

its mean

$$\cos^{-1}\left(\frac{7}{5\sqrt{2}}\right) = \tan^{-1}\left(\frac{1}{7}\right)$$

$$\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$\tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}}\right) = \tan^{-1}\left(\frac{\frac{21+4}{\cancel{28}}}{\frac{28-3}{\cancel{28}}}\right)$$

$$\tan^{-1}\left(\frac{25}{25}\right) = \tan^{-1}(1)$$

$$= \frac{\pi}{4} \quad \text{proved}$$

(XI) $\sin^{-1}(x) + \cos^{-1}(-x) = \frac{\pi}{2}$ $\boxed{-1<x<1}$

Sol Let

$$\alpha = \cos^{-1}(-x)$$

$$\cos\alpha = -x$$

Now we Find $\sin\alpha$.

$\cos\alpha = -\frac{x}{1}$

$B = -x$ $H = 1$

Use Phy Theorm

$H^2 = P^2 + B^2$

$(1)^2 = P^2 + (-x)^2$

$1 - x^2 = P^2$

$P = \sqrt{1-x^2}$

$\sin\alpha = \frac{\sqrt{1-x^2}}{1}$ $\alpha = \sin^{-1}\sqrt{1-x^2}$

OR

Use identity.

$\sin\alpha = \sqrt{1-\cos^2\alpha}$

$\sin\alpha = \sqrt{1-(-x)^2}$

$\sin\alpha = \sqrt{1-x^2}$

$\alpha = \sin^{-1}\sqrt{1-x^2}$

Now

$$\cos^{-1}(-x) = \sin^{-1}\sqrt{1-x^2}$$

$$\sin^{-1}x + \sin^{-1}\sqrt{1-x^2} = \frac{\pi}{2}$$

$$\sin^{-1}A + \sin^{-1}B = \sin^{-1}\left[A\sqrt{1-B^2} + B\sqrt{1-A^2}\right]$$

$$\sin^{-1}\left[x\left(\sqrt{1-\left(\sqrt{1-x^2}\right)^2}\right) + \sqrt{1-x^2}\left(\sqrt{1-(x)^2}\right)\right]$$

$$\sin^{-1}\left[x\sqrt{\cancel{1}-\cancel{1}+x^2} + \sqrt{1-x^2}\sqrt{1-x^2}\right]$$

$$\sin^{-1}\left[\cancel{x^2}+1-\cancel{x^2}\right] = \sin^{-1}(1) = \frac{\pi}{2}$$

Complete.