Exc 6.2

Angle b/w Lines represented by $ax^2+2hxy+by^2=0$

2nd order homogeneous eq

$$ax^2+2hxy+by^2=0$$

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$$

Q #2

Theorem

Second degree homogeneous equation $ax^2+2hxy+by^2=0$ represents pair of straight lines passing through origion (0,0)

Nature of Line

i) $h^2-ab > 0$ Real and distinct

ii) $h^2-ab = 0$ Real and coincident (Same Lines)

iii) $h^2-ab < 0$ Imaginary.

Q #3

Q #5

Eq of Line

$$(y-y_1) = m(x-x_1)$$

(x_1, y_1) = Given point

(x, y) = Fixed point

m = slope

- Slope intercept Form

$$y = mx + c$$

Q#1 Find the Lines represented by each of the following joint equations.

(i) $x^2+5xy+6y^2=0$

Sol

$$x^2+3xy+2xy+6y^2=0$$
$$x(x+3y)+2y(x+3y)=0$$
$$(x+2y)(x+3y)=0$$

$x+2y=0$ $\quad$ $x+3y=0$ $\quad$ required Lines.

(ii) $7x^2-2xy-9y^2=0$

Sol

$$7x^2-9xy+7xy-9y^2=0$$
$$x(7x-9y)+y(7x-9y)=0$$
$$(x+y)(7x-9y)=0$$

$x+y=0$ $\quad$ $7x-9y=0$ $\quad$ required Lines

(iii) $x^2+6xy=0$

Sol

$$x(x+6y)=0$$

$x=0$ $\quad$ $x+6y=0$ $\quad$ required Lines

(iv) $x^2-8xy+12y^2=0$

Sol

$$x^2-6xy-2xy+12y^2=0$$

$$x(x-6y)-2y(x-6y)=0$$

$$(x-2y)(x-6y)=0$$

$x-2y=0$ $\quad$ $x-6y=0$ required Lines

(v) $5x^2+3xy-2y^2=0$

Sol

$$5x^2+5xy-2xy-2y^2=0$$

$$5x(x+y)-2y(x+y)=0$$

$$(5x-2y)(x+y)=0$$

$5x-2y=0$ $\quad$ $x+y=0$ required Lines

(vi) vimp $x^2-3xy-y^2=0$

Sol

$$ax^2+bx+c=0$$

$a=1$ $\quad$ $b=-3y$ $\quad$ $c=-y^2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-3y)\pm\sqrt{(-3y)^2-4(1)(-y^2)}}{2(1)}$$

$$x=\frac{3y\pm\sqrt{9y^2+4y^2}}{2}$$

$$x = \frac{3y \pm \sqrt{13}\,y}{2}$$

$$x = \frac{3y + \sqrt{13}\,y}{2}$$

$$2x = 3y + \sqrt{13}\,y$$

$$2x - 3y - \sqrt{13}\,y = 0$$

$$2x - y(3 + \sqrt{13}) = 0$$

$$x = \frac{3y - \sqrt{13}\,y}{2}$$

$$2x = 3y - \sqrt{13}\,y$$

$$2x - 3y + \sqrt{13}\,y = 0$$

$$2x - y(3 - \sqrt{13}) = 0$$

required Lines.

Q#2 Find measure of acute angle (Less than 90°) b/w Lines represented by the following equations

1) $x^2 - y^2 = 0$

Compare with

$$ax^2 + 2hxy + by^2 = 0$$

$a = 1$, $2h = 0$, $b = -1$

$h = 0$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\tan\theta = \frac{2\sqrt{0 - (1)(-1)}}{1 + (-1)}$$

$$\tan\theta = \frac{2}{0} = \infty$$

$$\theta = \tan^{-1}(\infty)$$

$$\boxed{\theta = 90^\circ}$$ Ans.

This shows that Lines are Perpendicular to each other.

(ii) $x^2 + 5xy + 4y^2 = 0$

Sol: Compare with $ax^2 + 2hxy + by^2 = 0$

$$a = 1 \qquad 2h = 5 \qquad b = 4$$

$$h = \frac{5}{2}$$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\tan\theta = \frac{2\sqrt{(5/2)^2 - (1)(4)}}{1+4}$$

$$\tan\theta = \frac{2\sqrt{\frac{25}{4} - \frac{4}{1}}}{5}$$

$$\tan\theta = \frac{2\sqrt{9/4}}{5}$$

$$\tan\theta = \frac{\cancel{2} \cdot 3/\cancel{2}}{5}$$

$$\tan\theta = \frac{3}{5}$$

$$\theta = \tan^{-1}\left(\frac{3}{5}\right)$$

$$\boxed{\theta = 30.96^\circ}$$

(iii) $15x^2 - 19xy + 6y^2 = 0$

Sol Compare with $ax^2 + 2hxy + by^2 = 0$

$a = 15$ $\quad 2h = -19$ $\quad b = 6$

$h = -19/2$

$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$

$\tan\theta = \frac{2\sqrt{(-19/2)^2 - (15)(6)}}{15+6}$

$\tan\theta = \frac{2\sqrt{\frac{361}{4} - \frac{90}{1}}}{21}$

$\tan\theta = \frac{2\sqrt{\frac{361-360}{4}}}{21}$

$\tan\theta = \frac{\cancel{2} \cdot \frac{1}{\cancel{2}}}{21}$

$\tan\theta = \frac{1}{21}$

$\theta = \tan^{-1}\left(\frac{1}{21}\right)$

$\theta = 2.7^\circ$

(iv) $10x^2 - xy - 9y^2 = 0$

Sol Compare with $ax^2 + 2hxy + by^2 = 0$

$a = 10$ $\quad 2h = -1$ $\quad b = -9$

$h = -\frac{1}{2}$

$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$

$$\tan\theta = \frac{2\sqrt{\frac{1}{4}+\frac{90}{1}}}{10-9}$$

$$= \frac{2\sqrt{\frac{361}{4}}}{1}$$

$$= \cancel{2} \cdot \frac{19}{\cancel{2}}$$

$\tan\theta = 19$

$\theta = \tan^{-1}(19)$

$\boxed{\theta = 87^\circ}$

(v) $5x^2 - 3xy - 2y^2 = 0$

Sol:

Compare with $ax^2 + 2hxy + by^2 = 0$

$a = 5 \qquad 2h = -3 \qquad b = -2$

$h = \frac{-3}{2}$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\tan\theta = \frac{2\sqrt{\frac{9}{4}+10}}{5-2}$$

$$\tan\theta = \frac{\cancel{2}\,\frac{7}{\cancel{2}}}{3}$$

$$\tan\theta = \frac{7}{3}$$

$\theta = \tan^{-1}\left(\frac{7}{3}\right)$

$\boxed{\theta = 66.8^\circ}$

(vi) $7x^2 + 2xy - 9y^2 = 0$

Sol: Compare with $ax^2 + 2hxy + by^2 = 0$

$a = 7 \qquad 2h = 2 \qquad b = -9$

$h = 1$

$\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a+b}$

$\tan\theta = \dfrac{2\sqrt{1 + 63}}{-2}$

$\tan\theta = \dfrac{16}{-2}$

$\tan\theta = -8$

$\theta = \tan^{-1}(-8)$

$\theta = -82.87^\circ$

$\boxed{\theta = 82.87^\circ}$

Acute Angle.

Q#3 Show that the Lines represented following equations are real coincident or real distinct or imaginary.

(i) $x^2 + 4xy - 21y^2 = 0$

Sol: Compare with $ax^2 + 2hxy + by^2 = 0$

$a = 1 \qquad 2h = 4 \qquad b = -21$

$h = 2$

Now

$h^2 - ab$

$= (2)^2 - (1)(-21)$

$= 4 + 21$

$= 25 > 0$

So Lines are real and distinct.

(ii) $4x^2 - 12xy + 9y^2 = 0$

Sol. Compare with $ax^2 + 2hxy + by^2 = 0$

$a = 4 \quad 2h = -12 \quad b = 9$

$h = -6$

$= h^2 - ab$

$= (-6)^2 - (4)(9)$

$= 36 - 36$

$= 0$

So Lines are real and coincident.

(iii) $x^2 + xy + y^2 = 0$

Sol. Compare with $ax^2 + 2hxy + by^2 = 0$

$a = 1 \quad 2h = 1 \quad b = 1$

$h = \frac{1}{2}$

$= h^2 - ab$

$= \left(\frac{1}{2}\right)^2 - (1)(1)$

$= \frac{1}{4} - \frac{1}{1}$

$= \frac{-3}{4} < 0$

The Lines are imaginary.

(iv) $x^2 - 9y^2 = 0$

Compare with $ax^2 + 2hxy + by^2 = 0$

$a = 1$ $\quad 2h = 0$ $\quad b = -9$

$h = 0$

$= h^2 - ab$

$= (0)^2 - (1)(-9)$

$= 9 > 0$

The Lines are real and distinct.

Q#4

Show that the angle between Lines represented by the following equations is right angle.

(i) $x^2 + 5xy - y^2 = 0$

Sol

Compare with $ax^2 + 2hxy + by^2 = 0$

$a = 1$ $\quad b = -1$ $\quad 2h = 5$

$h = 5/2$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\tan\theta = \frac{2\sqrt{(5/2)^2 - (1)(-1)}}{1-1}$$

$$\tan\theta = \frac{2}{0} \Rightarrow \tan\theta = \infty$$

$$\theta = \tan^{-1}(\infty)$$

$$\theta = 90^\circ$$

The Lines are perpendicular to each other.

(iii) $x^2 - 2(\tan\theta)xy - y^2 = 0$

Sol: Compare with $ax^2 + 2hxy + by^2 = 0$

$a = 1 \qquad 2h = -2\tan\theta \qquad b = -1$

$$h = -\tan\theta$$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\tan\theta = \frac{2\sqrt{\tan^2\theta + 1}}{1-1}$$

$$\tan\theta = \infty \qquad \theta = \tan^{-1}(\infty)$$

$\theta = 90^\circ$

The Lines are Perpendicular to each other.

Q#5 Find a joint equation of the Lines through the origin (0,0) and Perpendicular to the Lines

i) $2x^2 - 7xy + 6y^2 = 0$

First we find Lines from given Eq

$$2x^2 - 4xy - 3xy + 6y^2 = 0$$

$$2x(x - 2y) - 3y(x - 2y) = 0$$

$$(x - 2y)(2x - 3y) = 0$$

$x - 2y = 0$ $\qquad\qquad$ $2x - 3y = 0$

First Consider $x - 2y = 0$

$x = 2y$

$y = \frac{1}{2}x + 0$

$y = mx + c$

$m = \frac{1}{2}$

Since required Line $\perp$ to given Line

(Slope of required Line)(Slope of Given Line) $= -1$

Slope of required Line $= \dfrac{-1}{\text{Slope of Given Line}}$

Slope of required ($\perp$) Line $= \dfrac{-1}{1/2}$

$= -2$

Eq of perpendicular (required) Line is
$O(0,0)$

$$(y - y_1) = m(x - x_1)$$

$$(y - 0) = -2(x - 0)$$

$$y = -2x$$

$$\boxed{2x + y = 0}$$

Now Consider $2x - 3y = 0$

$$-3y = -2x$$

$$y = \frac{2}{3}x + 0$$

$$y = mx + c$$

$m = \dfrac{2}{3}$ (slope of given Line)

Since required Line $\perp$ to given Line
(slope of required Line)(slope of given Line) $= -1$

Slope of required Line $= \dfrac{-1}{\text{slope of given Line}}$

$$= \frac{-1}{2/3} = -\frac{3}{2}$$

Eq of Perpendicular (required Line) is

$O(0,0)$

$$(y-y_1) = m(x-x_1)$$

$$(y-0) = -\frac{3}{2}(x-0)$$

$$2y = -3x$$

$$\boxed{3x+2y=0}$$

Joint equation

$$(2x+y)(3x+2y)=0$$

$$6x^2+4xy+3xy+2y^2=0$$

$$6x^2+7xy+2y^2=0$$

is required joint eq of Lines:

(ii) $x^2+17xy+60y^2=0$

Sol

First we Find Lines From given Eq

$$x^2+12xy+5xy+60y^2=0$$

$$x(x+12y)+5y(x+12y)=0$$

$$(x+5y)(x+12y)=0$$

$x+5y=0$	$x+12y=0$
$5y=-x$	$12y=-x$
$y=\frac{-1}{5}x+0$	$y=\frac{-1}{12}x+0$

$m = \frac{-1}{5}$ (Slope of given Line)

Since required Line $\perp$ to given Line.

(Slope of given Line)(Slope of req Line) = -1

Slope of req Line = $\frac{-1}{\text{Slope of given Line}}$

$= \frac{-1}{-1/5} = 5$

Eq of req perpendicular Line

$(y - y_1) = m(x - x_1)$

$(y - 0) = 5(x - 0)$

$y = 5x$

$5x - y = 0$

$m = -\frac{1}{12}$ (Slope of given Line)

Since req Line $\perp$ to given Line

(Slope of given Line)(Slope of req Line) = -1

Slope of req Line = $\frac{-1}{\text{Slope of given Line}}$

$= \frac{-1}{-1/12} = 12$

Eq of req perpendicular Line

$(y - y_1) = m(x - x_1)$

$(y - 0) = 12(x - 0)$

$y = 12x$

$12x - y = 0$

joint Eq is

$(5x - y)(12x - y) = 0$

$60x^2 - 5xy - 12xy + y^2 = 0$

$60x^2 - 17xy + y^2 = 0$

(iii) $3x^2 - 13xy - 10y^2 = 0$

Sol First we find Lines from given Eq

$3x^2 - 15xy + 2xy - 10y^2 = 0$

$$3x(x-5y)+2y(x-5y)=0$$
$$(3x+2y)(x-5y)=0$$

$3x+2y=0$

$2y=-3x$

$y=\frac{-3}{2}x+0$

$y=mx+C$

$m=-\frac{3}{2}$ [Slope of given Line]

Since req Line ⊥ to given

(Slope of given Line)(Slope of req Line) $=-1$

Slope of req Line $=\frac{-1}{\text{Slope of given Line}}$

$=\frac{-1}{-3/2}=\frac{2}{3}$

Eq of req ⊥ Line

$(y-y_1)=m(x-x_1)$

$(y-0)=\frac{2}{3}(x-0)$

$y=\frac{2}{3}x$

$2x-3y=0$

$x-5y=0$

$-5y=-x$

$y=\frac{1}{5}x+0$

$y=mx+C$

$m=\frac{1}{5}$ [Slope of given Line]

Since req Line ⊥ to given Line

(Slope of given Line)(Slope of req Line) $=-1$

Slope of req Line $=\frac{-1}{\text{Slope of given Line}}$

$=\frac{-1}{1/5}=-5$

Eq of req ⊥ Line

$(y-y_1)=m(x-x_1)$

$(y-0)=-5(x-0)$

$y=-5x$

$5x+y=0$

Joint Eq is

$$(2x-3y)(5x+y)=0$$
$$10x^2+2xy-15xy-3y^2=0$$
$$10x^2-13xy-3y^2=0 \quad \text{Ans.}$$

(iv) $x^2 + 3xy - 28y^2 = 0$

Sol. First we find Lines from given Eq

$$x^2 + 7xy - 4xy - 28y^2 = 0$$
$$x(x+7y) - 4y(x+7y) = 0$$
$$(x-4y)(x+7y) = 0$$

$$x - 4y = 0$$
$$-4y = -x$$
$$y = \frac{1}{4}x$$

$m = \frac{1}{4}$ [Slope of given Line]

(Slope of Given)(Slope of req) = -1

Slope of req = $\dfrac{-1}{\text{Slope of given}}$

$$= \frac{-1}{1/4} = -4$$

Eq of req ⊥ Line

$$(y - y_1) = m(x - x_1)$$
$$(y - 0) = -4(x - 0)$$
$$y = -4x$$
$$4x + y = 0$$

$$x + 7y = 0$$
$$7y = -x$$
$$y = \frac{-1}{7}x$$

$m = \frac{-1}{7}$ [Slope of given Line]

(Slope of given)(Slope of req) = -1

Slope of req = $\dfrac{-1}{\text{Slope of given}}$

$$= \frac{-1}{-1/7} = 7$$

Eq of req ⊥ Line

$$(y - y_1) = m(x - x_1)$$
$$(y - 0) = 7(x - 0)$$
$$y = 7x$$
$$7x - y = 0$$

Joint Eq is

$$(4x + y)(7x - y) = 0$$
$$28x^2 - 4xy + 7xy - y^2 = 0$$
$$28x^2 + 3xy - y^2 = 0$$

Complete.