Exc 6.3

Q#1 Three jet fighters are flying straight in different directions along the lines $2x - y - 14 = 0$, $x + y - 1 = 0$ and $3x + 2y - 7 = 0$. Check whether the jet fighters will pass through a single point or not. If yes, find that point.

Sol

$$2x - y - 14 = 0 \quad \text{—(i)}$$
$$x + y - 1 = 0 \quad \text{—(ii)}$$
$$3x + 2y - 7 = 0 \quad \text{—(iii)}$$

- Pass through a single point means concurrent. By taking determinant to check whether they meet at a point or not

$$\begin{vmatrix} 2 & -1 & -14 \\ 1 & 1 & -1 \\ 3 & 2 & -7 \end{vmatrix}$$

$$\therefore 2(-7+2) + 1(-7+3) - 14(2-3)$$
$$= -10 - 4 + 14$$
$$= 0$$

So these lines pass through a single point.

To calculate a point solve eq (i) and eq (ii)

$$2x - y = 14$$
$$x + y = 1$$
$$3x = 15$$

$$\boxed{x = 5}$$

Put $x = 5$ in (iii)

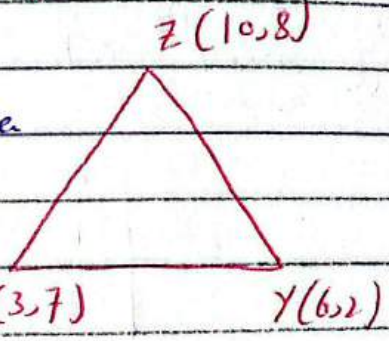
$$x + y - 1 \geq 0$$

$$5 + y - 1 \geq 0$$

$$y \geq -4$$

So the point of Concurrency from where three Fighter jets will pass is $(5, -4)$

Q#2 A Farmer owns a triangular shaped piece of land with corners at points $X(3,7)$ $Y(6,2)$ $Z(10,8)$. Calculate the cost of planting maize crops @ Rs. 300 Per Square unit.



Sol: First we Find the Area of triangular Land.

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$A = \frac{1}{2}\begin{vmatrix} 3 & 7 & 1 \\ 6 & 2 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[3(2-8) - 7(6-10) + 1(48-20)\right]$$

$$= \frac{1}{2}\left[-18 + 28 + 28\right] = \frac{38}{2} = 19$$

Area of Land = 19 Square Unit

For 1 Sq unit = 300

For 19 Sq unit = 300 × 19

= 5700 rupees.

Q#3 Hira is designing a triangular section of a roof with vertices at points P(4,1) Q(9,5) R(7,10). She needs to calculate the area of the section. Find the area.

Sol

Area of triangular region.

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$A = \frac{1}{2}\begin{vmatrix} 4 & 1 & 1 \\ 9 & 5 & 1 \\ 7 & 10 & 1 \end{vmatrix}$$

$$A = \frac{1}{2}\left[4(5-10) - 1(9-7) + 1(90-35)\right]$$

$$A = \frac{1}{2}\left[-20 - 2 + 55\right]$$

$$A = \frac{1}{2}\left[33\right] = 16.5 \text{ Square unit.}$$

Q#4 A Land Scaper is designing a triangular garden bed with vertices A(1,4) B(5,1) C(8,6). Calculate the cost of planting mango trees in the garden @ Rs 70 per sq unit.

Sol:

First we find the Area of triangular garden.

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$A = \frac{1}{2}\begin{vmatrix} 1 & 4 & 1 \\ 5 & 1 & 1 \\ 8 & 6 & 1 \end{vmatrix}$$

$$A = \frac{1}{2}\left[1(1-6) - 4(5-8) + 1(30-8)\right]$$

$$A = \frac{1}{2}\left[-5 + 12 + 22\right]$$

$$A = \frac{29}{2} = 14.5 \text{ square unit.}$$

For 1 square unit = 70 rupees

For 14.5 " " = 70×14.5

= 1015 rupees

Q#5 A civil engineer is tasked with designing a round about where three main roads converge. The equations of roads are $2x+y-11.5=0$, $x-4y+1=0$ & $3x-2y-12=0$. Find the coordinates of the point to design the round about.

Sol Given

$$2x+y-11.5=0 \quad -(i)$$
$$x-4y+1=0 \quad -(ii)$$
$$3x-2y-12=0 \quad -(iii)$$

First we check above Lines are concurrent or not.

$$\begin{vmatrix} 2 & 1 & -11.5 \\ 1 & -4 & 1 \\ 3 & -2 & -12 \end{vmatrix}$$

$$\therefore 2(48+2)-1(-12-3)-11.5(-2+12)$$

$$\therefore 100+15-115$$

$$= 0$$

So given Lins are concurrent. Also this shows that roads converge at same point.

For point Solving of (i) and (ii)

$$3x - 12y = -3$$
$$+3x \mp 2y = \pm 12$$
$$-10y = -15$$
$$y = \frac{15}{10} \qquad y = \frac{3}{2}$$

Put $y = \frac{3}{2}$ in (iii)

$$x - 4y + 1 = 0$$
$$x - 4\left(\frac{3}{2}\right) + 1 = 0$$
$$x - 6 + 1 = 0$$
$$x = 5$$

Hence $(5, \frac{3}{2})$ is the required point.

Q#6 Asad is arranging a Flash light in a marriage ceremony. The position of the Flash light is at the intersection of Lins $2x + y - 23 = 0$. Find the position of the Flashlight.

Sol

Given $2x + y - 23 = 0$ —(i)

$0.5x - y + 3 = 0$ —(ii)

$x - y - 1 = 0$ —(iii)

To Find Position of the Flash light we Find the point.

Solving (i) and (iii)

$$2x + y = 23$$
$$x - y = 1$$
$$3x = 24$$
$$\boxed{x = 8}$$

Put in (iii)

$$8 - y - 1 = 0$$
$$-y + 7 = 0$$
$$\boxed{y = 7}$$

$x - y = 1$
$8 - 7 = 1$
$1 = 1$
True.

So P(8, 7) is the position of flashlight.

Q#7 A surveyor is mapping out a triangular park where three straight walkways meet. The walkways are represented by the $x + y - 4 = 0$, $2x - y - 2 = 0$ $x - 2y + 2 = 0$ Find the coordinates of point of intersection of the walkways.

Sol Given

$$x + y - 4 = 0 \quad -(1)$$
$$2x - y - 2 = 0 \quad -(2)$$
$$x - 2y + 2 = 0 \quad -(3)$$

To find point of intersection solve above given equations.

Solving (i) and (ii)

$$x + y - 4 = 0$$
$$2x + y - 2 = 0$$
$$\overline{-x - 2 = 0}$$
$$3x - 6 = 0$$
$$3x = 6$$
$$x = 2$$

Put $x = 2$ in (i)

$$2 + y - 4 = 0$$
$$y - 2 = 0$$
$$y = 2$$

Hence point $(2, 2)$ is the point of intersection of walkway.

Check Put $(2, 2)$ in (i)

$$x + y - 4 = 0$$
$$2 + 2 - 4 = 0$$
$$0 = 0 \quad \text{True}$$

(8) An astronomer is studying the behavior of light rays passing through a converging lens. The lens focuses the rays at the origin. The equation of rays is given by $3x^2 - 4xy + y^2 = 0$. Find the path of individual light rays and angle b/w rays.

Sol: Given

$$3x^2 - 4xy + y^2 = 0$$

$$3x^2 - 3xy - xy + y^2 = 0$$

$$3x(x-y) - y(x-y) = 0$$

$$(x-y)(3x-y) = 0$$

$x - y = 0$ $\qquad$ $3x - y = 0$

are equation of individual rays.

Angle b/w rays:

$$3x^2 - 4xy + y^2 = 0$$

Compare with $ax^2 + 2hxy + by^2 = 0$

$a = 3 \qquad 2h = -4 \qquad b = 1$

$h = -2$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\tan\theta = \frac{2\sqrt{(-2)^2 - (3)(1)}}{3+1}$$

$$\tan\theta = \frac{2\sqrt{4-3}}{4} = \frac{1}{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right)$$

$$= 26.6^\circ \quad \text{Ans}$$

Q#9. A welder is designing a support structure for a building. The structure is made up of beams intersecting at origin. The equations of the lines representing the beams are shown by the joint eq $2x^3 - 8xy^2 = 0$. Find the equation of irons beams and angle b/w beams.

Sol

$$2x^3 - 8xy^2 = 0$$

$$2x(x^2 - 4y^2) = 0$$

$$2x(x^2 - (2y)^2) = 0$$

$$2x(x+2y)(x-2y) = 0$$

$2x = 0$ $\quad$ $x + 2y = 0$ $\quad$ $x - 2y = 0$

$x = 0$ $\quad$ $x + 2y = 0$ $\quad$ $x - 2y = 0$

required eq of beams.

Complete.