Chap # 08

Inverse Trigonometric Functions and their Graphs.

Exc 8.1

Domain and Range of inverse Trigonometric fns.

Inverse Trigonometric Functions	Domain	Range
$y = \sin^{-1} x$	$-1 \le x \le 1$ $\boxed{\text{or}}$ $x \in [-1, 1]$	$y \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$-1 \le x \le 1$ $\boxed{\text{or}}$ $x \in [-1, 1]$	$y \in [0, \pi]$
$y = \tan^{-1} x$	$x \in R$	$y \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	$x \in R$	$y \in (0, \pi)$
$y = \sec^{-1} x$	$x \in (-\infty, -1] \cup [1, +\infty)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$ or $[0, \pi]$ $y \neq \pi/2$
$y = \csc^{-1} x$	$x \in (-\infty, -1] \cup [1, +\infty)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ or $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ $y \neq 0$

Trigonometric Table

	0	30	45	60	90
sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

Q#1 Find the Principal values For each of the Following with out using Calculator

i) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $\qquad y \in [0, \pi]$

Sol

Let $y = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\cos y = \frac{\sqrt{3}}{2}$

Put $y = 30^\circ$ or $\boxed{y = \frac{\pi}{6}}$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

So $\boxed{\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}}$

(ii) $\sin^{-1}(1)$ $\qquad y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Sol Let $y = \sin^{-1}(1)$

$\sin y = 1$

Put $y = 90$ or $y = \frac{\pi}{2}$

$\sin\frac{\pi}{2} = 1$

So $\boxed{\sin^{-1}(1) = \frac{\pi}{2}}$

(iii) $\tan^{-1}(\sqrt{3})$ $\qquad y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Sol

Let $y = \tan^{-1}\sqrt{3}$

$\tan y = \sqrt{3}$

Put $y = 60^\circ$ or $y = \frac{\pi}{3}$

$\tan\frac{\pi}{3} = \sqrt{3}$

so $\boxed{\tan^{-1}\sqrt{3} = \frac{\pi}{3}}$

(iv) $\cot^{-1}\left(\frac{\sqrt{3}}{3}\right)$ $\qquad y \in (0, \pi)$

Sol

Let $y = \cot^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$\cot y = \frac{\sqrt{3}}{3} \Rightarrow \frac{1}{\tan y} = \frac{\sqrt{3}}{3}$

$\tan y = \frac{3}{\sqrt{3}}$

$\tan y = \frac{\sqrt{3}\,\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} \Rightarrow \tan y = \sqrt{3}$

$\tan\frac{\pi}{3} = \sqrt{3}$

$y = \pi/3$ $\qquad \boxed{\cot^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{3}}$

(v) $Sec^{-1}\left(\frac{2\sqrt{3}}{3}\right)$ $y \in [0, \pi]$ $y \neq \frac{\pi}{2}$

Sol $Sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$\frac{\sqrt{3}}{3}$ means $\frac{1}{\sqrt{3}}$

Let $y = Sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$Sec\,y = \frac{2}{\sqrt{3}}$

$\frac{1}{Cos\,y} = \frac{2}{\sqrt{3}} \Rightarrow Cos\,y = \frac{\sqrt{3}}{2}$

Put $y = 30^\circ$ or $\frac{\pi}{6}$

$Cos\,\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

So $\boxed{Sec^{-1}\left(\frac{2\sqrt{3}}{3}\right) = \frac{\pi}{6}}$

(vi) $Cosec^{-1}(-\sqrt{2})$ $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $y \neq 0$

Sol Let $y = Cosec^{-1}(-\sqrt{2})$

$Cosec\,y = -\sqrt{2}$

$\frac{1}{Sin\,y} = -\sqrt{2}$

$Sin\,y = -\frac{1}{\sqrt{2}}$

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$y = -\frac{\pi}{4}$ or $y = -45^\circ$

$\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

So $\boxed{\csc^{-1}(-\sqrt{2}) = -\frac{\pi}{4}}$

(VII) $\cos^{-1}\left(-\frac{1}{2}\right)$ $y \in [0, \pi]$

Sol

Let $y = \cos^{-1}\left(-\frac{1}{2}\right)$

$\cos y = -\frac{1}{2}$

$y = 120^\circ$ or $\frac{2\pi}{3}$

$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

So $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

2nd Method

$\cos y = -\frac{1}{2}$

$-\cos y = \frac{1}{2}$

$\cos(\pi - y) = \frac{1}{2}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\pi - y = \frac{\pi}{3}$

$y = \frac{2\pi}{3}$

(VIII) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Sol

Let $y = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$$\tan y = \frac{1}{\sqrt{3}}$$

Put $y = \frac{\pi}{6}$ or 30°

$$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

So $\boxed{\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}}$

(ix) $\operatorname{cosec}^{-1}(-2)$ $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $y \neq 0$

Sol: Let $y = \operatorname{cosec}^{-1}(-2)$

$$\operatorname{cosec} y = -2$$

$$\frac{1}{\sin y} = -2 \Rightarrow \sin y = -\frac{1}{2}$$

Put $y = -\frac{\pi}{6}$

$$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$$

So $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

or

$$\boxed{\operatorname{cosec}^{-1}(-2) = -\frac{\pi}{6}}$$

Q#2 Find Sum of Principal values of following inverse trigonometric Expressions without using Calculator.

(i) $\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

Sol

$\tan^{-1}(1)$	$\cos^{-1}\left(\frac{-1}{2}\right)$	$\sin^{-1}\left(\frac{-1}{2}\right)$
Let $y = \tan^{-1}(1)$	Let $y = \cos^{-1}\left(\frac{-1}{2}\right)$	Let $y = \sin^{-1}\left(\frac{-1}{2}\right)$
$\tan y = 1$	$\cos y = -\frac{1}{2}$ * 2nd Method $\boxed{\text{Q\#1 vii}}$ Same	$\sin y = -\frac{1}{2}$
Put $y = \frac{\pi}{4}$	$y = 120^\circ$ or $\frac{2\pi}{3}$	$y = -\frac{\pi}{6}$
$\tan\frac{\pi}{4} = 1$	$\cos\frac{2\pi}{3} = \frac{-1}{2}$	$\sin\left(\frac{-\pi}{6}\right) = -\frac{1}{2}$
$\tan^{-1}(1) = \frac{\pi}{4}$	$\cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$	$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$
$= \frac{\pi}{4}$	$= \frac{2\pi}{3}$	$= -\frac{\pi}{6}$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

$$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

(ii) $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

Sol

$\cos^{-1}\left(\frac{1}{2}\right)$	$\sin^{-1}\left(\frac{1}{2}\right)$
Put $y = \cos^{-1}\left(\frac{1}{2}\right)$	Put $y = \sin^{-1}\left(\frac{1}{2}\right)$
$\cos y = \frac{1}{2}$	$\sin y = \frac{1}{2}$
Put $y = 60^\circ$ or $\frac{\pi}{3}$	Put $y = 30^\circ$ or $\frac{\pi}{6}$
$\cos\frac{\pi}{3} = \frac{1}{2}$	$\sin\frac{\pi}{6} = \frac{1}{2}$
$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$	$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$
$= \frac{\pi}{3}$	$= \frac{\pi}{6}$

$$= \frac{\pi}{3} + 2\cdot\frac{\pi}{6}$$

$$= \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3} \quad \text{Ans.}$$

(iii) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

Sol

$\tan^{-1}\sqrt{3}$ $\qquad$ $\sec^{-1}(-2)$

Let $y = \tan^{-1}\sqrt{3}$	Let $y = \sec^{-1}(-2)$
$\tan y = \sqrt{3}$	$\sec y = -2$
Put $y = 60^\circ$ or $\frac{\pi}{3}$	$\frac{1}{\cos y} = -2$
$\tan\frac{\pi}{3} = \sqrt{3}$	$\cos y = \frac{-1}{2}$
$\tan^{-1}\sqrt{3} = \frac{\pi}{3}$	Put $y = \frac{2\pi}{3}$ or 120°
$= \frac{\pi}{3}$	$\cos\frac{2\pi}{3} = \frac{-1}{2}$
	$\frac{2\pi}{3} = \cos^{-1}\left(\frac{-1}{2}\right)$
	$\sec^{-1}(-2) = \frac{2\pi}{3}$
	$= \frac{2\pi}{3}$

2nd Method Q # 1 Part vii or

$$= \frac{\pi}{3} - \frac{2\pi}{3} = \frac{\pi - 2\pi}{3} = \frac{-\pi}{3}$$

(iv) $\cot^{-1}(-\sqrt{3}) + \operatorname{cosec}^{-1}(-2) - \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

Sol

$$\cot^{-1}(-\sqrt{3}) + \operatorname{cosec}^{-1}(-2) - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

Let $y = \cot^{-1}(-\sqrt{3})$	Let $y = \operatorname{cosec}^{-1}(-2)$	Let $y = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\cot y = -\sqrt{3}$	$\operatorname{cosec} y = -2$	$\cos y = \frac{1}{\sqrt{2}}$
$\frac{1}{\tan y} = -\sqrt{3}$	$\frac{1}{\sin y} = -2$	$y = \frac{\pi}{4}$ or 45°
$\tan y = -\frac{1}{\sqrt{3}}$	$\sin y = -\frac{1}{2}$	$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$
$\tan\left(\frac{-\pi}{6}\right) = -\frac{1}{\sqrt{3}}$	$\sin\left(\frac{-\pi}{6}\right) = -\frac{1}{2}$	$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6}$	$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$	$= \frac{\pi}{4}$
$\cot^{-1}(-\sqrt{3}) = -\frac{\pi}{6}$	$\operatorname{cosec}^{-1}(-2) = -\frac{\pi}{6}$	
$-\frac{\pi}{6}$	$= -\frac{\pi}{6}$	

$$= -\frac{\pi}{6} - \frac{\pi}{6} - \frac{\pi}{4}$$

$$= \frac{-2\pi - 2\pi - 3\pi}{12} = -\frac{7\pi}{12} \text{ Ans.}$$

Q#3 Find the exact real number value of each without using calculator.

(i) $\cos^{-1}\left(\sin\frac{\pi}{4}\right)$

Sol/ $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Let $y = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\cos y = \frac{1}{\sqrt{2}}$

Put $y = \frac{\pi}{4}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\boxed{\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}}$ Ans.

(ii) $\sin^{-1}\left[\cos\left(\frac{-2\pi}{3}\right)\right]$

Sol/ $= \sin^{-1}\left[\cos(-120^\circ)\right]$

$= \sin^{-1}\left[\frac{-1}{2}\right]$

Let $y = \sin^{-1}\left(\frac{-1}{2}\right)$

$\sin y = -\frac{1}{2}$

Put $y = -\frac{\pi}{6}$

$\sin\left(\frac{-\pi}{6}\right) = -\frac{1}{2}$

So Ans $\boxed{y = -\frac{\pi}{6}}$

2nd Method

Let $y = \sin^{-1}\left[\cos(-2\pi/3)\right]$

$\sin y = \cos\left(\frac{-2\pi}{3}\right)$ $\because \cos(-\theta) = \cos\theta$

$\sin y = \cos\left(\frac{2\pi}{3}\right)$

$\sin y = \cos\left(\pi - \frac{\pi}{3}\right)$ using Allied Angles.

$\sin y = -\cos\frac{\pi}{3}$

$\sin y = -\frac{1}{2}$

put $y = -\frac{\pi}{6}$

$\sin\left(\frac{-\pi}{6}\right) = \frac{-1}{2}$

So $\boxed{y = -\frac{\pi}{6}}$

2nd Method

(iii) $\cos^{-1}\left(\sin\frac{11\pi}{6}\right)$

Sol $= \cos^{-1}[\sin 330^\circ]$

$= \cos^{-1}\left(-\frac{1}{2}\right)$

or 2nd Method Q#1 vii

Let $y = \cos^{-1}\left(-\frac{1}{2}\right)$

$\cos y = \frac{-1}{2}$

Put $y = 120^\circ$ or $\frac{2\pi}{3}$

$\cos\frac{2\pi}{3} = \frac{-1}{2}$

$\boxed{y = \frac{2\pi}{3}}$ An

$y = \cos^{-1}\left(\sin\frac{11\pi}{6}\right)$

$\cos y = \sin\left(2\pi - \frac{\pi}{6}\right)$ Use Allied Angle

$\cos y = -\sin\frac{\pi}{6}$

$\cos y = -\frac{1}{2}$

Put $y = \frac{2\pi}{3}$ or 120°

$\cos\frac{2\pi}{3} = \frac{-1}{2}$

So $\boxed{y = \frac{2\pi}{3}}$ An.

(iv) $\cos^{-1}\left(\cos\frac{\pi}{6}\right)$

Sol Put

$y = \cos^{-1}\left(\cos\frac{\pi}{6}\right)$

$\cos y = \cos\frac{\pi}{6}$

$\cos y = \frac{\sqrt{3}}{2}$

Put $y = \frac{\pi}{6}$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

So $\boxed{y = \frac{\pi}{6}}$ Ans

2nd Method

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right)$

Since Property

$\cos(\cos^{-1}x) = x$

So

$\cos^{-1}\left(\cos\frac{\pi}{6}\right) = \frac{\pi}{6}$

Important Point

$\sin\theta = \frac{P}{H}$

$\cos\theta = \frac{B}{H}$ $\qquad H^2 = P^2 + B^2$

$\tan\theta = \frac{P}{B}$

(V) $\sin\left(\tan^{-1}\frac{3}{4}\right)$

Sol Let $y = \tan^{-1}\frac{3}{4}$

$\tan y = \frac{3}{4}$

$\boxed{P = 3}$ $\boxed{B = 4}$

Using Phythagoras Theorem

$H^2 = P^2 + B^2$

$H^2 = (3)^2 + (4)^2$

$H^2 = 9 + 16$

$H^2 = 25$

$\boxed{H = 5}$

Now

$\sin y = \frac{P}{H}$

$\sin y = \frac{3}{5}$

Putting value of θ

$$\boxed{\sin\left(\tan^{-1}\frac{3}{4}\right) = \frac{3}{5}}$$

Ans.

(vi) $\cos\left(2\sin^{-1}\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$

Sol We write

$= \cos\left(2\sin^{-1}\frac{1}{\sqrt{2}}\right)$

Let $y = \sin^{-1}\frac{1}{\sqrt{2}}$

$\sin y = \frac{1}{\sqrt{2}}$

Put $y = \frac{\pi}{4}$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$

So $= \cos\left(2\cdot\frac{\pi}{4}\right)$

$= \cos\left(\frac{\pi}{2}\right)$

$= 0$ Ans.

(vii) $\sin\left[2\sin^{-1}\frac{4}{5}\right]$

$\sin 2y = 2\sin y\cos y$

Sol

Put $y = \sin^{-1}\frac{4}{5}$

$\boxed{\sin y = \frac{4}{5}}$

$P = 4 \quad H = 5$

Using Phy Theorem.

$$H^2 = P^2 + B^2$$

$$(5)^2 = (4)^2 = B^2$$

$$25 - 16 = B^2$$

$$\boxed{B = 3}$$

$$\boxed{\cos y = \frac{3}{5}}$$

Now

$$\sin\left[2 \sin^{-1} \frac{4}{5}\right] \qquad \because y = \sin^{-1} \frac{4}{5}$$

$$\sin 2y = 2 \sin y \cos y$$

$$= 2 \cdot \frac{4}{5} \cdot \frac{3}{5}$$

$$\sin\left[2 \sin^{-1} \frac{4}{5}\right] = \frac{24}{25} \quad \text{Ans.}$$

(VIII) $\cos\left(\sin^{-1} \frac{5}{13}\right)$

Sol Let $y = \sin^{-1} \frac{5}{13}$

$$\sin y = \frac{5}{13}$$

$P = 5 \quad H = 13$

Using Phy Theorem

$$H^2 = P^2 + B^2$$
$$(13)^2 = (5)^2 + B^2$$
$$169 - 25 = B^2$$
$$B^2 = 144$$
$$\boxed{B = 12}$$
$$\cos y = \frac{B}{H} = \frac{12}{13}$$
$$\boxed{\cos y = \frac{12}{13}}$$
Putting value of y
$$\cos\left(\sin^{-1}\frac{5}{13}\right) = \frac{12}{13}$$

(ix) $\sin\left(\cos^{-1}\left(\frac{-3}{5}\right)\right)$

Sol Let $y = \cos^{-1}\left(\frac{-3}{5}\right)$

$\cos y = \frac{-3}{5}$ As $\cos y = \frac{B}{H}$

$B = -3$ $H = 5$

Using Phy Theorem
$$H^2 = P^2 + B^2$$
$$(5)^2 = P^2 + (-3)^2$$
$$25 - 9 = P^2$$
$$\boxed{P = 4}$$

So $\sin\theta = \frac{P}{H}$

$\sin\theta = \frac{4}{5}$

So

$\sin\left(\cos^{-1}\left(-\frac{3}{5}\right)\right) = \frac{4}{5}$ Ans.

Vimp

(x) $\sin\left(\sin^{-1}\frac{2}{3} + \cos^{-1}\frac{1}{2}\right)$

R.W

$\sin(\alpha+\beta) =$

$\sin\alpha\cos\beta + \cos\alpha\sin\beta$

Sol

Let

$\alpha = \sin^{-1}\frac{2}{3}$

$\sin\alpha = \frac{2}{3}$

$P = 2 \quad H = 3$

Using Phy Theorem

$H^2 = P^2 + B^2$

$(3)^2 = (2)^2 + B^2$

$9 - 4 = B^2$

$5 = B^2$

$B = \sqrt{5}$

$\cos\alpha = \frac{B}{H}$

Let

$\beta = \cos^{-1}\frac{1}{2}$

$\cos\beta = \frac{1}{2}$

$B = 1 \quad H = 2$

Using Phy Theorem

$H^2 = P^2 + B^2$

$(2)^2 = P^2 + (1)^2$

$4 - 1 = P^2$

$P^2 = 3$

$P = \sqrt{3}$

$\sin\beta = \frac{P}{H}$

$\cos\alpha = \frac{\sqrt{5}}{3}$ $\sin\beta = \frac{\sqrt{3}}{2}$

$= \sin\left(\sin^{-1}\frac{2}{3} + \cos^{-1}\frac{1}{2}\right)$

$= \sin(\alpha + \beta)$

$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}} + \frac{\sqrt{5}}{3}\cdot\frac{\sqrt{3}}{2}$

$= \frac{1}{3} + \frac{\sqrt{15}}{6}$

$= \frac{2+\sqrt{15}}{6}$ Ans.

(xi) $\cos\left(\sin^{-1}\frac{3}{4} + \cos^{-1}\frac{5}{13}\right)$ $\cos(\alpha+\beta) =$ $\cos\alpha\cos\beta - \sin\alpha\sin\beta$

Sol

$\alpha = \sin^{-1}\frac{3}{4}$ $\beta = \cos^{-1}\frac{5}{13}$

$\sin\alpha = \frac{3}{4}$ $\cos\beta = \frac{5}{13}$

$P = 3$ $H = 4$ $B = 5$ $H = 13$

Using Phy Theorem Using Phy Theorem

$H^2 = P^2 + B^2$ $H^2 = P^2 + B^2$

$(4)^2 = (3)^2 + B^2$

$16 - 9 = B^2$

$7 = B^2$

$B = \sqrt{7}$

$\cos\alpha = \frac{B}{H}$

$\cos\alpha = \frac{\sqrt{7}}{4}$

$(13)^2 = P^2 + (5)^2$

$169 - 25 = P^2$

$P^2 = 144$

$P = 12$

$\sin\beta = \frac{P}{H}$

$\sin\beta = \frac{12}{13}$

$= \cos\left(\sin^{-1}\frac{3}{4} + \cos^{-1}\frac{5}{13}\right)$

$= \cos(\alpha + \beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{\sqrt{7}}{4} \cdot \frac{5}{13} - \frac{3}{4} \cdot \frac{12}{13}$

$= \frac{5\sqrt{7}}{52} - \frac{36}{52}$

$\frac{5\sqrt{7} - 36}{52}$ Ans.

(xii)

$\cos\left[\sec^{-1}(3) + \tan^{-1} 2\right]$

Sol $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

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$\alpha = \sec^{-1}(3)$

$\sec\alpha = 3$

$\boxed{\cos\alpha = \frac{1}{3}}$

$B = 1 \quad H = 3$

Using Phy Theorem

$H^2 = P^2 + B^2$

$(3)^2 = P^2 + (1)^2$

$9 - 1 = P^2$

$P = \sqrt{8}$

$P = \sqrt{8}$

$\sin\alpha = \frac{P}{H}$

$\boxed{\sin\alpha = \frac{\sqrt{8}}{3}}$

$\beta = \tan^{-1}(2)$

$\tan\beta = \frac{2}{1}$

$P = 2 \quad B = 1$

Using Phy Theorem

$H^2 = P^2 + B^2$

$H^2 = (2)^2 + (1)^2$

$H^2 = 4 + 1$

$H = \sqrt{5}$

$\sin\beta = \frac{P}{H}$

$\boxed{\sin\beta = \frac{2}{\sqrt{5}}}$

$\cos\beta = \frac{B}{H}$

$\boxed{\cos\beta = \frac{1}{\sqrt{5}}}$

$\cos\left[\sec^{-1}3 + \tan^{-1}2\right]$

$\cos\left[\alpha + \beta\right]$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= \frac{1}{3} \cdot \frac{1}{\sqrt{5}} - \frac{\sqrt{8}}{3} \cdot \frac{2}{\sqrt{5}}$$

$$= \frac{1}{3\sqrt{5}} - \frac{2\sqrt{8}}{3\sqrt{5}}$$

$$= \frac{1 - 2\sqrt{8}}{3\sqrt{5}} \quad \text{Ans:}$$

Q#4 Use a Calculator to evaluate the following as real number to three decimal places:

(i)

$$\cos^{-1}\left(\frac{3}{5}\right) = \sin^{-1}(?)$$

Sol:

Let $y = \cos^{-1}\frac{3}{5}$

$\cos y = \frac{3}{5}$ $\quad B = 3 \quad H = 5$

Using Phy Theorem

$$H^2 = P^2 + B^2$$

$$(5)^2 = P^2 + (3)^2$$

$$25 - 9 = P^2$$

$$P^2 = 16$$

$$P = 4$$

Now

$\sin y = \frac{P}{H} \Rightarrow \sin y = \frac{4}{5}$

$$y = \sin^{-1}\left(\frac{4}{5}\right)$$

As $\cos^{-1}\frac{3}{5} = y$

$$\cos^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{4}{5}\right)$$

$$= 53.130 \quad \text{Ans.}$$

(ii) $\sin^{-1}\left(\frac{2}{3}\right) = \cos^{-1}(\quad)$

Sol Let $y = \sin^{-1}\frac{2}{3}$

$\sin y = \frac{2}{3}$ $\quad P = 2 \quad H = 3$

Using Phy Theorem.

$$H^2 = P^2 + B^2$$

$$(3)^2 = (2)^2 + B^2$$

$$9 - 4 = B^2$$

$$B = \sqrt{5}$$

$\cos y = \frac{B}{H}$ $\quad \cos y = \frac{\sqrt{5}}{3}$

$$y = \cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$$

As

$$\sin^{-1}\frac{2}{3} = y$$

$$\sin^{-1}\left(\frac{2}{3}\right) = \cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$$

$$= 41.810^\circ \quad \text{Ans}$$

(iii) $\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right) = -\cos^{-1}(?)$

Sol

Let $-y = \sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)$ — (i)

$\cancel{-}y = \cancel{-}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$y = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\sin y = \frac{1}{\sqrt{5}}$ $\quad P = 1 \quad H = \sqrt{5}$

Using Phy Theorem

$$H^2 = P^2 + B^2$$

$$(\sqrt{5})^2 = (1)^2 + B^2$$

$$5 - 1 = B^2$$

$$B = 2$$

$\cos y = \frac{B}{H} \Rightarrow \cos y = \frac{2}{\sqrt{5}}$

$$y = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

Eq (i) becomes:

$$\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right) = -\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

$$= -26.565^\circ \quad \text{Ans}$$

(IV) $\tan^{-1}\left(\frac{1}{4}\right) = \cos^{-1}(\quad)$

Sol

Let $y = \tan^{-1}\left(\frac{1}{4}\right)$ —(i)

$\tan y = \frac{1}{4}$

$P = 1 \qquad B = 4$

using Phy Theorem

$H^2 = P^2 + B^2$

$H^2 = (1)^2 + (4)^2$

$H^2 = 1 + 16$

$H = \sqrt{17}$

$\cos y = \frac{B}{H} \Rightarrow \cos y = \frac{4}{\sqrt{17}}$

$y = \cos^{-1}\left(\frac{4}{\sqrt{17}}\right)$

So eq (i) becomes

$\tan^{-1}\left(\frac{1}{4}\right) = \cos^{-1}\left(\frac{4}{\sqrt{17}}\right)$

$= 14.036^\circ$ Ans.

(V) $\tan^{-1}(-1.2) = -\cos^{-1}(\quad)$

Sol Put

$-y = \tan^{-1}(-1.2)$ —(i)

$\cancel{-}y = \cancel{-}\tan^{-1}(1.2)$

$y = \tan^{-1}(1.2)$

$\tan y = \frac{1.2}{1}$

$P = 1.2 \qquad B = 1$

Using Phy Theorem

$$H^2 = P^2 + B^2$$

$$H^2 = (1.2)^2 + (1)^2$$

$$H^2 = 1.44 + 1$$

$$H^2 = 2.44$$

$$H = 1.56$$

$\cos y = \frac{B}{H} \Rightarrow \cos y = \frac{1}{1.56}$

$$y = \cos^{-1}\left(\frac{1}{1.56}\right)$$

$$y = \cos^{-1}(0.640)$$

Put in (i)

$$\tan^{-1}(-1.2) = -\cos^{-1}(0.640)$$

$$\tan^{-1}(-1.2) = -50.194 \quad \text{An}$$

(vi)

$$\cot^{-1}\left(\frac{-3}{4}\right) = -\sin^{-1}(\quad)$$

Sol

Put $-y = \cot^{-1}\left(-\frac{3}{4}\right)$ —— (1)

$$\cancel{-}y = \cancel{-}\cot^{-1}\left(\frac{3}{4}\right)$$

$$y = \cot^{-1}\left(\frac{3}{4}\right)$$

$\cot y = \frac{3}{4}$

$\tan y = \frac{4}{3}$ $\quad P = 4 \quad B = 3$

Using Phy Theorem

$$H^2 = P^2 + B^2$$

$$H^2 = (4)^2 + (3)^2$$

$$= 16 + 9$$

$$H^2 = 25$$

$$H = 5$$

$\sin y = \frac{P}{H} \Rightarrow \sin y \;\; \frac{4}{5}$

$$y = \sin^{-1}\left(\frac{4}{5}\right)$$

Eq (i) becomes

$\cot^{-1}\left(\frac{-3}{4}\right) = -y$

$\cot^{-1}\left(\frac{-3}{4}\right) = -\sin^{-1}\left(\frac{4}{5}\right)$

$$= -53.130^\circ \quad \text{An}$$

(viii) $\sec^{-1}(2.041) = \tan^{-1}(\quad)$

Sol

Let $y = \sec^{-1}(2.041)$ — (1)

$\sec y = 2.041$

$\cos y = \dfrac{1}{2.041}$

$\cos y = \dfrac{B}{H}$

$B = 1 \qquad H = 2.041$

Using Phy Theorem.

$H^2 = P^2 + B^2$

$(2.041)^2 = P^2 + (1)^2$

$(2.041)^2 - 1 = P^2$

$P^2 = 3.165$

$P = 1.779$

$\tan y = \dfrac{P}{B} = \tan y = \dfrac{1.779}{1}$

$y = \tan^{-1}(1.779)$

Put in (i)

$\sec^{-1}(2.041) = \tan^{-1}(1.779)$

$\sec^{-1}(2.041) = 60.662°$ Ans.

(viii) $\sec^{-1}(-\sqrt{5}) = \cot^{-1}(\quad)$

Sol put

$$y = \sec^{-1}(-\sqrt{5}) \quad \text{--- (1)}$$

$$\sec y = -\sqrt{5}$$

$$\cos y = -\frac{1}{\sqrt{5}}$$

$B = -1 \qquad H = \sqrt{5}$

using phy Theorem

$$H^2 = P^2 + B^2$$

$$(\sqrt{5})^2 = P^2 + (-1)^2$$

$$5 - 1 = P^2$$

$$P = 2$$

Now

$$\cot y = \frac{B}{P} = \frac{-1}{2}$$

$$y = \cot^{-1}\left(\frac{-1}{2}\right)$$

Put in (1)

$$\sec^{-1}(-\sqrt{5}) = \cot^{-1}\left(\frac{-1}{2}\right)$$

$$\sec^{-1}(-\sqrt{5}) = \tan^{-1}(-2)$$

$$= -63.435^\circ \text{ Ans.}$$

$y = \cot^{-1}\left(\frac{-1}{2}\right)$

$\cot y = \frac{-1}{2}$

$\tan y = -2$

$y = \tan^{-1}(-2)$

(IX) $\operatorname{Cosec}^{-1}(1.172) = \sin^{-1}(\quad)$

Sol Put $y = \operatorname{Cosec}^{-1}(1.172)$ —(i)

$\operatorname{Cosec} y = 1.172$

$\dfrac{1}{\sin y} = 1.172$

$\sin y = \dfrac{1}{1.172}$

$y = \sin^{-1}\left(\dfrac{1}{1.172}\right)$

Put in (i)

$\operatorname{Cosec}^{-1}(1.172) = y$

$\operatorname{Cosec}^{-1}(1.172) = \sin^{-1}\left(\dfrac{1}{1.172}\right)$

$= \sin^{-1}(0.853)$

$= 58.566^{\circ}$ Ans.

(X)

$\operatorname{Cosec}^{-1}\left(-\dfrac{5}{3}\right) = \tan^{-1}(\quad)$

Sol Let $y = \operatorname{Cosec}^{-1}\left(\dfrac{-5}{3}\right)$ —(1)

$\operatorname{Cosec} y = -\dfrac{5}{3}$

$\sin y = -\dfrac{3}{5}$

$\cancel{B} = \cancel{-8}$ $\cancel{H} =$

$P = -3 \quad H = 5$

Using Phy Theorem

$H^2 = P^2 + B^2$

$(5)^2 = (-3)^2 + B^2$

$25 - 9 = B^2$

$B = 4$

$\tan y = \frac{P}{B} = \frac{-3}{4}$

$y = \tan^{-1}\left(\frac{-3}{4}\right)$

eq (i) becomes

$\operatorname{cosec}^{-1}\left(\frac{-5}{3}\right) = y$

$\operatorname{cosec}^{-1}\left(\frac{-5}{3}\right) = \tan^{-1}\left(\frac{-3}{4}\right)$

$\operatorname{cosec}^{-1}\left(\frac{-5}{3}\right) = -36.870^\circ$ Ans.

Complete.