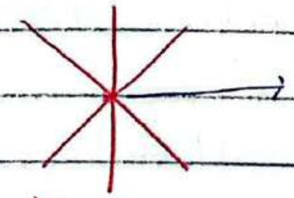
Chap # 06

Concurrent Lines:

Three or more Lines are said to be Concurrent If they Pass throught Same Points. that Point is called point of Concurrency.



This point is Called point of Concurrency.

These above Lines are Concurrent

Mathematically: If

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$
$$a_3x + b_3y + c_3 = 0$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

then Lines are Concurrent.

Eq of Line

$$y - y_1 = m(x - x_1)$$

where

(x_1, y_1) = Given Points

(x, y) = Fixed Points

m = Slope

$m = \tan\theta$ (Angle Given)

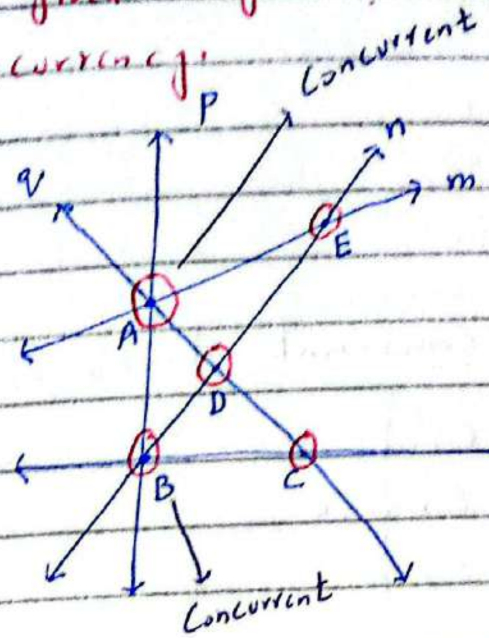
Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ (If two points given)

$y = mx + c$ (Line given)

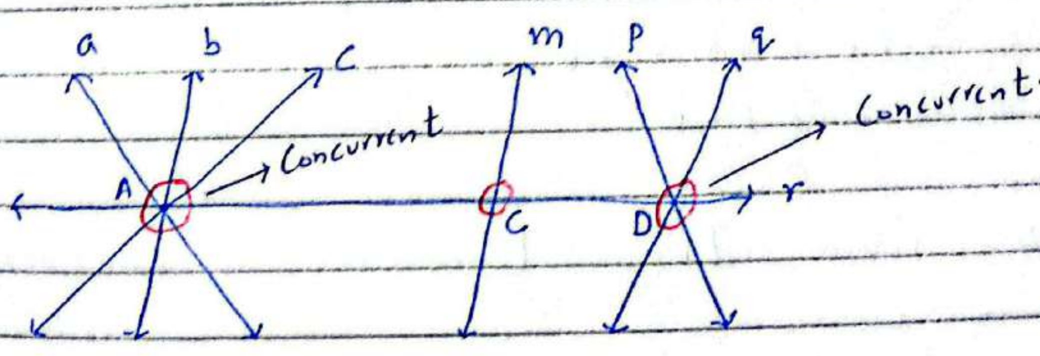
Q#1. Which sets of Lines are Concurrent in the given figure? Also, tell the point of Concurrency.

(i)



- p, q and m are Concurrent at A.
- n, m are ~~Concurrent~~ intersect at E.
- q, n " intersect at D.
- p, n, L " concurrent at B.
- L, q " intersect at C.

(ii)



- a, b, c, r are Concurrent at A.
- m, r " intersect at C
- p, q, r " concurrent at D

Q#2 Check whether the Lines are Concurrent or not:

i) $L_1 = 3x - 4y - 13 = 0$
$L_2 = 8x - 11y - 33 = 0$
$L_3 = 2x - 3y - 7 = 0$

Sol Condition for Concurrent Lines If

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 3 & -4 & -13 \\ 8 & -11 & -33 \\ 2 & -3 & -7 \end{vmatrix}$$

$= 3(77 - 99) + 4(-56 + 66) - 13(-24 + 22)$

$= 3(-22) + 4(10) - 13(-2)$
$= -66 + 40 + 26$
$= -66 + 66$
$= 0$

So Given three Lines are Concurrent.

(ii) $L_1 = x + 2y - 4 = 0$
$L_2 = x - y - 1 = 0$
$L_3 = 4x + 5y - 13 = 0$

Sol

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & -4 \\ 1 & -1 & -1 \\ 4 & 5 & -13 \end{vmatrix}$$

$= 1(13+5) - 2(-13+4) - 4(5+4)$

$= 18 + 18 - 36$

$= 0$

So Given three Lines are Concurrent.

Q #3 Determine the value of 'a' If the Line $2x - y + 3$, $x - y = 0$, $3x + ay + 1 = 0$ are concurrent.

Sol

As given three Lines are Concurrent

So

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & -1 & 3 \\ 1 & -1 & 0 \\ 3 & a & 1 \end{vmatrix} = 0$$

$= 2(-1-0) + 1(1-0) + 3(a+3) = 0$

$-2 + 1 + 3a + 9 = 0$

$8 + 3a = 0$

$\boxed{a = \frac{-8}{3}}$ Ans.

IMPORTANT POINTS

- **Slope = m**

- General Form $ax + by + c = 0$

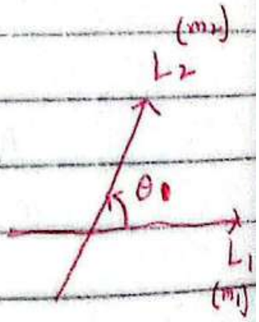
$$m = \frac{-a}{b} = -\frac{\text{coeff of } x}{\text{coeff of } y}.$$

- $y = mx + c$ (Slope Intercept Form)

$$y = \underbrace{m}_{\text{slope}}\overset{\text{axis}}{x} + \underbrace{c}_{\text{intercept point}}$$

- To Find Angle b/w L_1 and L_2

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$



- If two Lines are $\perp$ than $m_1 \cdot m_2 = -1$

- If two Lines are $\parallel$ than $m_1 = m_2$

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Q#4 Show that the Lines

$L_1 = x + y + 1 = 0$

$L_2 = x - y + 1 = 0$ and X-axis

$L_3 = y = 0$

are Concurrent.

Sol

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$= 1(0-1) - 1(0-0) + 1(1-0)$

$= -1 + 1$

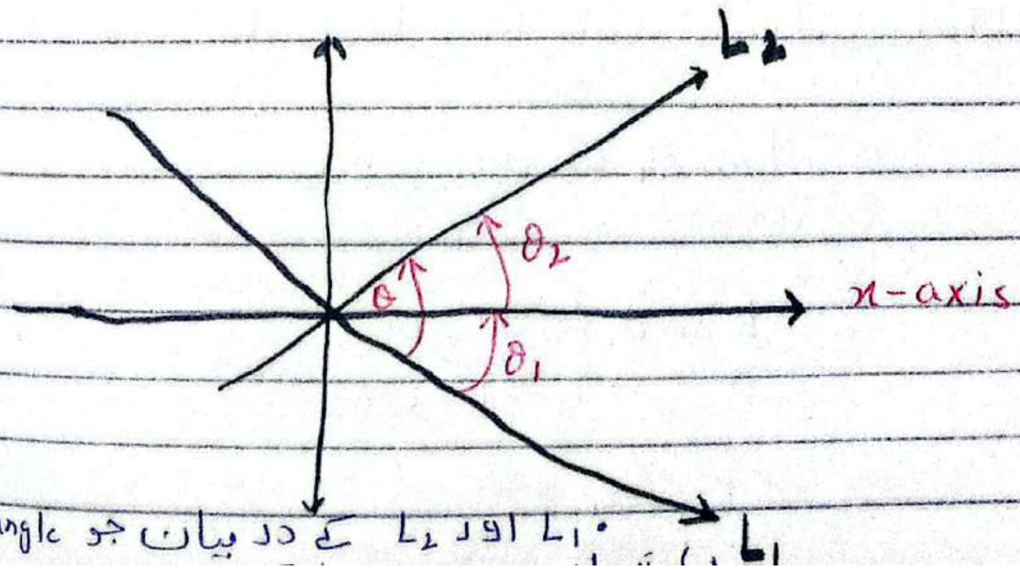
$= 0$

So the given three Lines are Concurrent.

(b) Also prove that x-axis bisects the angle b/w $x+y+1=0$ and $x-y+1=0$.

$L_1 = x + y + 1 = 0$

$L_2 = x - y + 1 = 0$



L_1 اور L_2 کے درمیان جو angle ہے

$L_1 = x+y+1=0$	$L_2 = x-y+1=0$	X-axis $y=0$
Slope $= m_1$	Slope $= m_2$	Slope $= m_3$
$m_1 = \frac{-a}{b}$	$m_2 = \frac{-a}{b}$	$m_3 = \frac{-a}{b}$
$m_1 = -\frac{1}{1}$	$m_2 = \frac{-1}{-1}$	$m_3 = \frac{-0}{1}$
$m_1 = -1$	$m_2 = 1$	$m_3 = 0$

- First Find angle b/w L_1 and L_2

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$\tan\theta = \frac{1-(-1)}{1+(-1)(1)} = \frac{1+1}{1-1} = \frac{2}{0} = \infty$$

$$\tan\theta = \infty$$
$$\theta = \tan^{-1}(\infty)$$
$$\boxed{\theta = 90^\circ}$$

- Now Find angle b/w L_1 and X-axis

$$\tan\theta_1 = \frac{m_3 - m_1}{1 + m_3 m_1}$$

$$\tan\theta_1 = \frac{0+1}{1+0(-1)}$$

$$\tan\theta_1 = \frac{1}{1} \qquad \theta_1 = \tan^{-1}(1) = 45^\circ$$

Also Find angle b/w X-axis and L_2

$$\tan\theta_2 = \frac{m_2 - m_3}{1 + m_2 m_3}$$

$$\tan\theta_2 = \frac{1 - 0}{1 + 1\cdot 0} = \frac{1}{1} = 1$$

$$\tan\theta_2 = 1$$

$$\theta_2 = \tan^{-1}(1)$$

$$\theta_2 = 45^\circ \quad (1)$$

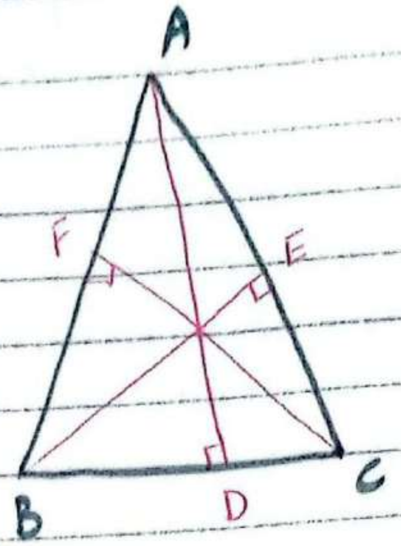
Hence It is proved that X-axis bisect the angle b/w L_1 and L_2.

v.imp

Q#5 Find the (1) equations of altitudes and point (2) of concurrency of the triangle ABC when A(4, -2) B(5, 5) and C(-1, 3). What is the name (3) of point of concurrency?

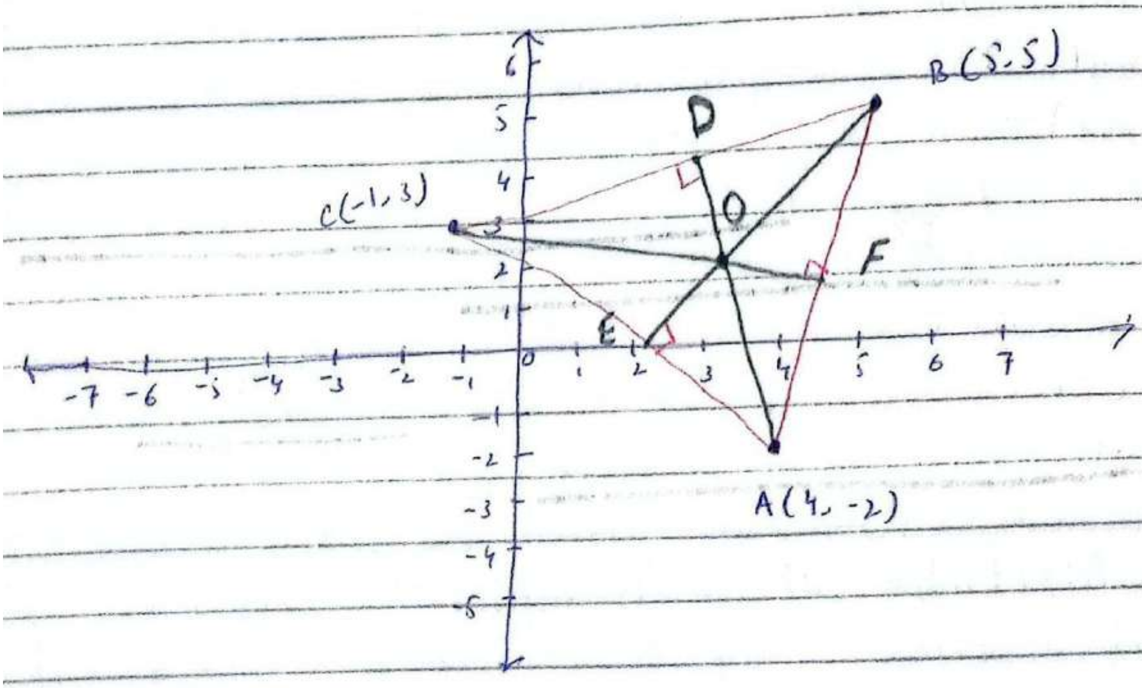
Altitudes of a Triangle: An altitude of a triangle is a perpendicular line drawn from a vertex to the opposite side. It represents the height of the triangle from that vertex.

Sol



AD, BE and CF are altitudes of triangle.

① Given A(4, -2) B(5, 5) C(-1, 3)



1) Eq of altitudes = ?

(a) Equation of altitude AD

(b) Eq of altitude BE

(c) Eq of altitude CF

(a) Eq of altitude AD

$$y - y_1 = m(x - x_1) \quad = [\text{Eq of AD}]$$

Point $(x_1, y_1) = (4, -2)$

$$\boxed{x_1 = 4 \qquad y_1 = -2}$$

Slope of BC $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3 - 5}{-1 - 5} = \dfrac{-2}{-6}$

$B(5, 5)$ $C(-1, 3)$

x_1 y_1 $\quad$ x_2 y_2

$$= 1/3$$

Since Line AD $\perp$ BC So

$$(\text{Slope of AD})(\text{Slope of BC}) = -1$$

$$\text{Slope of AD} \cdot \frac{1}{3} = -1$$

$$\boxed{\text{Slope of AD} = -3}$$

Eq of altitude AD is

$$y - y_1 = m(x - x_1)$$

$$y + 2 = -3(x - 4)$$

$$y + 2 = -3x + 12$$

$$3x + y + 2 - 12 = 0$$

$$3x + y - 10 = 0$$

required Eq of altitude AD.

(b) Eq of altitude BE

$(y-y_1) = m(x-x_1)$ [Eq of BE]

Point $(x_1, y_1) = (5, 5)$

$\boxed{x_1 = 5 \quad y_1 = 5}$

Slope of AC $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$A(4, -2)$ $C(-1, 3)$

$x_1 \; y_1$ $x_2 \; y_2$

$= \dfrac{3+2}{-1-4} = \dfrac{5}{-5} = -1$

Since Line BE is $\perp$ to AC So

(Slope of BE)(Slope of AC) $= -1$

Slope of BE $= \dfrac{-1}{\text{Slope of AC}}$

$= \dfrac{-1}{-1}$ $\boxed{= 1 = \text{Slope of BE}}$

Eq of altitude BE is

$(y-y_1) = m(x-x_1)$

$(y-5) = 1(x-5)$

$x - \cancel{5} - y + \cancel{5} = 0$

$x - y = 0$

required eq of altitude BE.

(c) Eq of altitude CF

$(y-y_1) = m(x-x_1)$ [Eq of CF]

Point $(x_1, y_1) = (-1, 3)$

$\boxed{x_1 = -1 \quad y_1 = 3}$

Slope of AB $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5+2}{5-4}$

A(4, -2) B(5, 5)

x_1, y_1 x_2, y_2

$= 7$

Since Line CF is $\perp$ to AB So

(Slope of CF) (Slope of AB) $= -1$

Slope of CF $= \dfrac{-1}{\text{slope of AB}} = \dfrac{-1}{7}$

Eq of altitude CF is

$(y-y_1) = m(x-x_1)$

$(y-3) = -\dfrac{1}{7}(x-(-1))$

$y-3 = \dfrac{-1}{7}(x+1)$

$7y - 21 = -x - 1$

$x + 7y - 20 = 0$

required Eq of altitude CF

2

Three Eq of altitudes are.

$$3x + y - 10 = 0 \quad \text{—(i)}$$
$$x - y = 0 \quad \text{—(ii)}$$
$$x + 7y - 20 = 0 \quad \text{—(iii)}$$

Now we find point of concurrency using any two equations.

From eq (ii)

$x = y$ —(iv) Put in (i)

$$3x + x - 10 = 0$$
$$4x - 10 = 0$$
$$\boxed{x = 10/4} \qquad x = \frac{5}{2}$$

put in (iv)

$$y = \frac{5}{2}$$

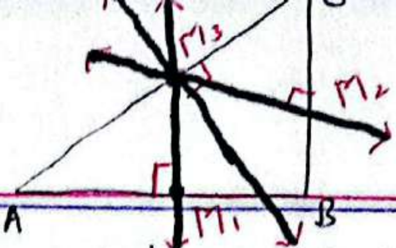
So $O(x, y) = O\left(\frac{5}{2}, \frac{5}{2}\right)$ is the point of concurrency.

3 The point of concurrency of altitudes of a triangle is called orthocentre.

Right Bisector (Perpendicular Bisector)

A right bisector of triangle is a line that

i) Divides a side into two equal parts.

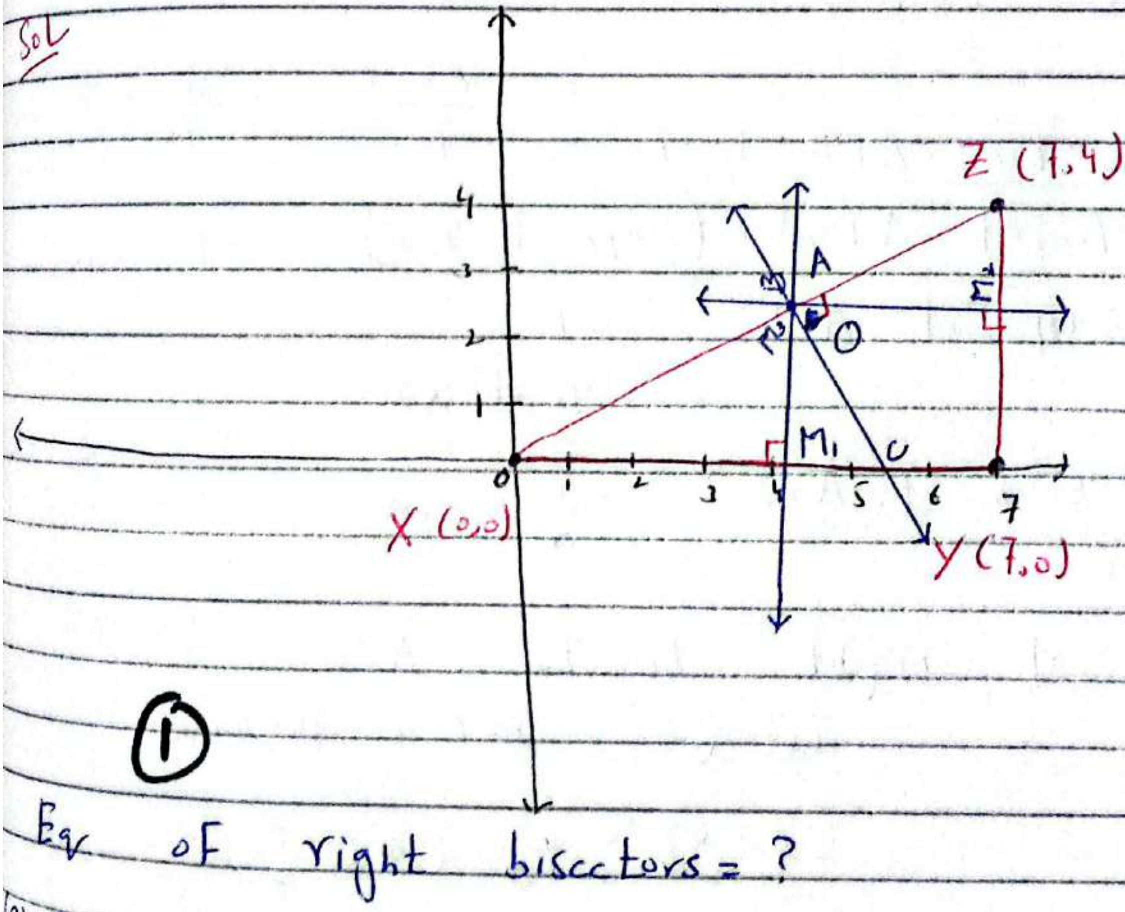


(iii) is perpendicular to that side.

Note:
Three perpendicular bisector of a triangle meet at a point is called circumcentre.

Q#6
Find the equations of right bisectors and their point of concurrency in the triangle XYZ when X(0,0) Y(7,0) Z(7,4). What is the name of point of concurrency.

Sol



(i)

Eq of right bisectors = ?

(a) Eq of right bisector XY = ? AM_1
(b) Eq of right bisector YZ = ? BM_2
(c) Eq of right bisector XZ = ? CM_3

(a) Eq of right bisector XY (AM_1)

$(y - y_1) = m(x - x_1)$

Mid point of $XY = M_1 = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$X(0,0)$ $Y(7,0)$

$= \left(\frac{0+7}{2}, \frac{0+0}{2}\right)$

$\boxed{M_1 = \left(\frac{7}{2}, 0\right)}$

Now slope of $XY = \frac{y_2 - y_1}{x_2 - x_1}$

$X(0,0)$ $Y(7,0)$

$= \frac{0-0}{7-0} = \frac{0}{7} = 0$

Since Line $AM_1 \perp$ to XY

$(\text{Slope of } AM_1) \cdot (\text{Slope of } XY) = -1$

Slope of $AM_1 = \frac{-1}{\text{Slope of } XY}$

Slope of $AM_1 = \frac{-1}{0} = \infty$

Eq of right bisector AM_1

$(y - y_1) = m(x - x_1)$

$(y - 0) = \frac{1}{0}\left(x - \frac{7}{2}\right)$

$x - \frac{7}{2} = 0$ $\quad 2x - 7 = 0$

is required eq of right bisector AM_1.

(b) Eq of right biscetor YZ (BM_2)

$$(y-y_1) = m(x-x_1)$$

Mid point of YZ = $M_2 = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Y(7,0) Z(7,4)

$$M_2 = \left(\frac{7+7}{2}, \frac{0+4}{2}\right)$$

$$\boxed{M_2 = (7, 2)}$$

Now slope of YZ = $\frac{y_2-y_1}{x_2-x_1}$

Y(7,0) Z(7,4)

$$= \frac{4-0}{7-7} = \infty$$

Since Line BM_2 ⊥ to YZ

(Slope of BM_2) (Slope of YZ) = -1

Slope of BM_2 = $\frac{-1}{\text{slope of YZ}}$

Slope of BM_2 = $\frac{-1}{\infty} = 0$

Eq of right biscetor BM_2

$$(y-y_1) = m(x-x_1)$$

$$(y-2) = 0(x-7)$$

$$y-2=0$$

is required eq of right biscetor BM_2

Eq of right bisector XZ (CM_3)

$$(y-y_1) = m(x-x_1)$$

Mid point of XZ = $M_3 = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

X (0, 0) Z (7, 4)

$$M_3 = \left(\frac{0+7}{2}, \frac{0+4}{2}\right)$$

$$\boxed{M_3 = \left(\frac{7}{2}, 2\right)}$$

Now slope of XZ = $\frac{y_2-y_1}{x_2-x_1}$

X (0, 0) Z (7, 4)

Slope of XZ $= \frac{4-0}{7-0} = \frac{4}{7}$

Since line $CM_3 \perp$ to XZ

(Slope of CM_3) (Slope of XZ) = -1

Slope of $CM_3 \cdot \frac{4}{7} = -1$

$$\boxed{\text{Slope of } CM_3 = -\frac{7}{4}}$$

Eq of right bisector CM_3

$$(y-y_1) = m(x-x_1)$$

$$(y-2) = \frac{-7}{4}\left(x-\frac{7}{2}\right)$$

$$4y-8 = -7x+\frac{49}{2}$$

$$4y - 8 = \frac{-14x + 49}{2}$$

$$8y - 16 = -14x + 49$$

$$14x + 8y - 16 - 49 = 0$$

$$14x + 8y - 65 = 0$$

(2) Three Eqs of right bisector are

$$2x - 7 = 0 \quad \text{—(i)}$$
$$y - 2 = 0 \quad \text{—(ii)}$$
$$14x + 8y - 65 = 0 \quad \text{—(iii)}$$

Now we Find point of Concurrency using any two equations

$$2x - 7 = 0$$

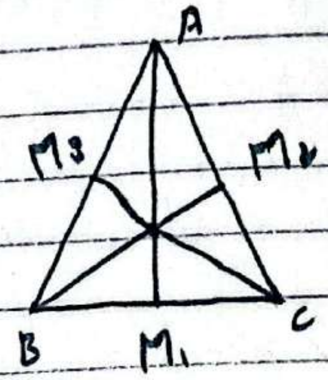
$$x = \frac{7}{2} \qquad y = 2$$

$$\left(\frac{7}{2}, 2\right)$$

So $O(x,y) = O\left(\frac{7}{2}, 2\right)$ is the point of Concurrency.

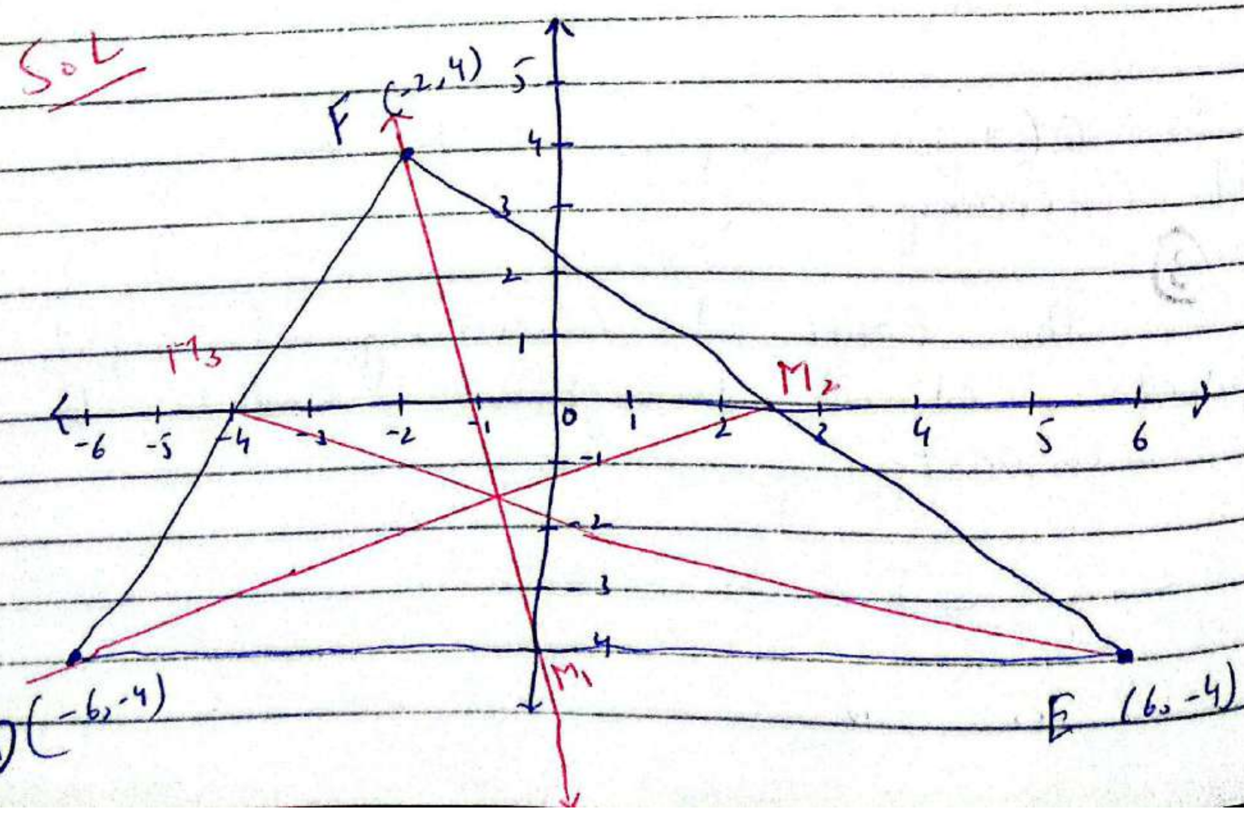
(3) The Point of Concurrency of right bisectors of a triangle is called the circum centre.

Median of a triangle:- A median of a triangle is a line segment that joins a vertex of the triangle to the mid point of the opposite side.



Q #7 Find the equation of medians and their point of concurrency of the triangle DEF when D(-6, -4) E(6, -4) F(-2, 4). What is the name of point of concurrency.

Sol



We have to Find

i) Eq of median FM_1 = ?
ii) Eq of median DM_2 = ?
iii) Eq of median EM_3 = ?

(a) Eq of Median FM_1

$$(y - y_1) = m(x - x_1)$$

Mid point of DE = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

D(-6, -4) E(6, -4)

$$= \left(\frac{-6+6}{2}, \frac{-4-4}{2}\right)$$

$$M_1 = (0, -4)$$

Now Slope of $FM_1 = \frac{y_2 - y_1}{x_2 - x_1} = m$

F(-2, 4) M_1(0, -4)

$$m = \frac{-4-4}{0+2} = -4$$

Eq of median FM_1 (F($\overset{x_1}{-2}$, $\overset{y_1}{4}$))

$$(y - y_1) = m(x - x_1)$$

$$(y - 4) = -4(x + 2)$$

$$y - 4 = -4x - 8$$

$$4x + y - 4 + 8 = 0$$

$$4x + y + 4 = 0$$

is req Eq of median FM_1

(b) Eq of Median DM_2

$$(y-y_1) = m(x-x_1)$$

Mid point of $EF = M_2 = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$E(6,-4)$ $F(-2,4)$

$$M_2 = \left(\frac{6-2}{2}, \frac{-4+4}{2}\right)$$

$$M_2 = (2, 0)$$

Now slope of $DM_2 = m = \frac{y_2-y_1}{x_2-x_1}$

$D(-6,-4)$ $M_2(2,0)$

$$m = \frac{0+4}{2+6}$$

$$m = \frac{1}{2}$$

Eq of median DM_2 $(D(-6,-4))$

$$(y-y_1) = m(x-x_1)$$

$$(y+4) = \frac{1}{2}(x+6)$$

$$2y+8 = x+6$$

$$x-2y-8+6 = 0$$

$$\boxed{x-2y-2=0}$$

is required eq of median DM_2

(C) Eq of Median EM_3

$$(y - y_1) = m(x - x_1)$$

Midpoint of DF $= M_3 = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$D(-6, -4)$ $F(-2, 4)$

$$M_3 = \left(\frac{-6-2}{2}, \frac{-4+4}{2}\right)$$

$$M_3 = (-4, 0)$$

Now Slope of $EM_3 =$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$E(6, -4)$ $M_3(-4, 0)$

$$m = \frac{0 + 4}{-4 - 6}$$

$$m = \frac{4}{-10} = \frac{-2}{5}$$

Eq of median EM_3 $E(6, -4)$

$$(y - y_1) = m(x - x_1)$$

$$(y + 4) = \frac{-2}{5}(x - 6)$$

$$5y + 20 = -2x + 12$$

$$2x + 5y + 20 - 12 = 0$$

$$\boxed{2x + 5y + 8 = 0}$$

Three eq of medians are

$$4x + y + 4 = 0 \quad \text{---(i)}$$
$$x - 2y - 2 = 0 \quad \text{---(ii)}$$
$$2x + 5y + 8 = 0 \quad \text{---(iii)}$$

Now we find point of Concurrency.

$$8x + \cancel{2y} + 8 = 0$$
$$x \cancel{- 2y} - 2 = 0$$
$$\overline{9x + 6 = 0}$$

$$x = \frac{-6}{9} = \frac{-2}{3}$$

$$x = \frac{-2}{3}$$

Put in (ii)

$$\frac{-2}{3} - 2y - 2 = 0$$
$$-2y = 2 + \frac{2}{3}$$
$$-2y = \frac{8}{3} \Rightarrow y = \frac{\cancel{8}^{4}}{3 \cdot \cancel{-2}}$$
$$y = \frac{-4}{3}$$

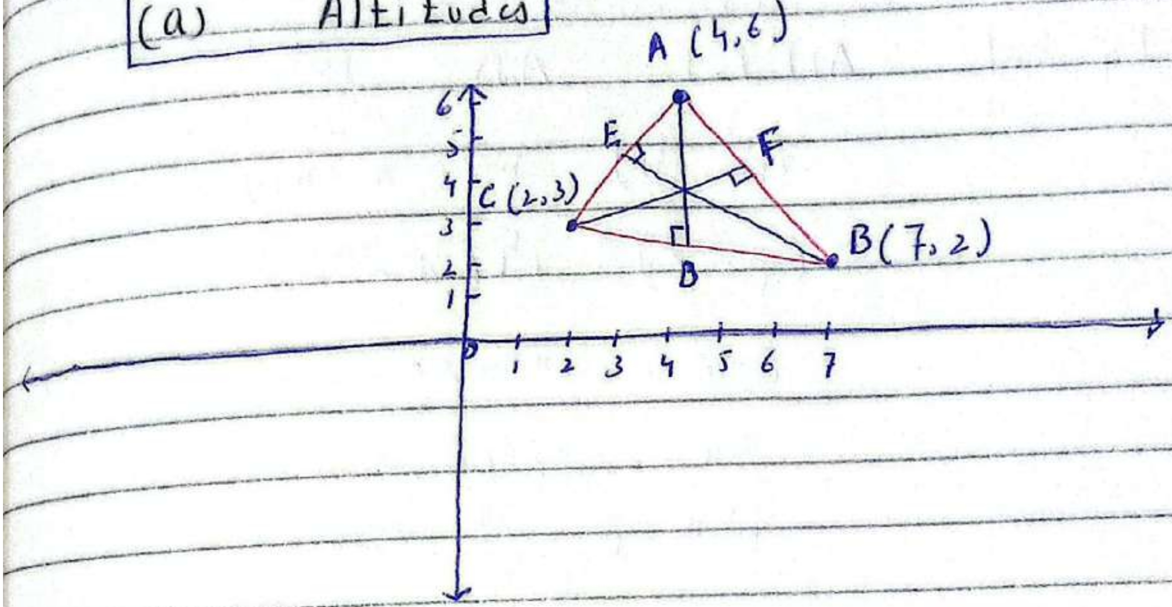
So $\left(\frac{-2}{3}, \frac{-4}{3}\right)$ is the Point of Concurrency.

The point of concurrency of medians of a triangle is called centroid.

Q#8 Prove that (a) altitudes (b) right bisectors and (c) medians of the following triangles are concurrent.

1) A(4,6) B(7,2) C(2,3)

$\boxed{\text{(a) Altitudes}}$



(a) Eq of altitude AD
(b) Eq of altitude BE
(c) Eq of altitude CF

Eq of altitude AD

$(y-y_1) = m(x-x_1)$ = [Eq of AD]

Point $(x_1, y_1) = (4, 6)$ $\boxed{x_1 = 4 \quad y_1 = 6}$

Slope of BC $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-2}{2-7} = \dfrac{1}{-5}$

$B(7,2)$ $C(2,3)$

x_1 y_1 x_2 y_2

$= -\dfrac{1}{5}$

Since Line AD $\perp$ BC So

Slope of AD . Slope of BC = -1

Slope of AD . $\frac{-1}{5} = -1$

Slope of AD = 5

Eq of Altitude AD is

$$(y - y_1) = m(x - x_1)$$

$$(y - 6) = +5(x - 4)$$

$$y - 6 = 5x - 20$$

$$5x - 20 - y + 6 = 0$$

$$\boxed{5x - y - 14 = 0}$$

Eq of Altitude CF

$(y - y_1) = m(x - x_1)$ [Eq of CF]

Point $(x_1, y_1) = (7, 3)$ $x_1 = 7, y_1 = 3$

Slope of AB = $\frac{y_2 - y_1}{x_2 - x_1} = m$

A(4, 6) B(7, 2)
x_1 y_1 x_2 y_2

$$= \frac{2 - 6}{7 - 4} = \frac{-4}{3}.$$

Since Line AB $\perp$ CF

(Slope of CF) (slope of AB) = -1

Slope of CF . $\frac{-4}{3} = -1$

Slope of CF = $\frac{3}{4}$

Eq of altitude CF is

$$(y - y_1) = m(x - x_1)$$

$$(y - 3) = \frac{3}{4}(x - 2)$$

$$4y - 12 = 3x - 6$$

$$3x - 6 - 4y + 12 = 0$$

$$\boxed{3x - 4y + 6 = 0}$$

Eq of Altitude BE

$(y - y_1) = m(x - x_1)$ [Eq of BE]

Point $(x_1, y_1) = (7, 2)$ $x_1 = 7, y_1 = 2$

Slope of AC = $\frac{y_2 - y_1}{x_2 - x_1}$

A(4, 6) C(2, 3)

$$= \frac{3 - 6}{2 - 4} = \frac{-3}{-2} = \frac{3}{2}$$

Since Line AC $\perp$ BE

$(\text{Slope of } AC)(\text{slope of } BE) = -1$

$\text{Slope of } BE = \dfrac{-1}{\text{Slope of } AC}$

$\text{Slope of } BE = \dfrac{-1}{3/2} = \dfrac{-2}{3}$

Eq of Altitude BE is

$(y - y_1) = m(x - x_1)$

$(y - 2) = \dfrac{-2}{3}(x - 7)$

$3y - 6 = -2x + 14$

$2x + 3y - 6 - 14 = 0$

$\boxed{2x + 3y - 20 = 0}$

Eq of altitude AD $= 5x - y - 14 = 0$
Eq of " CF $= 3x - 4y + 6 = 0$
Eq of " BE $= 2x + 3y - 20 = 0$

Now we show that these altitudes are concurrent.

$$\begin{vmatrix} 5 & -1 & -14 \\ 3 & -4 & 6 \\ 2 & 3 & -20 \end{vmatrix} = 0$$

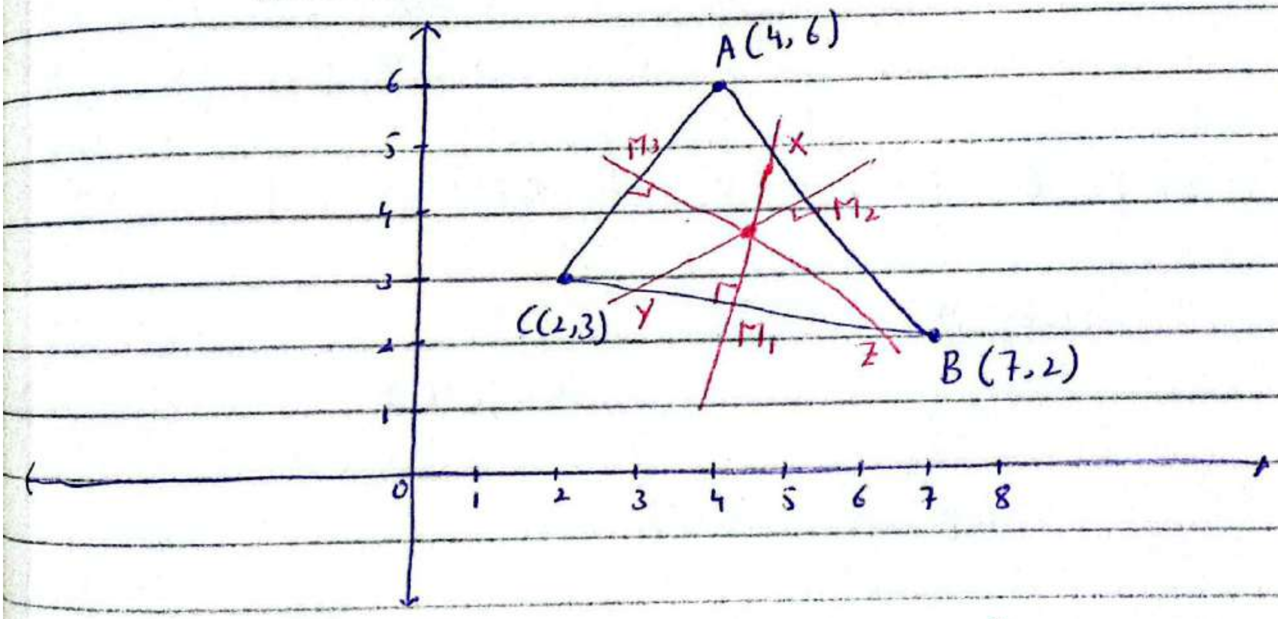
$\Rightarrow 5(80-18)+1(-60-12)-14(9+8)=0$

$\Rightarrow 310-72-238=0$

$\Rightarrow 0=0$

This shows that altitude of triangle are concurrent.

(b) Right Bisectors



Eq of right bisector $XM_1 = ?$

Eq " " " $YM_2 = ?$

Eq " " " $ZM_3 = ?$

Eq of right Bisector XM_1

$$(y-y_1)=m(x-x_1)$$

Mid point of $BC = M_1 = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$B(7,2)$ $C(2,3)$

$$M_1 = \left(\frac{7+2}{2}, \frac{2+3}{2}\right)$$

$$M_1 = \left(\frac{9}{2}, \frac{5}{2}\right)$$

$$\boxed{x_1 = 9/2 \qquad y_1 = 5/2}$$

Now slope of BC $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$B(7, 2)$ $C(2, 3)$
x_1 y_1 x_2 y_2

$$= \frac{3-2}{2-7} = \frac{1}{-5} = \frac{-1}{5}$$

Slope of BC . Slope of $XM_1 = -1$

$$\text{Slope of } XM_1 = \frac{-1}{\text{Slope of BC}} = \frac{-1}{-1/5} = 5$$

Slope of $XM_1 = 5$

Eq of right biscctor XM_1

$$(y - y_1) = m(x - x_1)$$

$$\left(y - \frac{5}{2}\right) = 5\left(x - \frac{9}{2}\right)$$

$$\frac{2y-5}{\cancel{2}} = 5\left(\frac{2x-9}{\cancel{2}}\right)$$

$$2y - 5 = 10x - 45$$

$$10x - 45 - 2y + 5 = 0$$

$$\boxed{10x - 2y - 40 = 0}$$

Eq of right bisector YM_2

$(y-y_1) = m(x-x_1)$

Midpoint of AB $= M_2 = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

A(4,6) B(7,2)

$= \left(\frac{4+7}{2}, \frac{6+2}{2}\right)$

$M_2 = \left(\frac{11}{2}, 4\right)$

$\boxed{x_1 = \frac{11}{2}, \quad y_1 = 4}$

Now slope of AB $= \frac{y_2 - y_1}{x_2 - x_1}$

A(4,6) B(7,2)

$= \frac{2-6}{7-4} = \frac{-4}{3}$

slope of AB · slope of YM_2 $= -1$

Slope of $YM_2 = \frac{-1}{\text{slope of AB}}$

Slope of $YM_2 = \frac{-1}{-4/3} = \frac{3}{4}$

Eq of right biscetor YM_2

$(y-y_1) = m(x-x_1)$

$(y-4) = \frac{3}{4}\left(x - \frac{11}{2}\right)$

$4y - 16 = 3\left(\frac{2x-11}{2}\right)$

$8y - 32 = 6x - 33$

$6x - 33 - 8y + 32 = 0$

$$\boxed{6x - 8y - 1 = 0}$$

Eq of right bisector ZM_3

$(y - y_1) = m(x - x_1)$

Midpoint of AC $= M_3 = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$A(4,6)$ $C(2,3)$

$= \left(\frac{4+2}{2}, \frac{6+3}{2}\right)$

$= \left(3, \frac{9}{2}\right)$

$$\boxed{x_1 = 3 \quad y_1 = \frac{9}{2}}$$

Now slope of AC $= \frac{y_2 - y_1}{x_2 - x_1}$

$A(4,6)$ $C(2,3)$

$= \frac{3-6}{2-4} = \frac{-3}{-2} = \frac{3}{2}$

slope of AC. Slope of $ZM_3 = -1$

Slope of $ZM_3 = \frac{-1}{\text{Slope of AC}}$

$= \frac{-1}{3/2}$

Slope of $ZM_3 = \frac{-2}{3}$

Eq of right bisector ZM_3

$$(y - y_1) = m(x - x_1)$$

$$\left(y - \frac{9}{2}\right) = \frac{-2}{3}(x - 3)$$

$$\frac{2y - 9}{2} = \frac{-2}{3}(x - 3)$$

$$6y - 27 = -4x + 12$$

$$4x + 6y - 27 - 12 = 0$$

$$\boxed{4x + 6y - 39 = 0}$$

Eq of Right bisector $XM_1 = 10x - 2y - 40 = 0$
Eq " " " $YM_2 = 6x - 8y - 1 = 0$
Eq " " " $ZM_3 = 4x + 6y - 39 = 0$

Now we show that these right bisectors are concurrent.

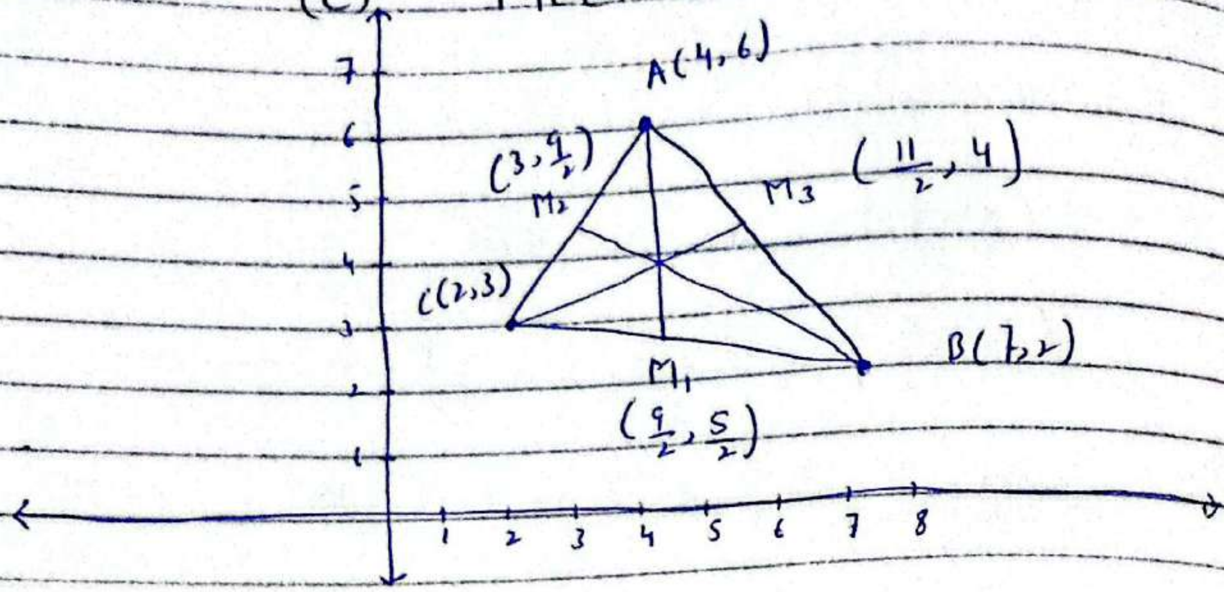
$$\begin{vmatrix} 10 & -2 & -40 \\ 6 & -8 & -1 \\ 4 & 6 & -39 \end{vmatrix} = 0$$

$$10(312 + 6) + 2(-234 + 4) - 40(36 + 32) = 0$$

$$3180 - 460 - 2720 = 0$$

This shows that right bisectors of triangle are concurrent

(C) Medians:



Eq of median AM_1 = ?
Eq " " BM_2 = ?
Eq " " CM_3 = ?

<u>Eq of Median AM_1</u>

$$(y - y_1) = m(x - x_1)$$

Midpoint of BC = $M_1 = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
B(7, 2) C(2, 3)

$$M_1 = \left(\frac{7+2}{2}, \frac{2+3}{2}\right)$$

$$M_1 = \left(\frac{9}{2}, \frac{5}{2}\right)$$

Now Slope of AM_1 = $\frac{y_2 - y_1}{x_2 - x_1}$
A(4, 6) $M_1\left(\frac{9}{2}, \frac{5}{2}\right)$

$$= \frac{5/2 - 6}{\frac{9}{2} - 4}$$

Slope of $AM_1 = -7$

Eq of median AM_1

$$(y-y_1) = m(x-x_1)$$

$$(y-6) = -7(x-4)$$

$$y-6 = -7x+28$$

$$7x+y-6-28=0$$

$$\boxed{7x+y-34=0}$$

Eq of median BM_2

$$(y-y_1) = m(x-x_1)$$

Mid point of $AC = M_2 = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$A(4,6)$ $C(2,3)$

$$M_2 = \left(\frac{4+2}{2}, \frac{6+3}{2}\right)$$

$$M_2 = \left(3, \frac{9}{2}\right)$$

Now Slope of $BM_2 = \frac{y_2-y_1}{x_2-x_1}$

$B(7,2)$ $M_2 = \left(3, \frac{9}{2}\right)$

$$= \frac{9/2 - 2}{3-7} = \frac{5/2}{-4/1}$$

$$m = \frac{5}{2} \times \frac{1}{-4} = \frac{-5}{8}$$

Slope of $BM_2 = \frac{-5}{}$

Eq of median BM_2 =

$$(y-y_1) = m(x-x_1)$$

$$(y-2) = \frac{-5}{8}(x-7)$$

$$8y-16 = -5x+35$$

$$5x+8y-16-35=0$$

$$\boxed{5x+8y-51=0}$$

Eq of Median CM_3

$$(y-y_1) = m(x-x_1)$$

Midpoint of AB = M_3 = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$A(4,6)$ $B(7,2)$

$$M_3 = \left(\frac{4+7}{2}, \frac{6+2}{2}\right)$$

$$M_3 = \left(\frac{11}{2}, \frac{8}{2}\right)$$

$$M_3 = \left(\frac{11}{2}, 4\right)$$

Now slope of CM_3 = $\frac{y_2-y_1}{x_2-x_1}$

$C(2,3)$ $M_3 = \left(\frac{11}{2}, 4\right)$

$$= \frac{4-3}{\frac{11}{2}-2} = \frac{1}{7/2} = \frac{2}{7}$$

Eq of median CM_3

$$(y-y_1) = m(x-x_1)$$

$$(y-3) = \frac{2}{7}(x-2)$$

$$7y-21 = 2x-4$$

$$2x-4-7y+21=0$$

$$\boxed{2x-7y+17=0}$$

Eq of median $AM_1 = 7x+y-34=0$
Eq " " $BM_2 = 5x+8y-51=0$
Eq " " $CM_3 = 2x-7y+17=0$

Now we show that these Medians are Concurrent.

$$\begin{vmatrix} 7 & 1 & -34 \\ 5 & 8 & -51 \\ 2 & -7 & 17 \end{vmatrix} = 0$$

$$\Rightarrow 7(136-357)-1(85+102)-34(-35-16)=0$$

$$\Rightarrow -1547-187+1734=0$$

$$\Rightarrow 0 = 0$$

This shows that Medians of triangle are Concurrent.

Q#8 2nd Part Do Your self.

Area of triangle ABC

For three points $A(x_1, y_1)$ $B(x_2, y_2)$ and $C(x_3, y_3)$

$$\text{Area of triangle } ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Q#9 Find the area of triangles

(i) $A(1,1)$ $B(4,5)$ $C(12,-1)$

Sol

$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 1 \\ 12 & -1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[1(5+1) - 1(4-12) + 1(-4-60)\right]$$

$$= \frac{1}{2}\left[6 + 8 - 64\right]$$

$$= \frac{-50}{2} = -25 = 25 \text{ sq. unit}$$

Area cannot be negative.

(ii) D (3,1) E (2,3) F (2,2)

Sol:

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 2 & 3 & 1 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[3(3-2) - 1(2-2) + 1(4-6)\right]$$

$$= \frac{1}{2}\left[3 - 0 - 2\right]$$

$$= \frac{1}{2} \text{ sq unit.}$$

Q#10 By Finding area of triangle, show that Points A (6,0) B (-3,6) C (3,2) are Collinear:

Sol:

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 6 & 0 & 1 \\ -3 & 6 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[6(6-2) - 0(-3-3) + 1(-6-18)\right]$$

$$= \frac{1}{2}\left[24 - 24\right] = 0$$

Since area of ΔABC is Zero thus all three points A, B and C are Collinear.

(ii) Using the Formula of area, Find x If the points $P(3,2)$ $Q(-1,x)$ $R(7,3)$ are collinear.

Sol: $P(3,2)$ $Q(-1,x)$ $R(7,3)$

The points P, Q, R will be Collinear If

Area of $\Delta PQR = 0$

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \times 2$$

$$\begin{vmatrix} 3 & 2 & 1 \\ -1 & x & 1 \\ 7 & 3 & 1 \end{vmatrix} = 0$$

$$3(x-3) - 2(-1-7) + 1(-3-7x) = 0$$

$$3x - 9 + 16 - 3 - 7x = 0$$
$$-4x + 4 = 0$$
$$-4x = -4$$
$$\boxed{x = 1}$$

(12) Vertices of a triangle are (3, -4) (4, h) and (2, 6). Find 'h' if area of the triangle is 10 square unit.

Sol A(3, -4) B(4, h) C(2, 6)

$h = ?$

Area of ΔABC = 10 square unit.

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 10$$

$$\begin{vmatrix} 3 & -4 & 1 \\ 4 & h & 1 \\ 2 & 6 & 1 \end{vmatrix} = 10 \times 2$$

$$3(h-6) + 4(4-2) + 1(24 - 2h) = 20$$

$$3h - 18 + 8 + 24 - 2h - 20 = 0$$

$$h - 6 = 0$$

$$\boxed{h = 6}$$

13 Find the area of Following Figures

(i) A(-3, -1) B(2, 3) C(2, -1)

Sol.

Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -3 & -1 & 1 \\ 2 & 3 & 1 \\ 2 & -1 & 1 \end{vmatrix}$

$= \frac{1}{2}\left[-3(3+1)+1(2-2)+1(-2-6)\right]$

$= \frac{1}{2}\left[-12+0-8\right]$

$= -10$ (Area cannot be negative)

$= 10$ square unit.

(ii) A(1, 3) B(-2, -3) C(4, -3)

Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ -2 & -3 & 1 \\ 4 & -3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\left[1(-3+3)-3(-2-4)+1(6+12)\right]$$

$$= \frac{1}{2}\left[0+18+18\right]$$

$= 18$ square unit.

(14) Find the area of triangle bounded by the lines

(i) $4x - 5y + 7 = 0$ — (i)
$x - 2 = 0$ — (ii)
$y + 1 = 0$ — (iii)

Sol

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Given lines but we need points first of all we find points from lines.

Solving eq (i) and (ii)

$x - 2 = 0 \qquad x = 2$

Put in (i)

$4(2) - 5y + 7 = 0$

$-5y = -8 - 7$

$-5y = -15$

$y = 3$

$P_1(x_1, y_1) = A(2, 3)$

Solving eq (ii) and eq (iii)

$y+1=0 \qquad y=-1$

and

$x-2=0 \qquad x=2$

So $P_2(x_2, y_2) = B(2, -1)$

Solving eq (i) and eq (iii)

$y+1=0 \qquad y=-1$

put in (i)

$4x-5(-1)+7=0$

$4x+5+7=0$

$4x=-12$

$x=-3$

So $P_3(x_3, y_3) = C(-3, -1)$

Now

$A(2,3) \quad B(2,-1) \quad C(-3,-1)$

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ 2 & -1 & 1 \\ -3 & -1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(-1+1) - 3(2+3) + 1(-2-3)\right]$$

$$= \frac{1}{2}\left[0 - 15 - 5\right]$$

$= \frac{-20}{2} = -10$ (Area cannot be negative)

$= 10$ Square unit

(iii) $x - 2y - 6 = 0$ —(i)

$3x - y + 3 = 0$ —(ii)

$2x + y - 4 = 0$ —(iii)

Sol

Solving eq (i) and eq (ii)

xing eq (ii) by 2

$x - 2y - 6 = 0$

$\pm 6x \mp 2y \pm 6 = 0$

$-5x - 12 = 0$

$-5x = 12$

$\boxed{x = \frac{-12}{5}}$

Put in (i)

$\frac{-12}{5} - 2y - 6 = 0$

$-2y = 6 + \frac{12}{5}$

$-2y = \frac{42}{5} \Rightarrow y = \frac{-42}{10}$

$y = \frac{-21}{5}$

Point $A\left(\frac{-12}{5}, \frac{-21}{5}\right)$

Solving eq (ii) and eq (iii)

$$3x - \cancel{y} + 3 = 0$$
$$2x + \cancel{y} - 4 = 0$$
$$5x - 1 = 0$$
$$x = \frac{1}{5}$$

Put in (iii)

$$2\left(\frac{1}{5}\right) + y - 4 = 0$$
$$y = 4 - \frac{2}{5}$$
$$y = \frac{18}{5}$$

Point $B\left(\frac{1}{5}, \frac{18}{5}\right)$

Solving eq (i) and eq (iii)
x in eq (iii) by 2

$$x - \cancel{2y} - 6 = 0$$
$$4x + \cancel{2y} - 8 = 0$$
$$5x - 14 = 0$$
$$x = \frac{14}{5}$$

Put in (i)

$$\frac{14}{5} - 2y - 6 = 0$$

$$-2y = 6 - \frac{14}{5}$$

$$-2y = \frac{16}{5} \Rightarrow y = \frac{\cancel{16}\,8}{5\times -\cancel{2}\,1}$$

$$y = -\frac{8}{5}$$

Point $C\left(\frac{14}{5}, -\frac{8}{5}\right)$

Now $A\left(\frac{-12}{5}, \frac{-21}{5}\right)$ $B\left(\frac{1}{5}, \frac{18}{5}\right)$ $C\left(\frac{14}{5}, \frac{-8}{5}\right)$

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} -12/5 & -21/5 & 1 \\ 1/5 & 18/5 & 1 \\ 14/5 & -8/5 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[\frac{-12}{5}\left(\frac{18}{5} + \frac{8}{5}\right) + \frac{21}{5}\left(\frac{1}{5} - \frac{14}{5}\right) + 1\left(\frac{-8}{25} - \frac{252}{25}\right)\right]$$

$$= \frac{1}{2}\left[\frac{-12}{5}\left(\frac{26}{5}\right) + \frac{21}{5}\left(\frac{-13}{5}\right) + 1\left(\frac{-260}{25}\right)\right]$$

$$= \frac{1}{2}\left[-\frac{312}{25} - \frac{273}{25} - \frac{260}{25}\right]$$

$$= \frac{1}{2}\left[\frac{-312 - 273 - 260}{25}\right]$$

$$= \frac{1}{2}\left[\frac{-845}{25}\right] \Rightarrow \frac{-\cancel{845}\,169}{\cancel{50}\,10} = \frac{-169}{10}$$

$$= \frac{169}{10} \text{ Square unit.}$$