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Merging man and maths

Exc 8.2

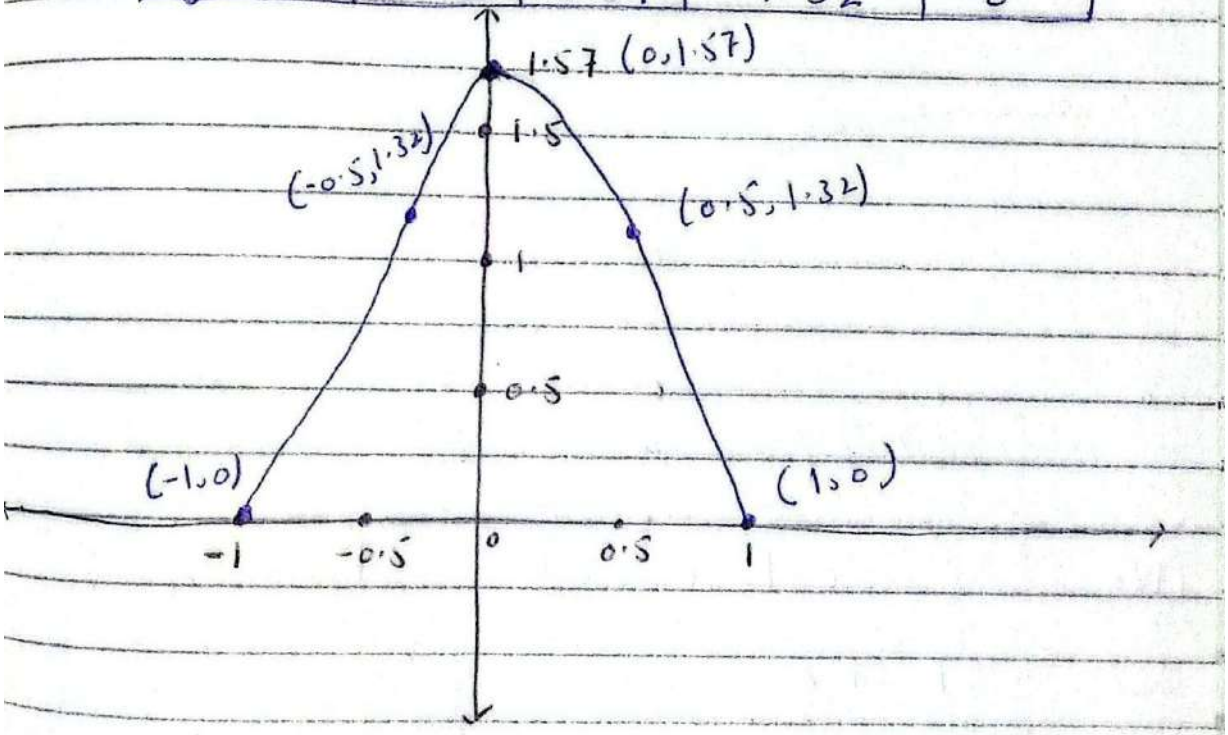
Q#1 Draw graph of each of the following functions against the intervals mentioned.

* Note: Calculator set in Radian Mode Must

i) $y = \cos^{-1}(x^2)$ for $x \in [-1, 1]$

Sol: Calculator Fix in 2 decimal places.

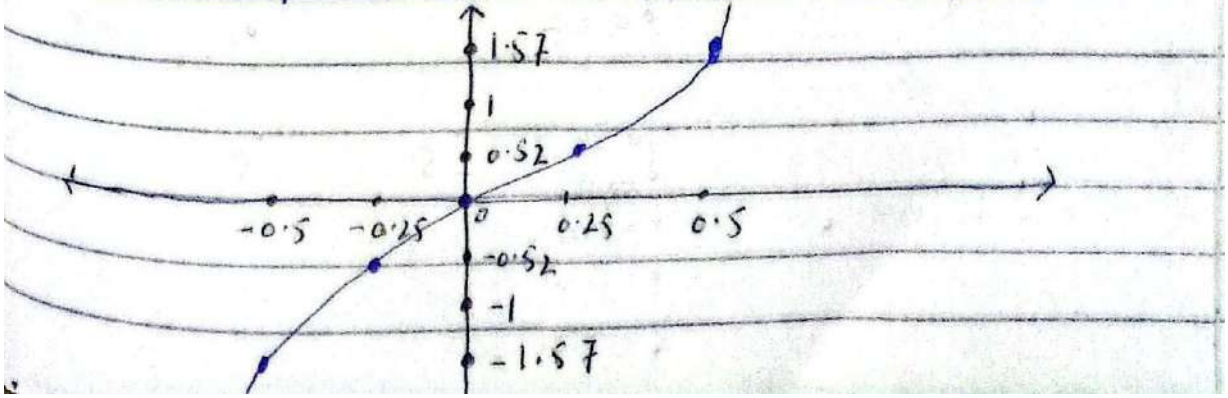
x	-1	-0.5	0	0.5	1
y	0	1.32	1.57	1.32	0



ii) $y = \sin^{-1}(2x)$ $\quad x \in \left[\frac{-1}{2}, \frac{1}{2}\right]$

Sol

x	-0.5	-0.25	0	0.25	0.5
y	-1.57	-0.52	0	0.52	1.57



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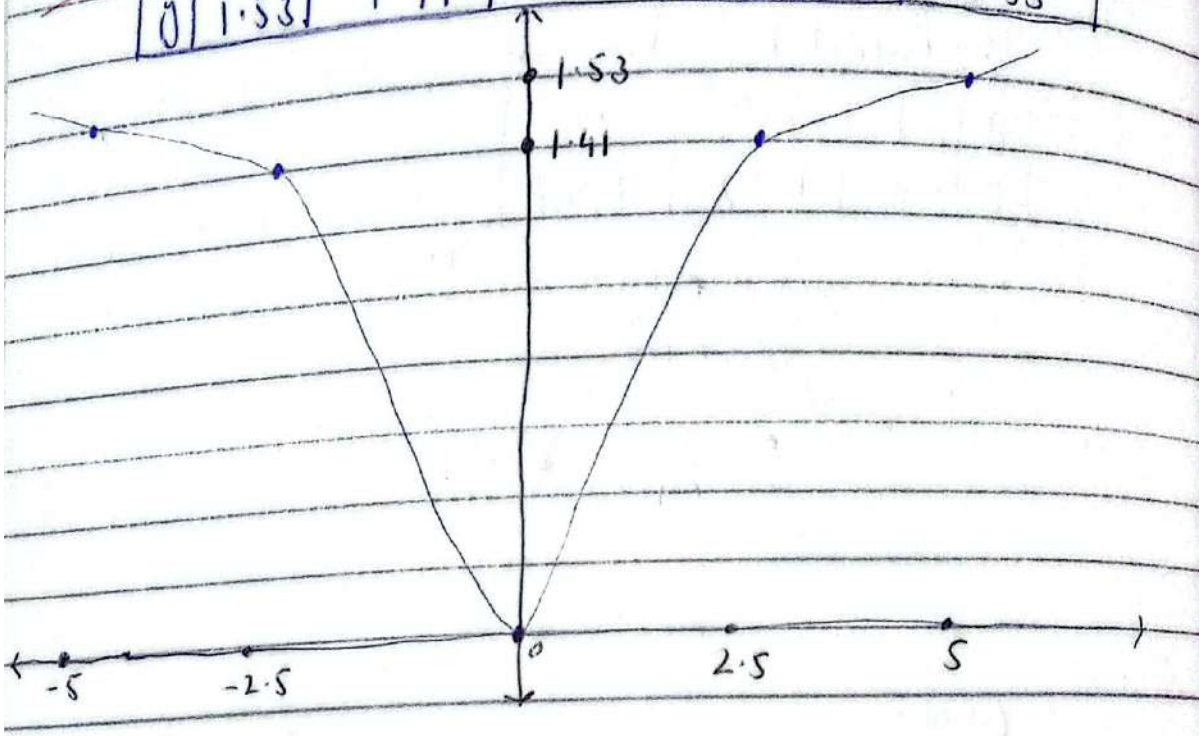
(iii) $y = \tan^{-1}(x^2)$ For $x \in R$

Sol

x	-5	-2.5	0	2.5	5
y	1.53	1.41	0	1.41	1.53

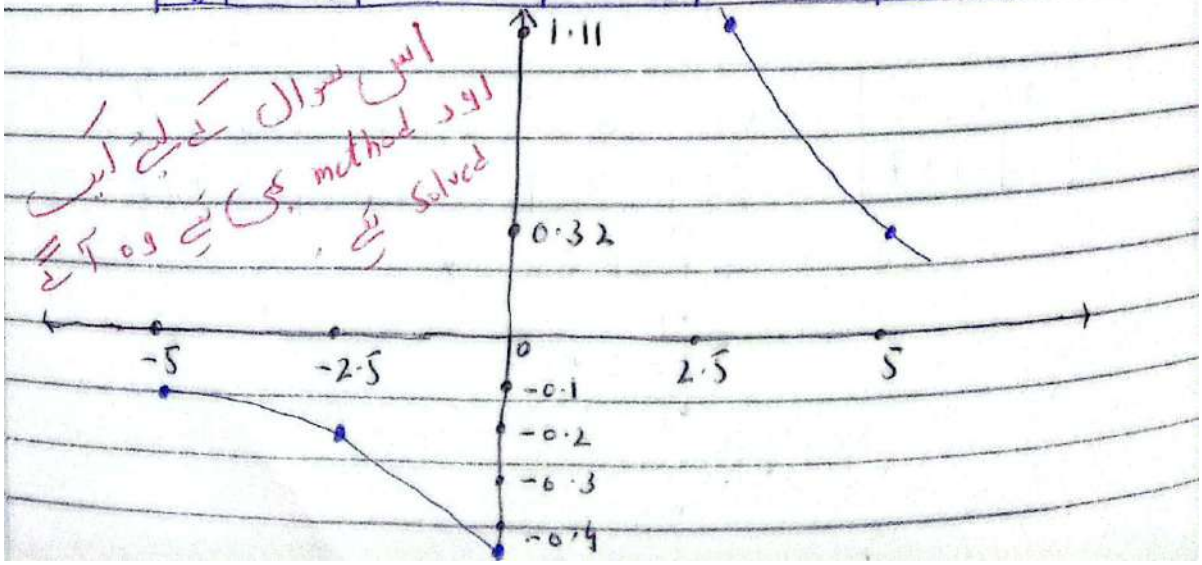


(iv) $y = \cot^{-1}(x-2)$ For $x \in R$

Sol $\cot^{-1}(x-2) = \tan^{-1}\left(\frac{1}{x-2}\right)$

x	-5	-2.5	0	2.5	5
y	-0.14	-0.22	-0.46	1.11	0.32

اس سوال کے لیے اب
اور method کی ضرورت ہے
solved ہے۔



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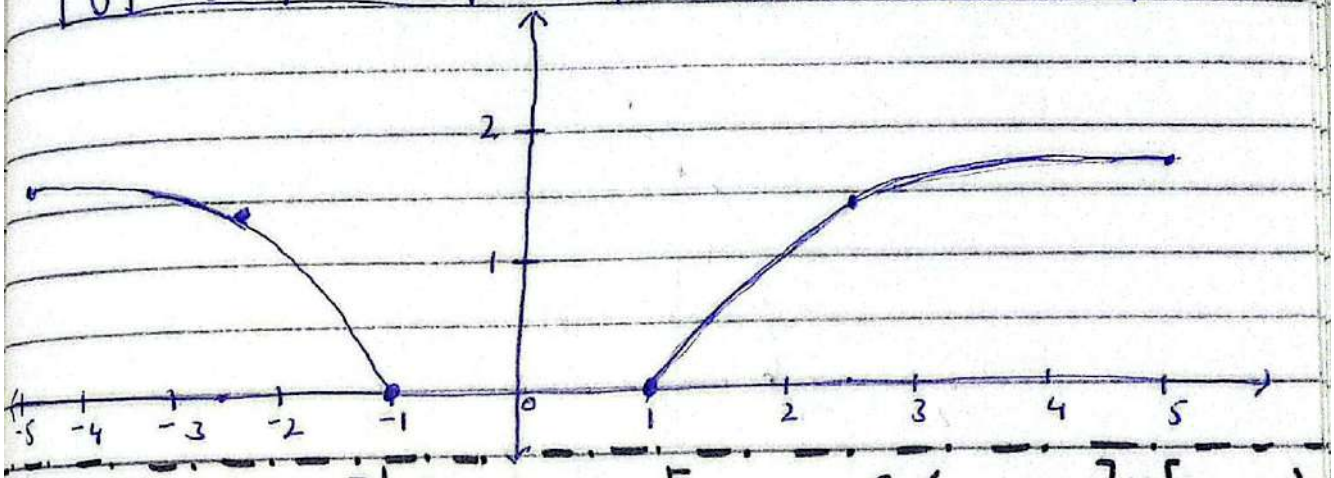
(V) $y = \sec^{-1}(x^2)$ for $x \in (-\infty, -1] \cup [1, +\infty)$

Sol

$$\sec^{-1}(x^2) = \cos^{-1}\left(\frac{1}{x^2}\right)$$

x	-5	-2.5	-1	1	2.5	5
y	1.53	1.41	0	0	1.41	1.53

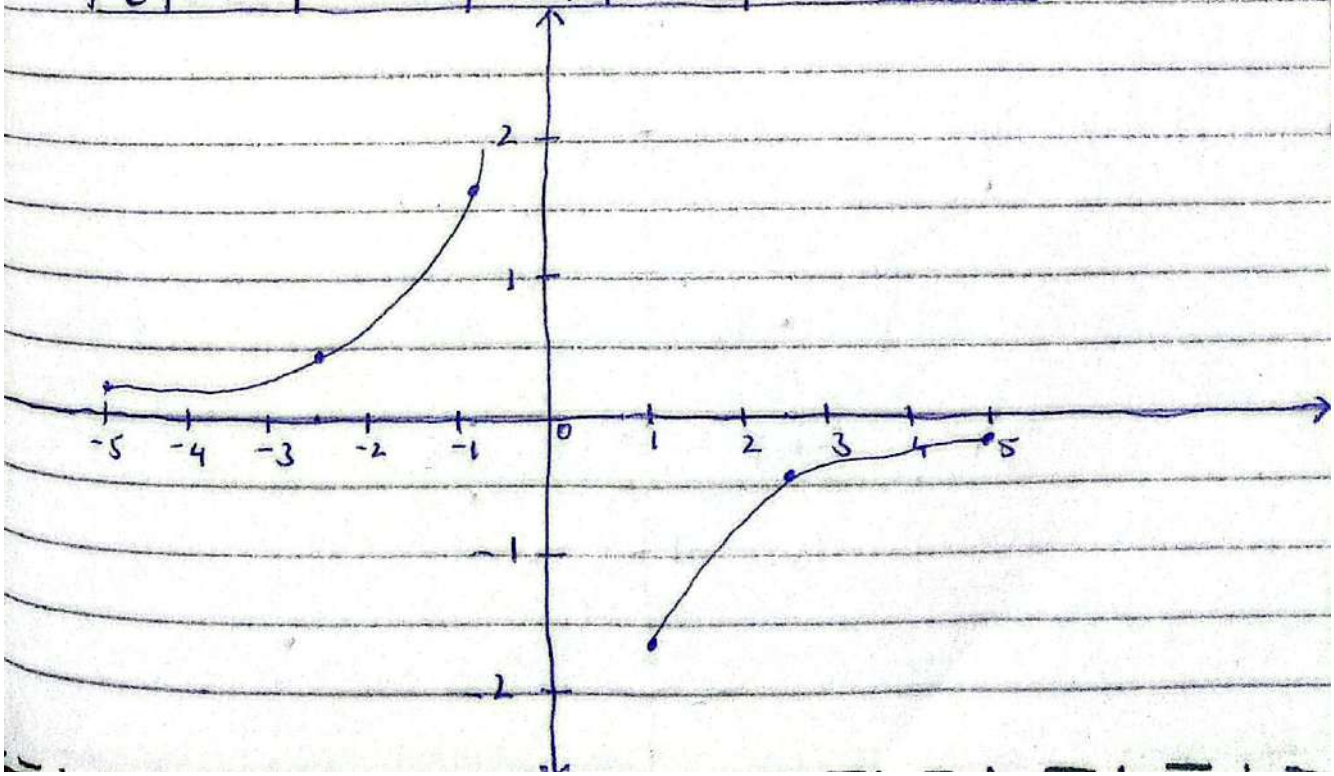


(VI) $y = \text{Cosec}^{-1}(-x)$ for $x \in (-\infty, -1] \cup [1, +\infty)$

Sol

$$\text{Cosec}^{-1}(-x) = \sin^{-1}\left(\frac{1}{-x}\right) = \sin^{-1}\left(\frac{-1}{x}\right)$$

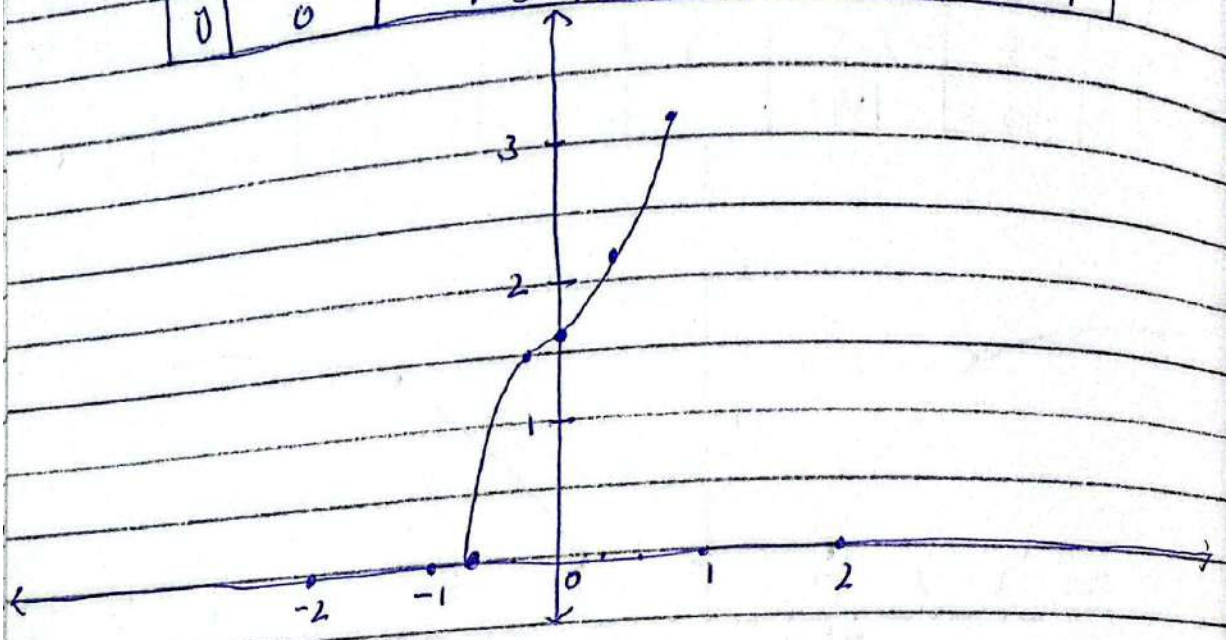
x	-5	-2.5	-1	1	2.5	5
y	0.20	0.41	1.57	-1.57	-0.41	-0.20



(vii) $y = \cos^{-1}(-2x)$ For $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

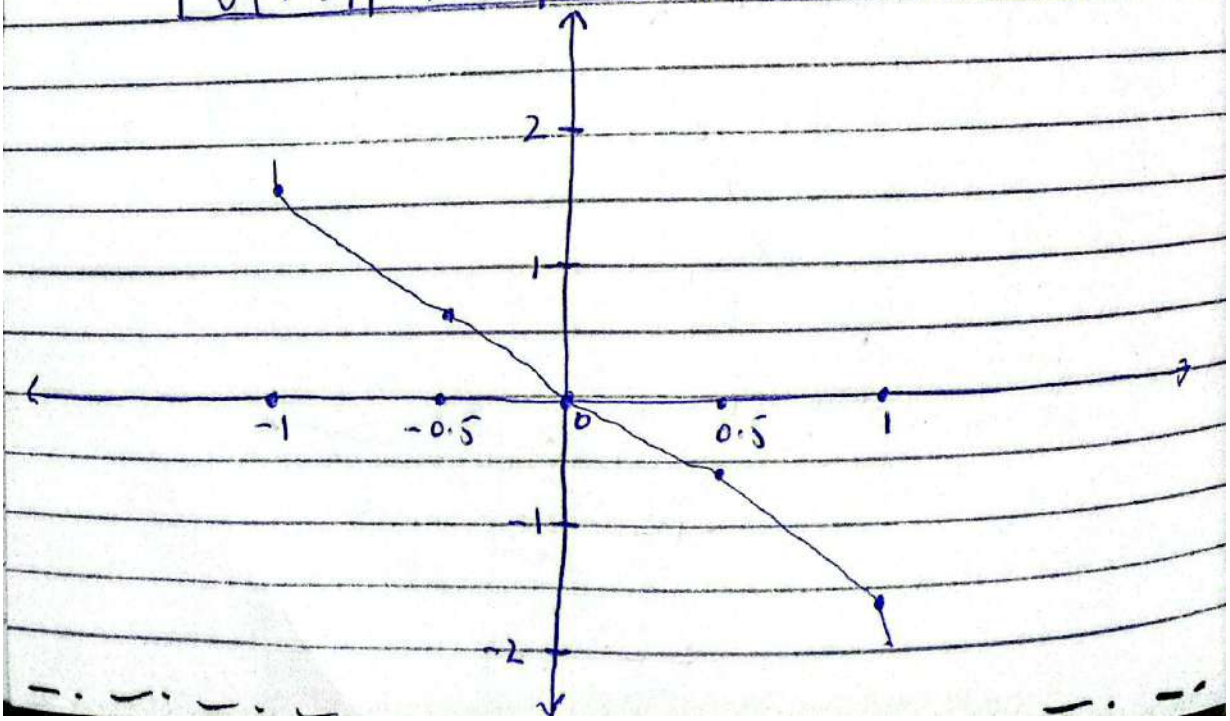
Sol

x	-0.5	-0.25	0	0.25	0.5
θ	0	1.5	1.57	2.09	3.14



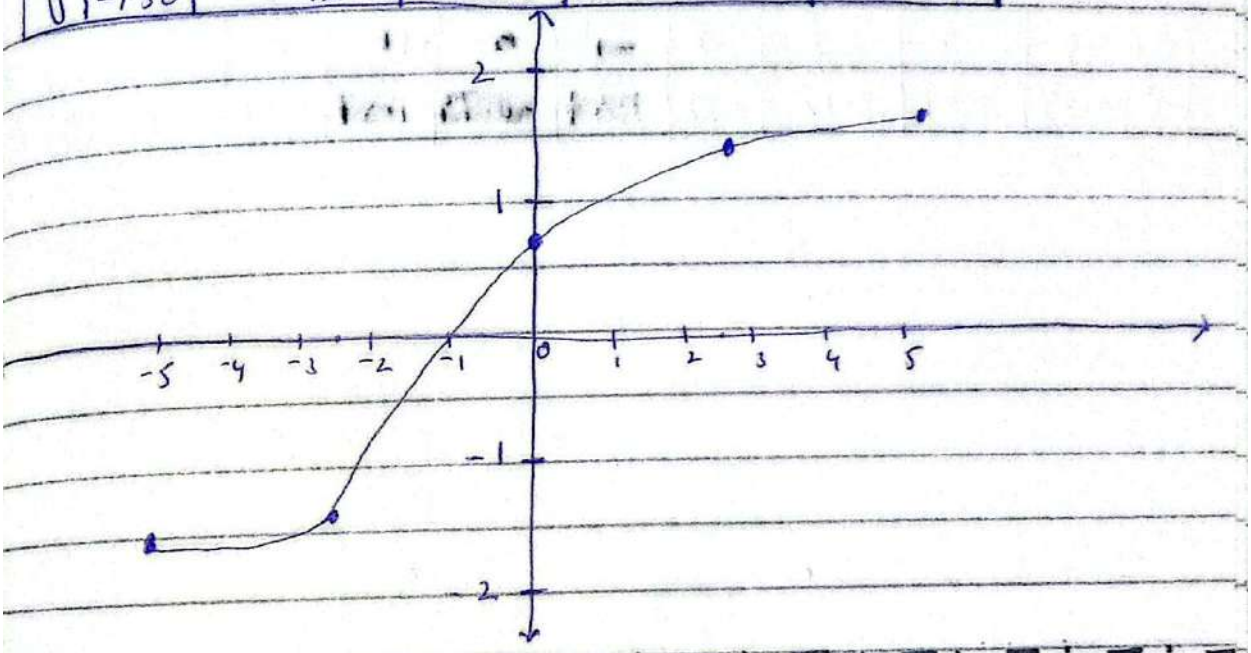
(viii) $y = \sin^{-1}(-x)$ For $x \in [-1, 1]$

x	-1	-0.5	0	0.5	1
θ	1.57	+0.52	0	-0.52	-1.57



(X) $y = \tan^{-1}(3x+1)$ For $x \in R$

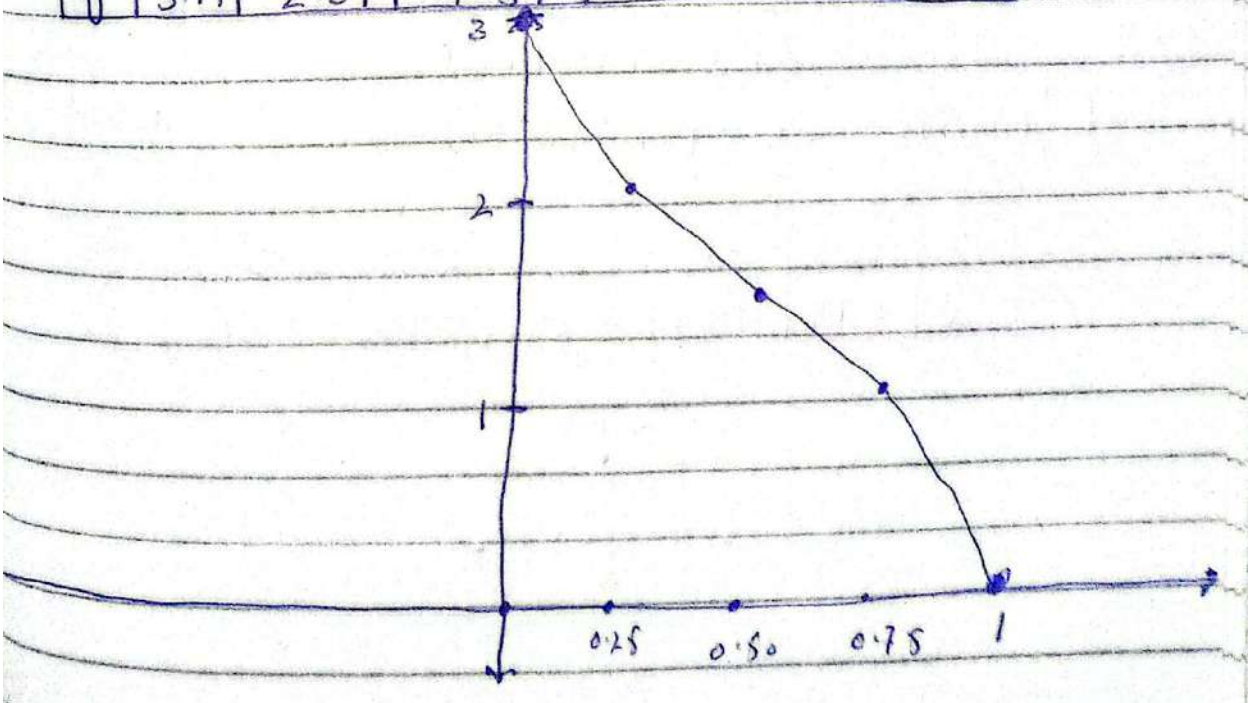
x	-5	-2.5	0	2.5	5
θ	-1.50	-1.42	0.79	1.45	1.51



(X) $y = \cos^{-1}(2x-1)$ For $x \in [0,1]$

Sol

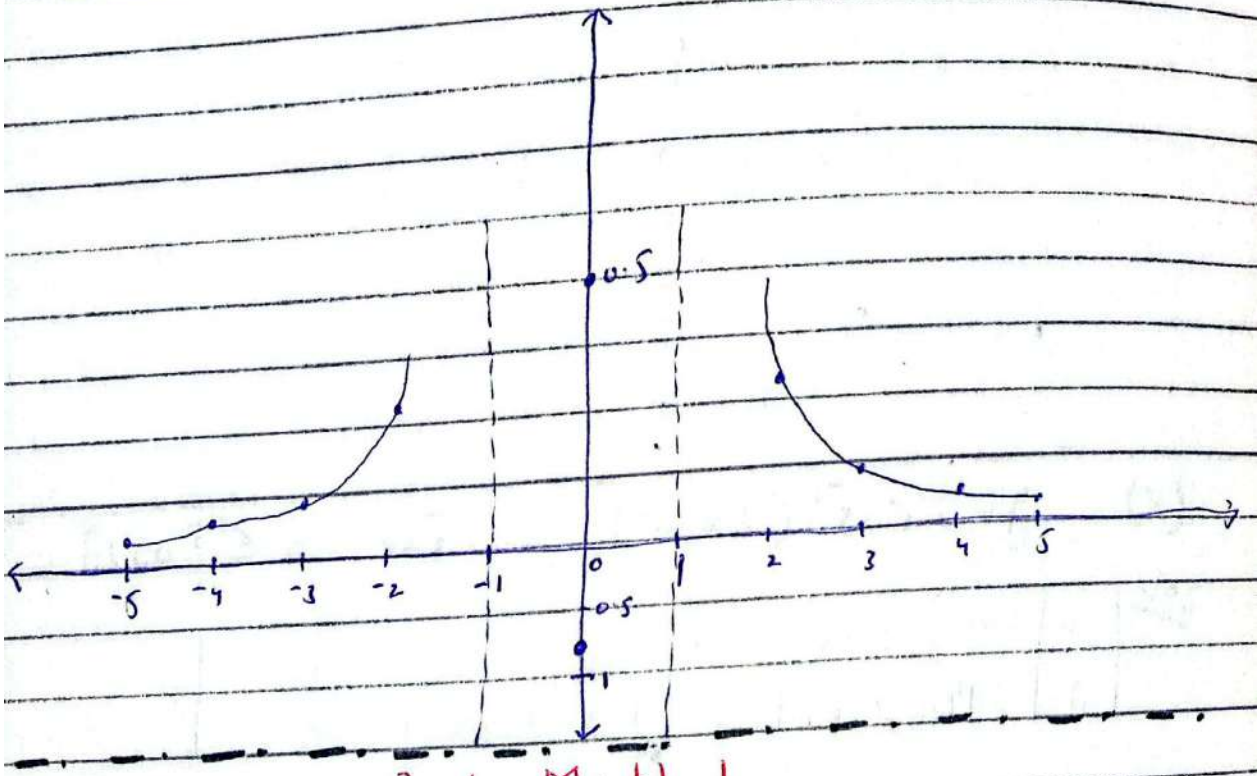
x	0	0.25	0.50	0.75	1
θ	3.14	2.09	1.57	1.05	0



(xi) $y = \cot^{-1}(x^2-1)$ for $x \in R$

Sol $\cot^{-1}(x^2-1) = \tan^{-1}\left(\frac{1}{x^2-1}\right)$

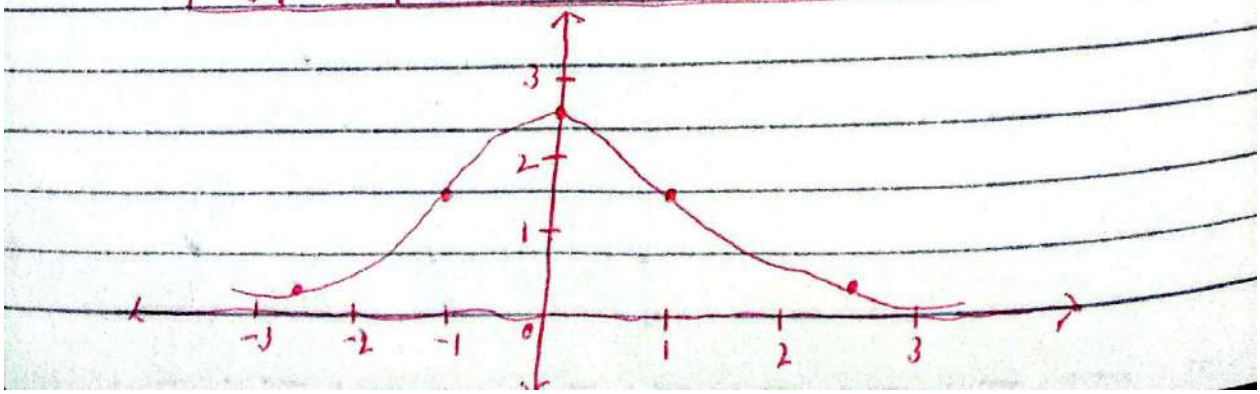
x	-5	-4	-3	-2	-1	0	+1	2	3	4	5
y	0.04	0.07	0.12	0.32	∞	[redacted]	∞	0.32	0.12	0.07	0.04



(xi) 2nd Method $x \in R$

$\cot^{-1}(x^2-1) = \frac{\pi}{2} - \tan^{-1}(x^2-1)$

x	-2.5	-1	0	1	2.5
y	0.19	1.57	2.36	1.57	0.19



2nd Method.

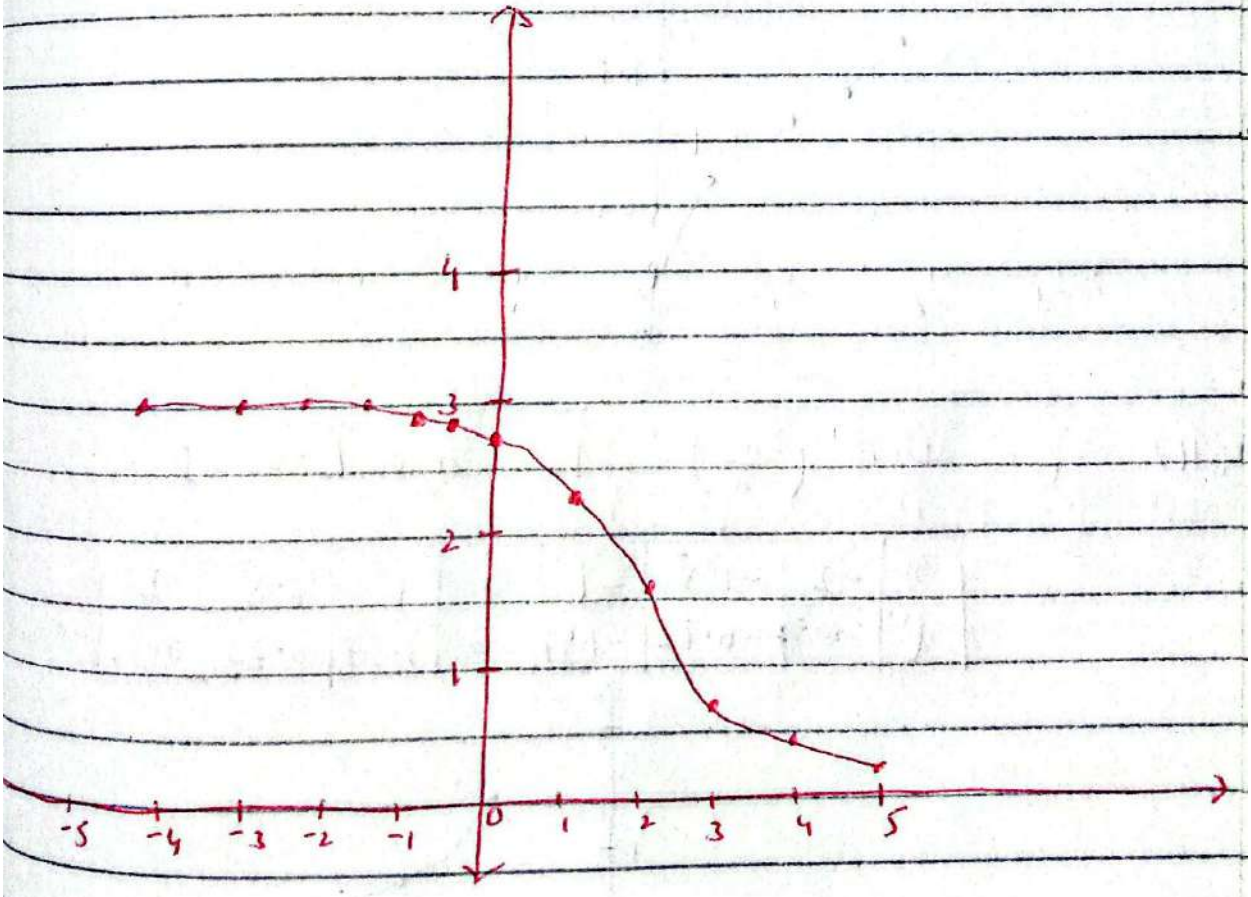
Q#1 (iv)

$$y = \cot^{-1}(x-2)$$

Sol

$$y = \cot^{-1}(x-2) = \frac{\pi}{2} - \tan^{-1}(x-2)$$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	3	2.98	2.94	2.9	2.82	2.68	2.36	1.57	0.79	0.46	0.12

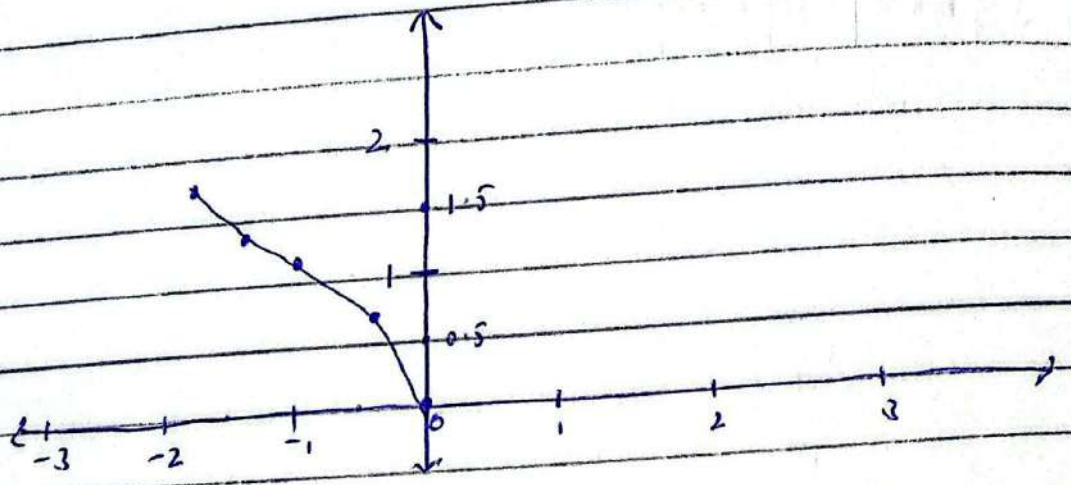


Q#2 Draw the Graph of each of the following Function against the Interval mentioned.

i) $y = \cos^{-1}\left(\frac{x+2}{2}\right)$ For $x \in [-2, 0]$

Sol

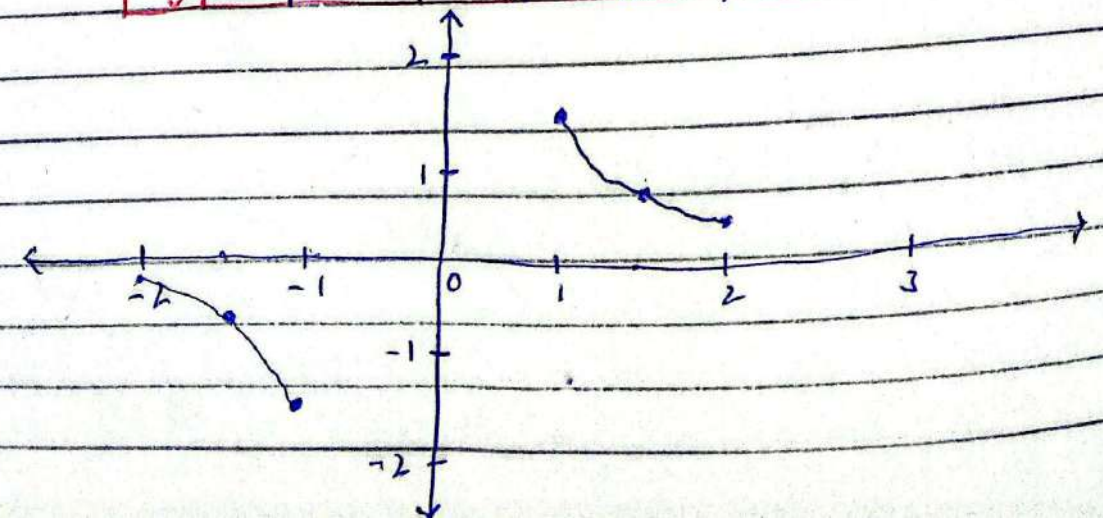
x	-2	-1.5	-1	-0.5	0
y	1.57	1.32	1.04	0.72	0



(ii) $y = \sin^{-1}\left(\frac{1}{x}\right)$ For $x \in [-2, 2]$

Sol

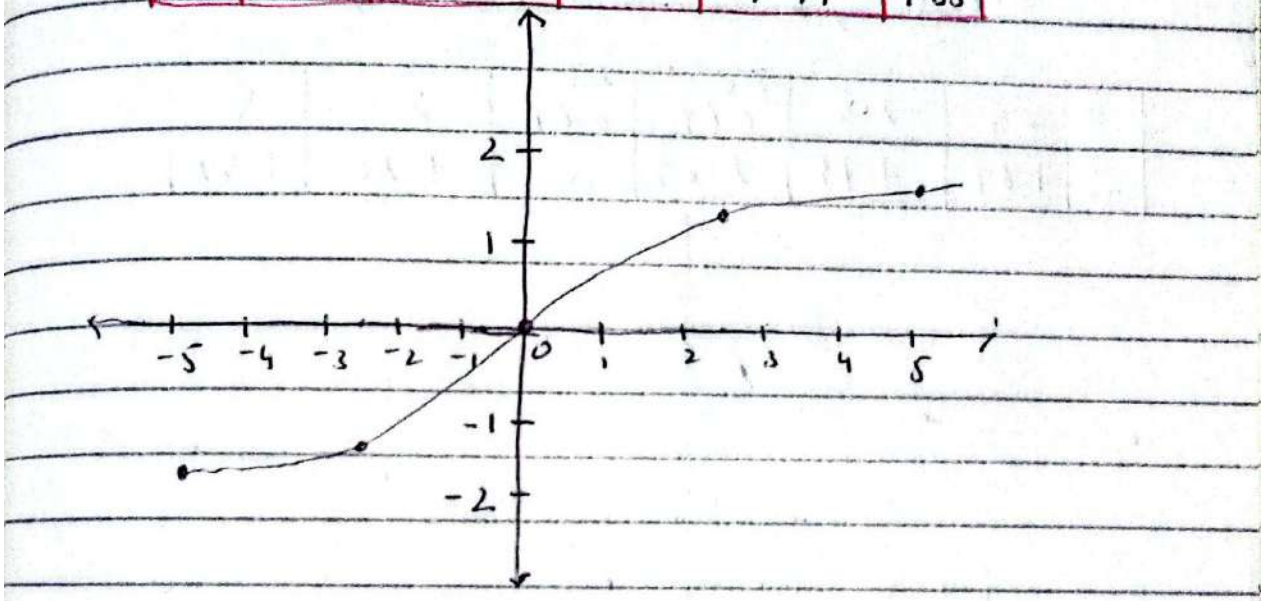
x	-2	-1.5	-1	0	1	1.5	2
y	-0.57	-0.73	-1.47	∞	1.57	0.73	0.52



(iii) $y = \tan^{-1}\left(\frac{x^3}{2}\right)$ for $x \in R$

Sol

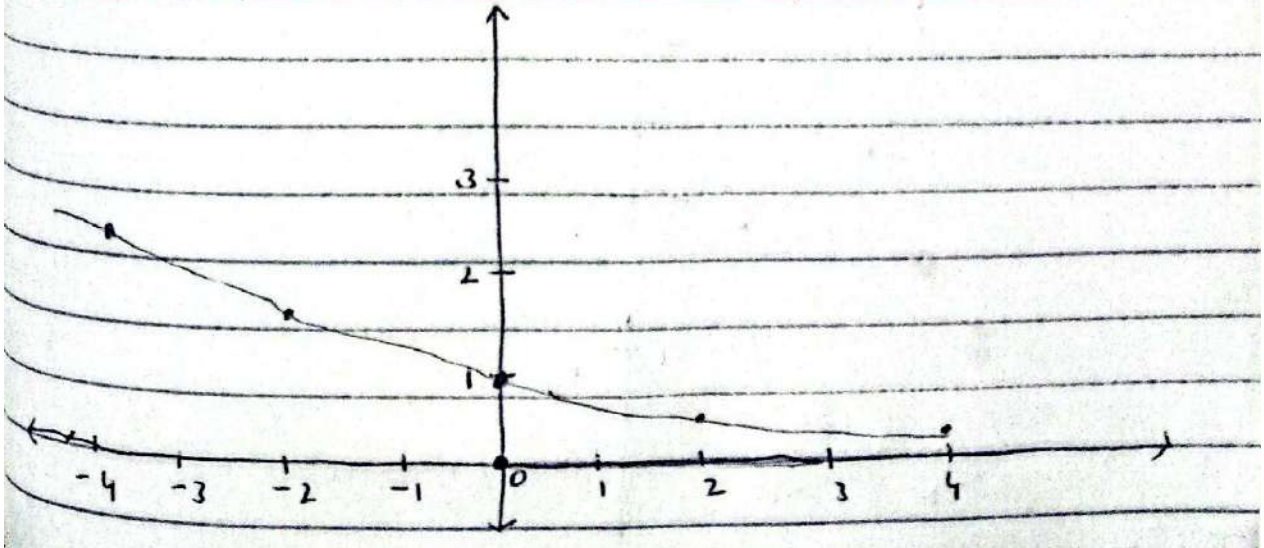
x	-5	-2.5	0	2.5	5
y	-1.55	-1.44	0	1.44	1.55



(iv) $y = \cot^{-1}\left(\frac{x+2}{3}\right)$ $x \in R$

$y = \cot^{-1}\left(\frac{x+2}{3}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{x+2}{3}\right)$

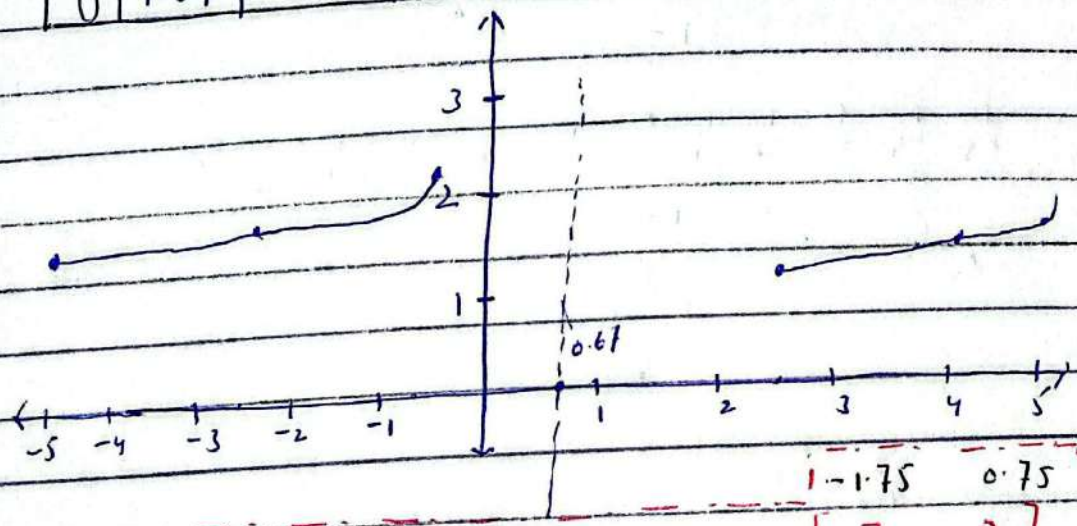
x	-4	-2	0	2	4
y	2.16	1.57	0.98	0.64	0.46



(V) $y = \sec^{-1}\left(\frac{3x-2}{2}\right)$ $\quad x \in \left(-\infty, \frac{-2}{3}\right] \cup \left[\frac{2}{3}, \infty\right)$

Sol $\quad \sec^{-1}\left(\frac{3x-2}{2}\right) = \cos^{-1}\left(\frac{2}{3x-2}\right)$

x	-5	-2.5	$-2/3$ -0.67	$2/3$ 0.67	2.5	5
y	1.69	1.78	2.09	∞	1.20	1.42

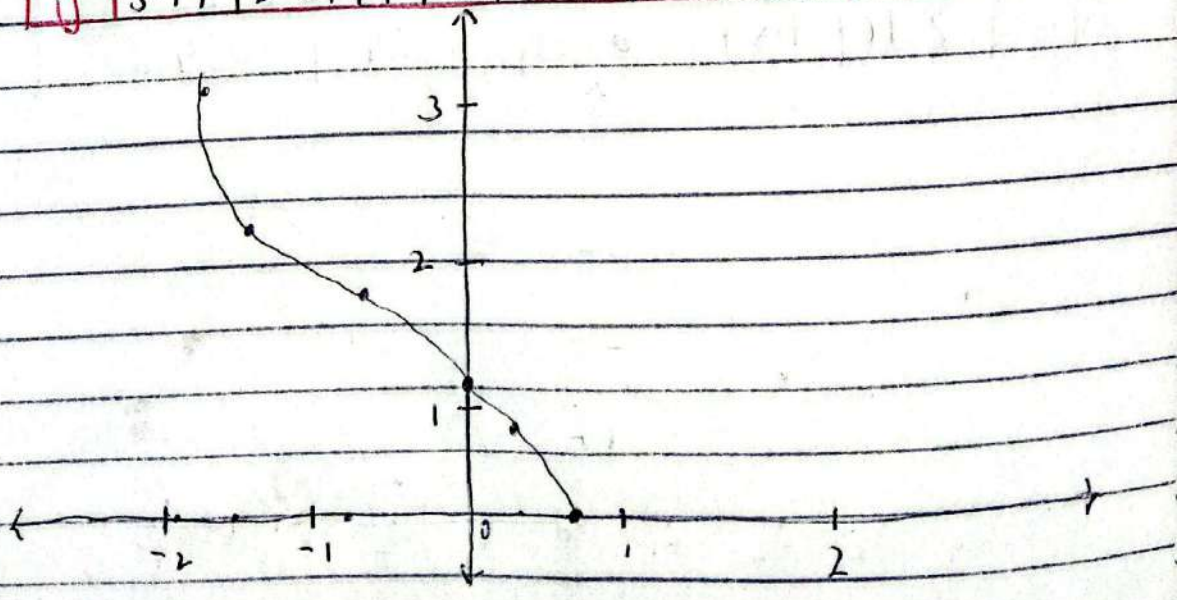


$1 \quad -1.75 \quad 0.75 \quad 1$

(VI) $y = \cos^{-1}\left(\frac{4x+2}{5}\right)$ $\quad x \in \left[\frac{-7}{4}, \frac{3}{4}\right]$

Sol

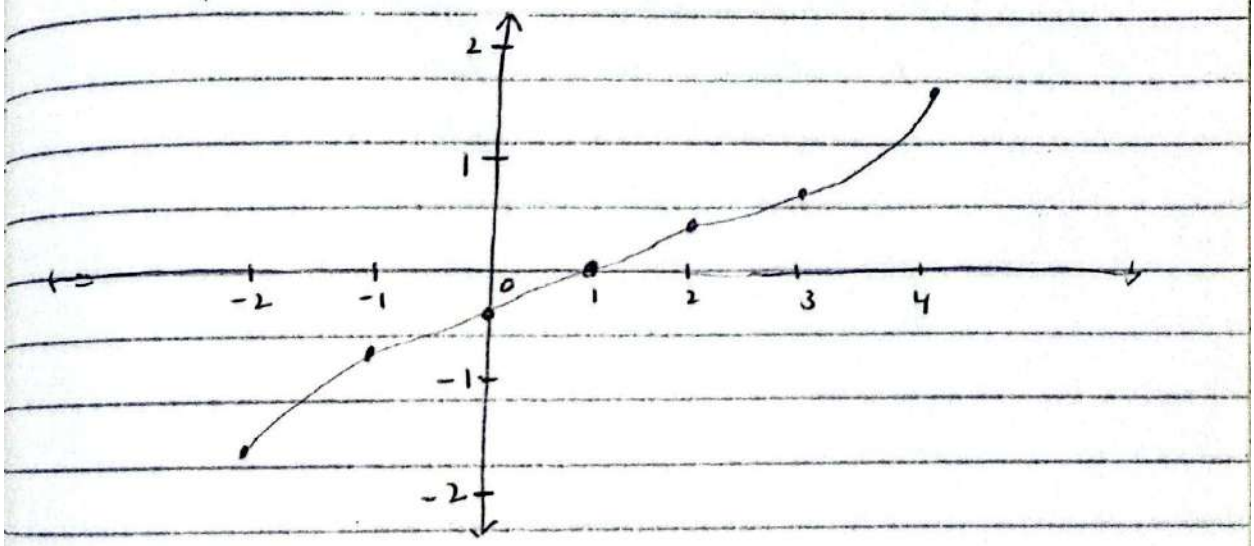
x	-1.75	-1.25	-0.75	-0.25	0	0.25	0.75
y	3.14	2.21	1.77	1.36	1.16	0.93	0



(VII) $y = \sin^{-1}\left(\frac{x-1}{3}\right)$ $x \in [-2, 4]$

Sol

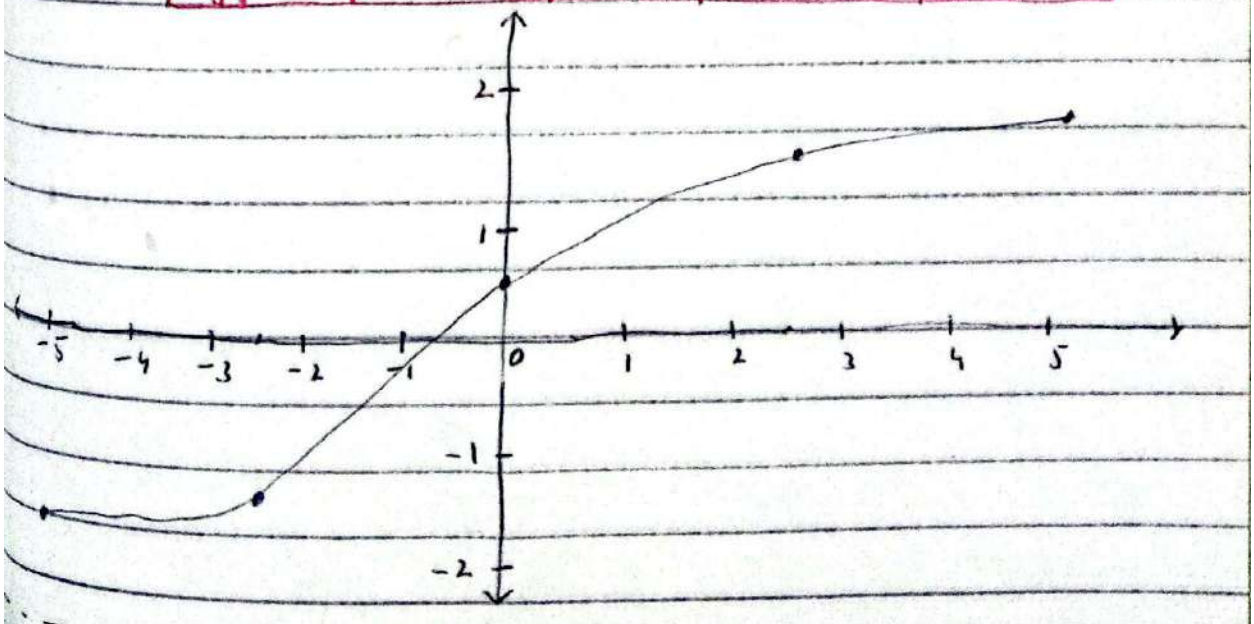
x	-2	-1	0	1	2	3	4
y	-1.57	-0.73	-0.34	0	0.34	0.73	1.57



(VIII) $y = \tan^{-1}\left(3x + \frac{1}{2}\right)$ $x \in R$

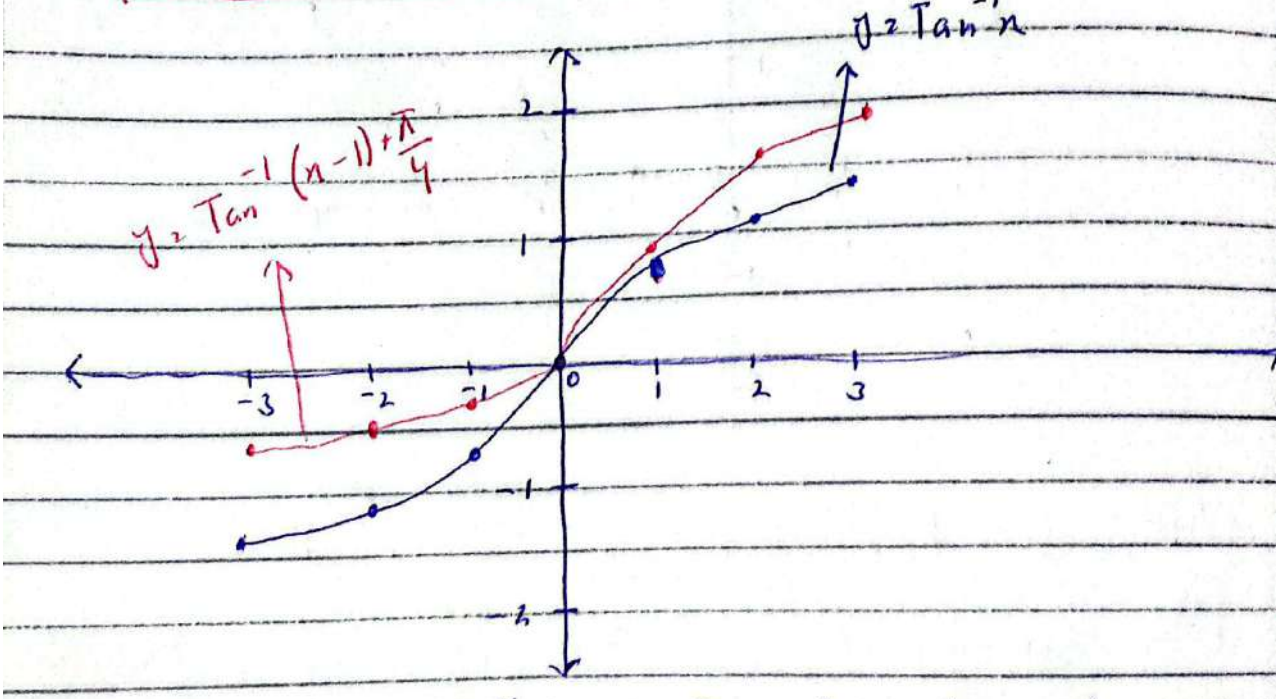
Sol

x	-5	-2.5	0	2.5	5
y	-1.51	-1.43	0.46	1.45	1.51



Q#3 (i) $y = \text{Tan}^{-1}(x-1) + \frac{\pi}{4}$ for $x \in R$

x	-3	-2	-1	0	1	2	3
$y = \text{Tan}^{-1}x$	-1.25	-1.1	-0.79	0	0.79	1.1	1.25
$y = \text{Tan}^{-1}(x-1) + \pi/4$	-0.54	-0.46	-0.32	0	0.79	1.57	1.89



The graph $y = \text{Tan}^{-1}(x-1) + \frac{\pi}{4}$ has been translated by 1 unit right side and $\frac{\pi}{4}$ upward. from the basic graph $y = \text{Tan}^{-1}x$.

(ii) $y = \sin^{-1}(x-2)$

Indicate the domain, range and Key Points

x	1	1.5	2	2.5	3
θ	-1.57	-0.52	0	0.52	1.57

- $\sin^{-1} x$

Domain = $[-1, 1]$

or

$-1 \leq x \leq 1$

Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

- $\sin^{-1}(x-2)$

Domain: $-1 \leq x-2 \leq 1$

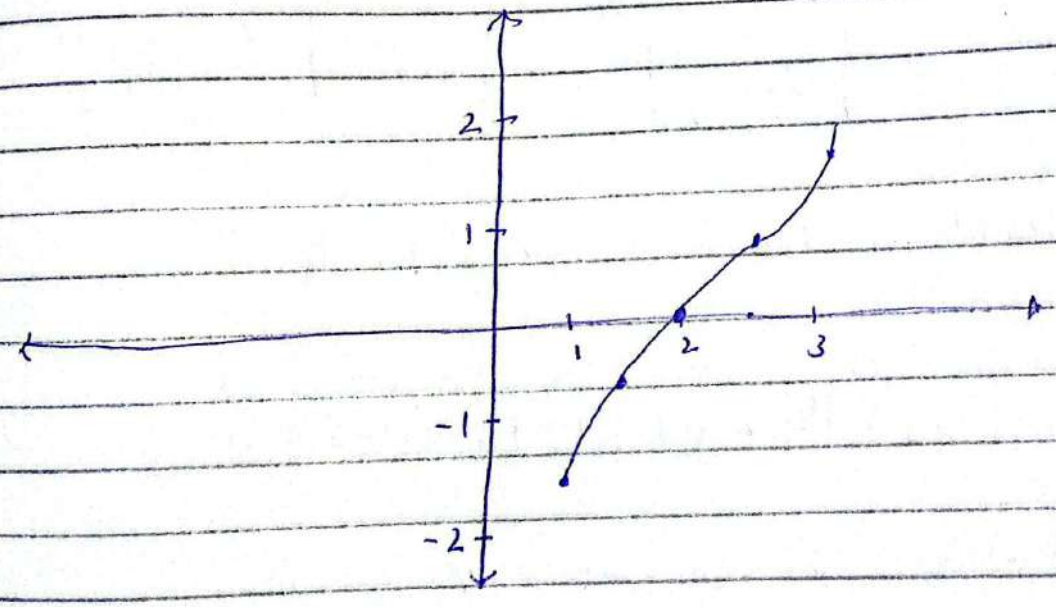
$-1+2 \leq x \leq 1+2$

$1 \leq x \leq 3$

$[1, 3]$

- $\sin^{-1}(x-2)$

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Key Point :-

i) Start $(1, -1.57)$

ii) End $(3, 1.57)$

iii) Midpoint $(2, 0)$

iv) Curve Path $(1.5, -0.52)$ $(2.5, 0.52)$

iii) $y = \cos^{-1}(2x) + \frac{\pi}{3}$

State the domain and range of the function transformation $y = \cos^{-1}x$

Sol

- $y = \cos^{-1}x$

Domain: $[-1, 1]$

$-1 \leq x \leq 1$

Range: $[0, \pi]$

- $y = \cos^{-1}(2x)$

Domain = $\frac{-1}{2} \leq \frac{2x}{2} \leq \frac{1}{2}$

$\frac{-1}{2} \leq x \leq \frac{1}{2}$

$\left[-\frac{1}{2}, \frac{1}{2}\right]$

$y = \cos^{-1}(2x) + \frac{\pi}{3}$

Range =

$\left[0 + \frac{\pi}{3}, \pi + \frac{\pi}{3}\right]$

$= \left[\frac{\pi}{3}, \frac{4\pi}{3}\right]$

Firstly Draw Graph of $y = \cos^{-1}x$

x	-1	-0.5	0	0.5	1
y	3.14	2.09	1.57	1.05	0

Now $y = \cos^{-1}2x + \pi/3$

x	-0.5	-0.25	0	0.25	0.5
y	4.19	3.14	2.62	2.09	1.05

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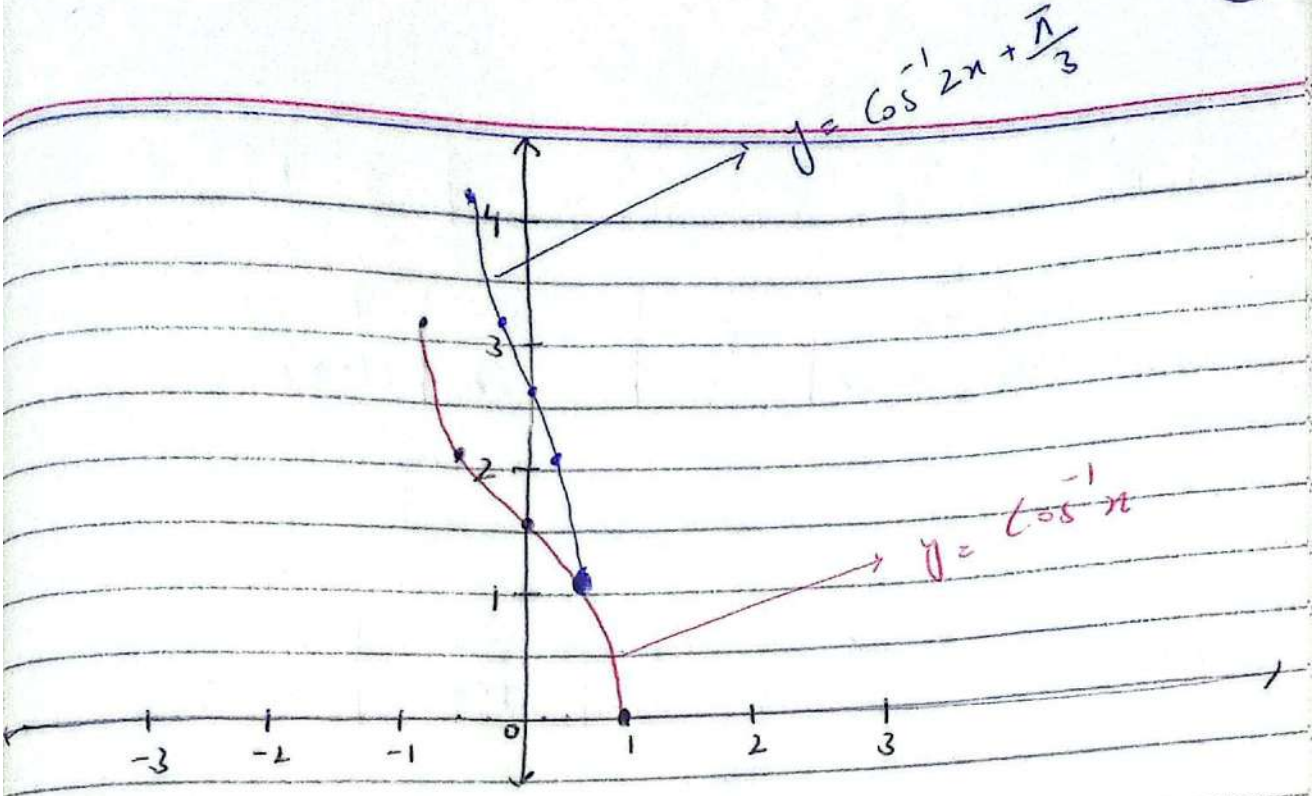


Translation

- Horizontal Compressed by 2 units.
- Vertical shift upwards by $\frac{\pi}{3}$ units.

(IV) $y = \sin^{-1}(2x) + \frac{\pi}{6}$

Sol

- $y = \sin^{-1} x$

Domain $= [-1, 1]$

$-1 \leq x \leq 1$

- $y = \sin^{-1}(2x)$

Domain: $-1 \leq 2x \leq 1$

$-\frac{1}{2} \leq x \leq \frac{1}{2}$

$\left[\frac{-1}{2}, \frac{1}{2}\right]$

Range: $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

- $y = \sin^{-1}(2x) + \frac{\pi}{6}$

Range:

$\left[\frac{-\pi}{2} + \frac{\pi}{6}, \frac{\pi}{2} + \frac{\pi}{6}\right]$

$= \left[\frac{-3\pi + \pi}{6}, \frac{3\pi + \pi}{6}\right]$

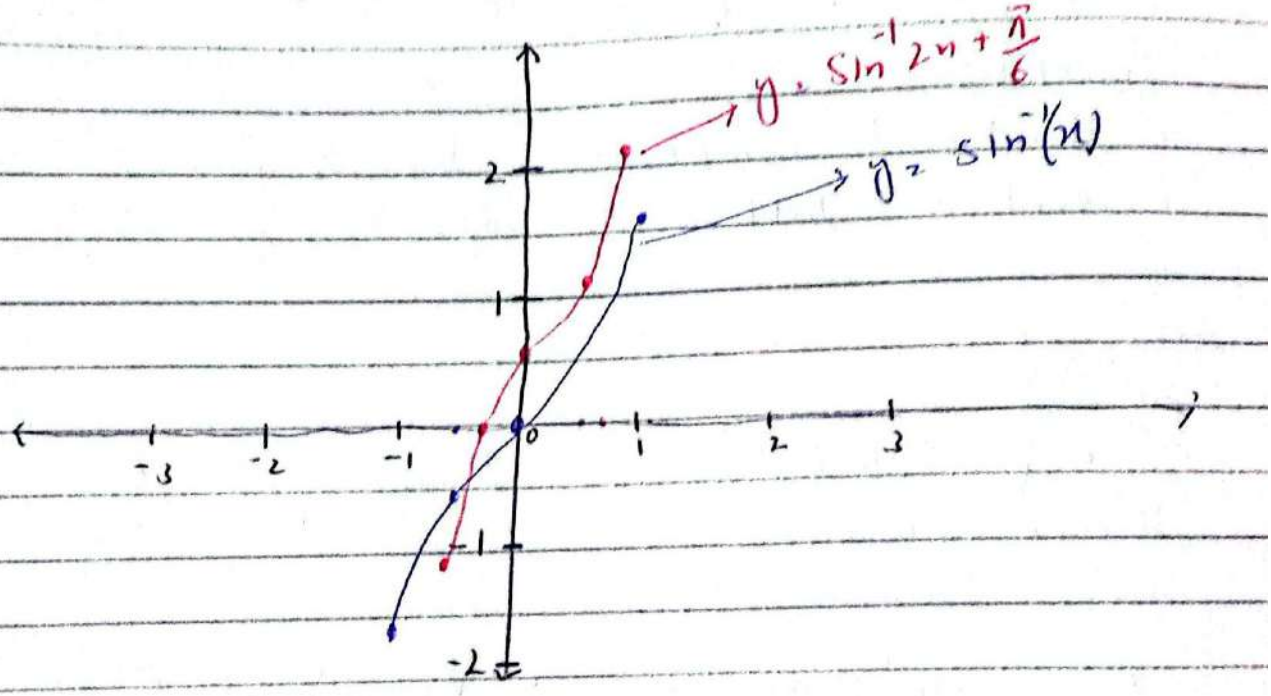
$= \left[\frac{-2\pi}{6}, \frac{4\pi}{6}\right]$

Table : $y = \sin^{-1} x$; $[-1, 1]$

x	-1	-0.5	0	0.5	1
y	-1.57	-0.52	0	0.52	1.57

Table : $y = \sin^{-1} 2x + \frac{\pi}{6}$; $[-0.5, 0.5]$

x	-0.5	-0.25	0	0.25	0.5
y	-1.05	0	0.52	1.05	2.09



- Horizontal Compressed by 2 units
- Vertical shift upwards $\frac{\pi}{6}$ units

Complete

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