Exc 5.2

Note:-

(i) For upward motion
- acceleration = $a = -g$
- Final velocity = $V_F = 0\ m/sec$

(ii) For downward motion.
- acceleration = $a = +g$.
- initial velocity = $0\ m/sec$.

(iii)
a = acceleration = $g = 9.8\ m/sec^2$

اگر m/sec^2 کو ft/sec^2 میں تبدیل کرنا ہو تو 3.2 سے ضرب دیں گے، کیونکہ $1m = 3.2ft$ ہوتا ہے۔

$= 9.8 \times 3.2\ ft/sec^2 \quad \Rightarrow 1m = 3.2ft.$

$a = 32\ feet/sec^2$

Formulas

i) Velocity : $V = \int a\,dt$

ii) Distance : $S = \int V\,dt$

Q #1 A projectile is Launched vertically upward from an initial height of 129ft with an initial velocity of 87ft/sec.

Sol

Given

Height = S_1 = 129 ft

Initial velocity = V_i = 87 ft/sec

i) What are position, velocity and acceleration function?

Position = $S(t)$ = ?

velocity = $V(t)$ = ?

acceleration = $a(t)$ = ?

Acceleration

For upward motion.

$$a = -g$$

$$a = -9.8\ m/sec^2$$

- Change m/sec^2 to ft/sec^2 (Question requirement)

$$a = -9.8 \times 3.2\ ft/sec^2$$

$$\boxed{a = -32\ ft/sec^2}$$

Velocity :

$$V = \int a \, dt$$

$$V = \int -g \, dt \qquad \because a = -g \text{ upward motion}$$

$$V = -g \int 1 \, dt$$

$$V = -gt + C_1 \quad \text{——(i)}$$

initially $t = 0$, $V_i = 87 \text{ ft/sec}$ (Given)

$$87 = -g(0) + C_1$$

$$C_1 = 87 \quad \text{Put in (i)}$$

$$\boxed{V(t) = -gt + 87} \text{ or } \boxed{V(t) = -32t + 87}$$

Position (Distance)

$$S = \int v \, dt$$

$$S = \int (87 - gt) \, dt$$

$$S = 87t - g\frac{t^2}{2} + C_2 \quad \text{— (ii)}$$

- At $t = 0$, Height $= S_1 = 129$

$$129 = 87(0) - 0 + C_2$$

$C_2 = 129$ Put in iii

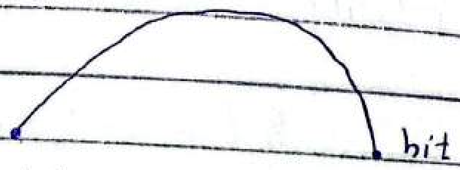
$$\boxed{S = 87t - \frac{1}{2} g t^2 + 129} \quad \text{or}$$

$$S = 87t - \frac{1}{2}(32)t^2 + 129$$

$$\boxed{S = 87t - 16t^2 + 129}$$

(ii) When will the Projectile hit the ground.

$t = ?$



جب Projectile ground کو hit کرے گا تو اس کا Zero distance/displacement ہو گا
When projectile hits the ground then its height (distance) from surface of earth will be Zero

$$S = 0$$

$$87t - 16t^2 + 129 = 0$$

$$16t^2 - 87t - 129 = 0$$

$a = 16 \quad b = -87 \quad c = -129$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-87) \pm \sqrt{(-87)^2 - 4(16)(-129)}}{2(16)}$$

$$t = \frac{87 \pm \sqrt{15825}}{32}$$

$\boxed{t = 6.65 \text{ sec}}$

• At this time projectile hit the ground.

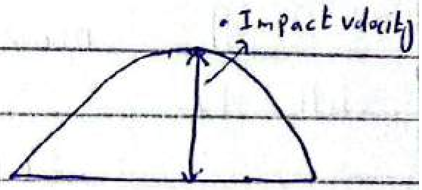
$t = \frac{-38.2}{32} = -1.212$

(time cannot be -ve) Ignore.

(iii) What is its impact velocity?

It is a velocity when projectile hit top to bottom on a ground. Now

$V = -32t + 87$



$V = -32(6.65) + 87$

$V = -125.8 \text{ ft/sec}$ (projectile downward)

(iv) When will the projectile reach its maximum height.

The projectile will reach its maximum height then velocity = $V = 0$

$$V(t) = -32t + 87$$

put $v = 0$

$$87 - 32t = 0$$

$$-32t = -87$$

$$t = \frac{87}{32} \text{ sec.}$$

$$\boxed{t = 2.72 \text{ sec}}$$

(v) What is the maximum height of Projectile.

Sol

The maximum height at $t = 2.72$ sec will be

$$S(t) = 87t - 16t^2 + 129$$

$$S(2.72) = 87(2.72) - 16(2.72)^2 + 129$$

Maximum Height = 247.3 ft Ans.

Q #2

An object has its position defined by $S = t^3 - 9t^2 + 24t + 20$ in feet.

Sol

Given

$$S = t^3 - 9t^2 + 24t + 20 \quad \text{--- (i)}$$

i) What are the velocity and acceleration function?

Velocity:

$$V = \frac{ds}{dt}$$

Diff (i) w.r. to t

$$V = \frac{ds}{dt} = 3t^2 - 18t + 24$$

$$\boxed{V = 3t^2 - 18t + 24} \text{ --- (ii)}$$

Acceleration:

$$a = \frac{dV}{dt}$$

$$V = 3t^2 - 18t + 24$$

Diff w.r. to t

$$a = \frac{dV}{dt} = 6t - 18$$

$$\boxed{a = 6t - 18}$$

(ii) What are the position and velocity of the object when its acceleration is $-6.5 ft/sec^2$?

$S(t) = ?$ $V(t) = ?$

Given $a = -6.5 ft/sec^2$

As $a = 6t - 18$

$$6t - 18 = -6.5$$

$$6t = 18 - 6.5$$

$$6t = 11.5$$

$$t = \frac{11.5}{6}$$

$$t = 1.92 \text{ sec}$$

Now

$$V(t) = 3t^2 - 18t + 24$$

$$V(1.92) = 3(1.92)^2 - 18(1.92) + 24$$

$$\boxed{\text{velocity} = 0.5208 \text{ m/sec}}$$

Now

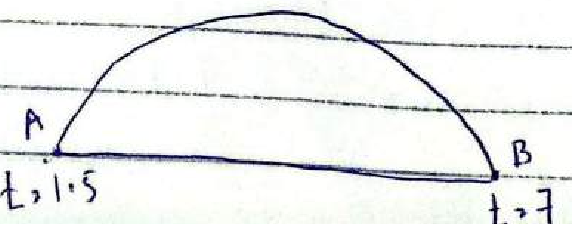
$$S(t) = t^3 - 9t^2 + 24t + 20$$

$$S(1.92) = (1.92)^3 - 9(1.92)^2 + 24(1.92) + 20$$

$$\boxed{S(1.92) = 39.98 \text{ Ft}}$$

(iii) Find the displacement and the total distance travelled by the projectile from $t = 1.5$ sec to $t = 7$ sec

Sol:



Displacement = $S(7) - S(1.5)$ — (A)

$S(t) = t^3 - 9t^2 + 24t + 20$

- $S(7) = (7)^3 - 9(7)^2 + 24(7) + 20$

$\boxed{S(7) = 90}$

- $S(1.5) = (1.5)^3 - 9(1.5)^2 + 24(1.5) + 20$

$\boxed{S(1.5) = 39.125}$

Displacement = $90 - 39.125$

Displacement = 50.87 ft

Total Distance =

$= S(7) + S(1.5)$

$= 90 + 39.125$

$= 129.125$ ft

2nd Method. To find the total distance Put

$V(t) = 0$

$3t^2 - 18t + 24 = 0$

$(t-2)(t-4) = 0$

$t = 2$ sec $t = 4$ sec

Distance $t = 1.5, 2, 4$ and 7 are

$S(1.5) = 39.13$

$S(2) = 40$

$S(4) = 36$

$S(7) = 90$

- Now distance from $t = 1.5$ sec to $t = 2$ sec

$|S(2) - S(1.5)| = |40 - 39.13| = 0.87$ ft

- Now distance from $t = 2$ sec to $t = 4$ sec

$|S(4) - S(2)| = |36 - 40| = 4$ ft

- Distance from $t = 4$ sec to $t = 7$ sec

$|S(7) - S(4)| = |90 - 36| = 54$ ft

Total distance ... 58.87 ft.

3) A Person is standing on top of the Minar-e-Pakistan and throws a ball directly upward with an initial velocity of 96ft/sec. The Minar Pakistan is 176 ft high.

Sol Given

Initial velocity = V_i = 96 ft/sc $t=0$

Height = S_1 = 176 Ft

i) What are the Functions for position, velocity and acceleration of the ball?

$S(t) = ?$

$V(t) = ?$

$a(t) = ?$

Acceleration since ball is thrown upward so

$$a = -g$$

$$a = -9.8\, m/sec^2$$

$$a = -9.8 \times 3.28\, ft/sec^2$$

$$\boxed{a = -32\, ft/sec^2}$$

Velocity

$$V = \int a\, dt$$

$V = \int -g \, dt$

$V = -gt + c$

initially $V = 96$ ft/sec $t = 0$

$96 = -g(0) + c$

$c = 96$

$\boxed{V = -32t + 96}$

Position

$S(t) = \int V \, dt$

$= \int (-32t + 96) \, dt$

$S(t) = -32 \frac{t^2}{2} + 96t + C$

$S(t) = -16t^2 + 96t + C$

initially $S(t) = 176$, $t = 0$

$176 = 0 + 0 + C \Rightarrow C = 176$

$S(t) = -16t^2 + 96t + 176$

(ii) When does the ball hit the ground and with what velocity?

When the ball hit the ground then

$$S(t) = 0$$

$$-16t^2 + 96t + 176 = 0$$

$$t^2 - 6t - 11 = 0$$

$a = 1, \quad b = -6 \quad c = -11$

$$t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-11)}}{2(1)}$$

$$t = \frac{6 \pm \sqrt{36 + 44}}{2}$$

$$t = \frac{6 \pm \sqrt{80}}{2}$$

$$t = \frac{6 + \sqrt{80}}{2}$$

$t = 7.5$ sec

$$t = \frac{6 - \sqrt{80}}{2}$$

$t = -1.472$

time cannot be negative

Velocity

$$V(t) = -32t + 96$$

At $t = 7.5$

$$V(7.5) = -32(7.5) + 96$$

$V = -144$ m/sec (−) show velocity downward direction.

(iii) How far does the ball travel during its flight?

Sol:

time = ?

velocity at highest point

$$V = 0$$

$$-32t + 96 = 0$$

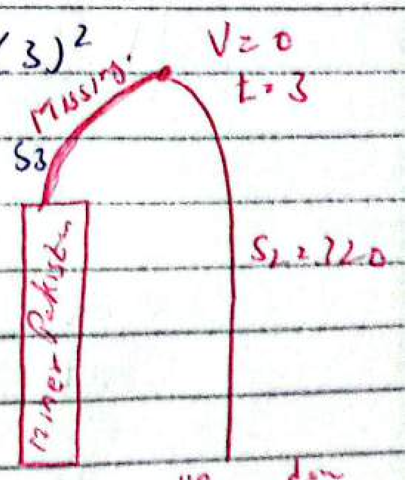
$$t = \frac{-96}{-32}$$

$$t = 3 \text{ sec}$$

Highest distance $= S(t) = 176 + 96t - 16t^2$

$$S(3) = 176 + 96(3) - 16(3)^2$$

$$S_2(3) = 320 \text{ ft}$$



Now

distance $= 320 - S_1$

$= 320 - 176$

$S_3 = 144$ ft

320+320

640−17

464

Total distance $= S_1 + S_3$

$= 320 + 144 = 464$ ft Ans

- **Increasing Function:-** A function F(x) is said to be increasing if $F'(x) > 0$ or $\frac{ds}{dt} > 0$
- **Decreasing Function:** A function F(x) is said to be decreasing if $F'(x) < 0$ or $\frac{ds}{dt} < 0$ or $\frac{dv}{dt} < 0$

Q #4 A Particle moves along a Line such that its position is

$$S = 2t^3 - 9t^2 + 12t - 4 \quad \text{For } t \geq 0$$

Sol

i) Find t for which the distance 'S' is increasing.

$$S = 2t^3 - 9t^2 + 12t - 4$$

Diff w.r.to 't'

$$V = \frac{ds}{dt} = \frac{d}{dt}(2t^3 - 9t^2 + 12t - 4)$$

$$\frac{ds}{dt} = 6t^2 - 18t + 12$$

$$\boxed{V = 6t^2 - 18t + 12}$$

Again Diff w.r.to t

$$\frac{dv}{dt} = \frac{d}{dt}(6t^2 - 18t + 12)$$

$$a = \frac{dv}{dt} = 12t - 18$$

$$\boxed{a = 12t - 18}$$

Distance will be increasing If

$$\frac{ds}{dt} > 0$$

So Here

$$\frac{ds}{dt} = V = 6t^2 - 18t + 12 \quad \text{---(i)}$$

Put $\frac{ds}{dt} = 0$

$$6t^2 - 18t + 12 = 0$$

$$t^2 - 3t + 2 = 0$$

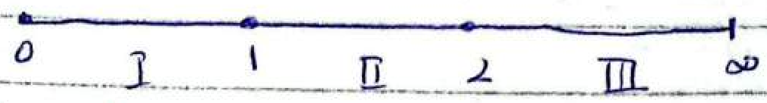
$$t^2 - 2t - t + 2 = 0$$

$$t(t-2) - 1(t-2) = 0$$

$$(t-1)(t-2) = 0$$

$t - 1 = 0$ $\qquad\qquad$ $t - 2 = 0$

$t = 1$ $\qquad\qquad$ $t = 2$



$x \geq 0$

So intervals are

$$[0,1), (1,2), (2,\infty)$$

Interval $[0,1)$

Put $t = \frac{1}{2}$

$$\frac{ds}{dt} = 6\left(\frac{1}{2}\right)^2 - 18\left(\frac{1}{2}\right) + 12$$

$$= \frac{6}{4} - 9 + 12$$

$$= \frac{3}{2} + 3$$

$$= \frac{9}{2} > 0 \qquad \frac{ds}{dt} > 0$$

So distance is increasing at $[0,1)$

Interval $(1,2)$

Put $t = \frac{3}{2}$ in (i)

$$V = \frac{ds}{dt} = 6t^2 - 18t + 12$$

$$= 6\left(\frac{3}{2}\right)^2 - 18\left(\frac{3}{2}\right) + 12$$

$$= \cancel{6}^{3} \cdot \frac{9}{\cancel{4}_{2}} - 9 \cdot 3 + 12$$

$$= \frac{27}{2} - 27 + 12$$

$$= \frac{27}{2} - \cancel{39}^{15}$$

$$= \frac{-3}{2} < 0 \qquad \frac{ds}{dt} < 0$$

So distance is decreasing at $(1,2)$

interval $(2, \infty)$

Put $t=3$ in (i)

$$\frac{ds}{dt} = 6t^2 - 18t + 12$$

$$= 6(3)^2 - 18(3) + 12$$

$$= 54 - 54 + 12$$

$$= 12 > 0 \qquad \frac{ds}{dt} > 0$$

So distance is increasing at $(2, \infty)$

So distance increasing in interval
$t \in [0,1] \cup (2, \infty)$

(iii) Find t for which the velocity is increasing.

- Velocity will be increasing If $\frac{dv}{dt} > 0$ Now

$$a = \frac{dv}{dt} = 12t - 18 \quad \text{— (iii)}$$

Put $\frac{dv}{dt} = 0$

$$12t - 18 = 0$$

$$12t = 18 \qquad t = \frac{18}{12}$$

$$t = \frac{3}{2}$$

$t \geq 0$

$0 \quad \frac{3}{2} \quad \infty$

So intervals are

$$\left[0, \frac{3}{2}\right), \left(\frac{3}{2}, \infty\right)$$

interval $\left[0, \frac{3}{2}\right)$

Put $t = 1$ in (2)

$$a = \frac{dv}{dt} = 12(1) - 18 = -6 < 0$$

$$\frac{dv}{dt} < 0$$

So velocity is decreasing at $\left[0, \frac{3}{2}\right)$

interval $\left(\frac{3}{2}, \infty\right)$

Put $t = 2$

$$a = \frac{dv}{dt} = 12(2) - 18 = 24 - 18 = 6 > 0$$

$$a = \frac{dv}{dt} > 0$$

So velocity is increasing at $\left(\frac{3}{2}, \infty\right)$

(iii) Find t for which the speed of particle is increasing.

There are two possibilities when speed of particles is increasing:

(i) IF $\underset{v > 0}{\dfrac{ds}{dt} > 0}$ and $\underset{a > 0}{\dfrac{dv}{dt} > 0}$; Speed increases

or

$V(t) > 0$ and $a(t) > 0$ "

(ii) IF $\dfrac{ds}{dt} < 0$ and $\dfrac{dv}{dt} < 0$; Speed ~~decreases~~ increases

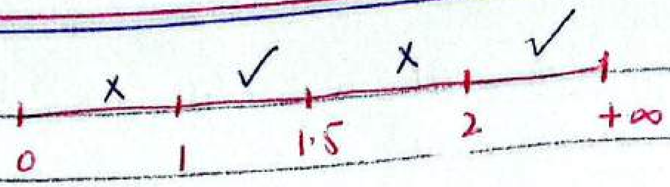
or

$V(t) < 0$ and $a(t) < 0$ "

All values of t getting after $\dfrac{ds}{dt} = 0$, $\dfrac{dv}{dt} = 0$ are

$a > 0$ $\quad\quad\quad\quad$ $V > 0$

$(1.5, \infty)$ $\quad$ $[0, 1)$ $\quad$ $(2, +\infty)$



So intervals are
[0, 1), (1, 1.5) (1.5, 2) (2, +∞)

Check [0, 1)

Velocity	Acceleration
$V(t) = 6t^2 - 18t + 12$	$a(t) = 12t - 18$
Put $t = 0.5$	Put $t = 0.5$
$6(0.5)^2 - 18(0.5) + 12$	$12(0.5) - 18$
$4.5 > 0$	$6 - 18$
+ve	$-12 < 0$
	−ve

Speed Decreasing in interval [0, 1).

Check (1, 1.5)

Velocity	Acceleration
$V(t) = 6t^2 - 18t + 12$	$a(t) = 12t - 18$
Put $t = 1.1$	Put $t = 1.1$
$= 6(1.1)^2 - 18(1.1) + 12$	$= 12(1.1) - 18$
$= -0.5 < 0$	$= -4.8 < 0$
− ve	−ve

Speed Increasing in interval $(1, 1.5)$

Check $(1.5, 2)$

Velocity	Acceleration
$v(t) = 6t^2 - 18t + 12$	$a(t) = 12t - 18$
Put $t = 1.6$	Put $t = 1.6$
$6(1.6)^2 - 18(1.6) + 12$	$= 12(1.6) - 18$
$= -1.4 < 0$	$= 37.2 > 0$
$-ve$	$+ve$

Speed decrease in interval $(1.5, 2)$

Check $(2, +\infty)$

Velocity	Acceleration
$v(t) = 6t^2 - 18t + 12$	$a(t) = 12t - 18$
Put $t = 3$	Put $t = 3$
$6(3)^2 - 18(3) + 12$	$= 12(3) - 18$
$= 54 - 54 + 12$	$= 18 > 0$
$12 > 0$	$+ve$
$+ve$	

Speed increasing in interval $(2, +\infty)$.

Hence speed will increase $(1, 1.5) \cup (2, +\infty)$

IV) Find the speed when $t = \frac{3}{2}$ s

$$V = \frac{ds}{dt} = 6t^2 - 18t + 12$$

$$V(1.5) = 6(1.5)^2 - 18(1.5) + 12$$

$$V(1.5) = -1.5$$

$$\text{Speed} = |-1.5|$$

Speed cannot be negative.

$$\boxed{\text{Speed} = 1.5}$$

(V) Find the total distance travelled in the time interval $[0, 4]$

At 1, 2 $V = 0$

$$d = |S(1) - S(0)| + |S(2) - S(1)| + |S(4) - S(2)|$$

Now

$$S(t) = 2t^3 - 9t^2 + 12t - 4$$

$$S(0) = -4$$

$$S(1) = 1$$

$$S(2) = 0$$

$$S(4) = 28$$

$= |1+4| + |0-1| + |28-0|$

$d = 5 + 1 + 28$

$d = 34$

Q#5 The Position of an object moving on a line is given by $S = 6t^2 - t^3$, $t \geq 0$ where S is in meters and t is in seconds.

Given $S(t) = 6t^2 - t^3$ —(1) $t \geq 0$

i) Determine the velocity and acceleration of the object at $t = 2$

Diff (1) w.r.to t

$$V = \frac{ds}{dt} = \frac{d}{dt}\, 6t^2 - t^3$$

$$V = \frac{ds}{dt} = 12t - 3t^2$$

$$V = 12t - 3t^2 \quad —(2)$$

At $t = 2$

$$V = 12(2) - 3(2)^2$$

$$V = 24 - 12$$

$$\boxed{V = 12\, m/s}$$

Diff (2) w.r.to t

$$a = \frac{dv}{dt} = \frac{d}{dt}(12t - 3t^2)$$

$$a = 12 - 6t$$

At $t = 2$

$$a = 12 - 6(2)$$

$$a = 12 - 12$$

$$\boxed{a = 0 \ m/sec}$$

(ii) At what time is the object at rest

The object at rest when

$$V = 0$$

$$12t - 3t^2 = 0$$

$$t(12 - 3t) = 0$$

$t = 0$

$$12 - 3t = 0$$

$$-3t = -12$$

$$t = 4$$

$t = 0, 4$

(iii) In which direction is the object moving at $t = 5$ sec

$V(t) = 12t - 3t^2$

$t = 5$

$= 12(5) - 3(5)^2$

$= 60 - 75$

$V = -15$ m/sec.

object is moving at backward direction.
or in negative direction.

(v) When is the object moving in a positive direction.

The object will move in a positive direction
If $V(t) > 0$ Now (increasing)

$V(t) = 12t - 3t^2$

Put $V(t) = 0$

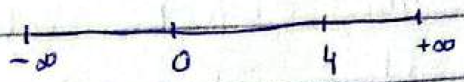
$12t - 3t^2 = 0$

$t(12 - 3t) = 0$

$t = 0 \qquad 12 - 3t = 0$

$t = 0 \qquad t = 4$

So intervals are



$[0, 4) \quad (4, \infty)$ Given $t \geq 0$

interval $[0, 4)$

Put $t = 1$ in (2)

$$V(1) = 12(1) - 3(1)^2$$
$$= 9 > 0$$

So velocity is increasing and positive at $[0, 4)$

interval $(4, \infty)$

Put $t = 5$ in (2)

$$V(t) = 12t - 3t^2$$
$$= 12(5) - 3(5)^2$$
$$= 60 - 75$$
$$= -15 < 0$$

So velocity is decreasing and negative at $(4, \infty)$.

(V) When does the object return to its initial position.

The object return to its initial position IF $S(t) = 0$

$$S(t) = 6t^2 - t^3$$

$$6t^2 - t^3 = 0$$
$$t^2(6 - t) = 0$$

$t = 0$ $\quad$ $t = 6$

Hence the object returns to its initial position at $t = 6$ sec.

6) A particle P moves on the x^+-axis. The acceleration of P in time t seconds, $t \geq 0$, is $a = (3t+5)\ m/sec^2$ in the positive x-direction. When $t = 0$, the velocity of P is 2 m/sec in the positive x-direction. When $t = T$, the velocity of P is 6 m/sec in the positive x-direction. Find the value of T.

Sol

Given

$$a = (3t+5)\ m/sec^2 \quad \text{—} (1)$$

when

$t = 0$, $\quad V = 2\ m/sec$

$t = T$, $\quad V = 6\ m/sec$

$$V = \int a \, dt$$

$$V = \int (3t+5)\, dt$$

$$V = 3\frac{t^2}{2} + 5t + C$$

At $t = 0$, $\quad V = 2\ m/sec$

$$2 = \frac{3(0)^2}{2} + 5(0) + C$$

$$2 = 0 + C_1$$

$$C_1 = 2$$

$$\boxed{V(t) = \frac{3t^2}{2} + 5t + 2} \quad -(2)$$

At $t = T$ $V = 6\,m/sec$

$$6 = \frac{3T^2}{2} + 5T + 2$$

$$12 = 3T^2 + 10T + 4$$

$$3T^2 + 10T + 4 - 12 = 0$$

$$3T^2 + 10T - 8 = 0$$

$$3T^2 + 12T - 2T - 8 = 0$$

$$3T(T+4) - 2(T+4) = 0$$

$$(T+4)(3T-2) = 0$$

$$T = -4 \qquad T = \frac{2}{3}$$

Q #7 A Particle P moves along the x^+-axis. At time t seconds the velocity of P is $V = (3t^2 - 4t + 3)\,m/s$. When $t = 0$, P is at the origin O.

Find the distance of P from O when P is moving with minimum velocity?

Sol Given

$$v(t) = 3t^2 - 4t + 3$$

$t = 0$ $\quad$ $S = 0$

Distance of P from origin O. = ?

Since particle is moving with minimum velocity when

acceleration $= 0$

$$a = 0$$

$$a = \frac{dv}{dt} = 0$$

$$\frac{d}{dt}(3t^2 - 4t + 3) = 0$$

$$6t - 4 = 0$$

$$t = \frac{4}{6}$$

$$t = \frac{2}{3}$$

Now

$$S = \int v \, dt$$

$$S = \int (3t^2 - 4t + 3) \, dt$$

$$S = 3\frac{t^3}{3} - 4\frac{t^2}{2} + 3t + C$$

$$S = t^3 - 2t^2 + 3t + C$$

At $t = 0$ $S = 0$

$$0 = 0 - 0 + 0 + C$$

$$C = 0$$

$$S = t^3 - 2t^2 + 3t$$

At $t = \frac{2}{3}$ velocity is minimum

$$S = \left(\frac{2}{3}\right)^3 - 2\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right)$$

$$= \frac{8}{27} - \frac{8}{9} + \frac{6}{3}$$

$$\boxed{S = \frac{38}{27} m}$$

Distance of P from origin is $\frac{38}{27}$

(8) A Particle P moves along the x-axis in a straight line so that at time t seconds the velocity of P is v m/sec, where

Sol. Given

$$V = \begin{cases} 10t - 2t^2 & 0 \le t \le 6 \\ -\dfrac{432}{t^2} & t > 6 \end{cases}$$

displacement = $S(t) = ?$

i) $t = 6$ sec

To find the displacement at $t = 6$

$$V(t) = 10t - 2t^2$$

$$S(t) = \int_0^6 V(t)\, dt$$

$$= \int_0^6 (10t - 2t^2)\, dt$$

$$= \frac{10t^2}{2} - \frac{2t^3}{3} \Big|_0^6$$

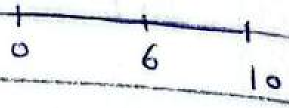
$$= 5t^2 - \frac{2}{3}t^3 \Big|_0^6$$

$$= \left(5 \cdot 36 - \frac{2}{\cancel{3}_{1}} \cdot \overset{72}{\cancel{216}}\right) - (0 - 0)$$

$$= 180 - 144$$

$$S(t) = 36\,m$$

ii) $t = 10$ sec



Given $V = -\dfrac{432}{t^2}$

$$S(t) = \int v\,dt$$

$$S(10) = 36 + \int_6^{10} -\frac{432}{t^2}\,dt$$

$$= 36 + \int_6^{10} -432\,t^{-2}\,dt$$

$$= 36 - 432\int_6^{10} t^{-2}\,dt$$

$$= 36 - 432 \left.\frac{t^{-1}}{-1}\right|_6^{10}$$

$$= 36 + \left.\frac{432}{t}\right|_6^{10}$$

$$= 36 + \left[\frac{432}{10} - \frac{432}{6}\right]$$

$S(10) = 36 + 43.2 - 72$

$S(10) = 7.2$

Q#9 A Particle P moves along the x^+-axis at time t seconds the velocity V of P is increasing in the direction of X-axis given by

$$V = \begin{cases} 8t - \frac{3}{2}t^2 & 0 \le t \le 4 \\ 16 - 2t & t > 4 \end{cases}$$

When $t = 0$, P is at the origin O. Find

(i)

Sol Given

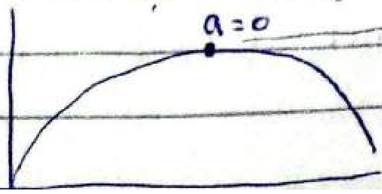
$$V(t) = \begin{cases} 8t - \frac{3}{2}t^2 & ; \quad 0 \le t \le 4 \\ 16 - 2t & ; \quad t > 4 \end{cases}$$

(i) the greatest speed of P in the interval $0 \le t \le 4$

Speed Greatest تب ہوتی ہے جب Zero acceleration کے لئے

جب speed Max or Min ہوگی تب $a = 0$ ہوگا

$a = 0 \rightarrow$ greatest speed.



For $0 \le t \le 4$

$$V(t) = 8t - \frac{3}{2}t^2$$

$t = ?$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(8t - \frac{3}{2}t^2\right)$$

$$a = 8 - \frac{3}{\cancel{2}} \cdot \cancel{2}t$$

$$a = 8 - 3t$$

Put $a = 0$

$$8 - 3t = 0$$

$$\cancel{-}3t = \cancel{-}8$$

$$t = \frac{8}{3} \text{ sec}$$

Greatest Speed =

$$V(t) = 8t - \frac{3}{2}t^2$$

$$V\left(\frac{8}{3}\right) = 8 \cdot \frac{8}{3} - \frac{3}{2}\left(\frac{8}{3}\right)^2$$

$$V\left(\frac{8}{3}\right) = \frac{64}{3} - \frac{3}{2} \cdot \frac{64}{9}$$

$$V\left(\frac{8}{3}\right) = \frac{32}{3} \text{ m/sec}$$

(ii) The distance of P from O when $t = 4$

For $t = 4$

$$V = 8t - \frac{3}{2}t^2$$

$$S = \int V \, dt$$

$$S = \int_0^4 \left(8t - \frac{3}{2}t^2\right) dt$$

$$4\,\frac{8 \cdot t^2}{\cancel{2}\,1} - \frac{\cancel{3}}{2} \cdot \frac{t^3}{\cancel{3}} \Big|_0^4$$

$$\left(4t^2 - \frac{t^3}{2}\right)\Big|_0^4$$

$$= 4(4)^2 - \frac{(4)^3}{2} - 0$$

$$= 64 - \frac{\overset{32}{64}}{\cancel{2}\,1}$$

$$\boxed{S = 32 \text{ m}}$$

(iii) the time at which P is instantaneously at rest for $t > 4$.

Particle rest پہ تب آئے گا جب velocity Zero ہو جائے گی

For $t > 4$

$$V = 16 - 2t$$

For rest $V = 0$

$$16 - 2t = 0$$
$$-2t = -16$$
$$t = 8\,sec$$

(IV) The total distance travelled by P in the first 10 sec of its motion:

Sol

$$V = 16 - 2t \qquad t \geq 4$$

$$S = \int v\,dt$$

$$S(t) = \int_0^4 v(t)\,dt + \int_4^8 v(t)\,dt + \int_8^{10} v(t)\,dt$$

$$= \int_0^4 \left(8t - \frac{3}{2}t^2\right) + \int_4^8 (16-2t)\,dt + \int_8^{10} (16-2t)\,dt$$

$$= 32 + 16t - \frac{2t^2}{2}\Big|_4^8 + 16t - \frac{2t^2}{2}\Big|_8^{10}$$

$$= 32 + \Big[\left[16(8) - (8)^2\right] - \left[16(4) - (4)^2\right] + \left[16(10) - (10)^2 - 16(8) - (8)^2\right]$$

$= 32 + [(128-64)-(64-16)] + [(160-100)-(128-64)]$

$= 32 + [64-48] + [60-64]$

$= |32| + |16| + |-4|$ ∵ Distance Cannot be negative.

$= 32+16+4$

$= 52m.$

Q#10 A Particle P moves along the x^+-axis The acceleration of P at time — — — — — — — — — — — — —

Given that $V=6$ when $t=0$ Find

Sol

Given

$a = 4t-8 \ m/sec^2$; $V=6$; $t=0$

(i) Velocity in terms t.

$a = 4t-8$

$V = \int a \, dt$

$V = \int (4t-8) \, dt$

$$V = 4\frac{t^2}{2} - 8t + C$$

initial condition

$V = 6 \quad t = 0$

$$6 = 4\frac{(0)}{2} - 8(0) + C$$

$$C = 6$$

$$\boxed{V = 2t^2 - 8t + 6}$$

(iii) The distance between the two points where P is instantaneously at rest.

To find the distance between the two points where P is instantaneously at rest.

$$V(t) = 0$$

$$2t^2 - 8t + 6 = 0$$

$$2(t^2 - 4t + 3) = 0$$

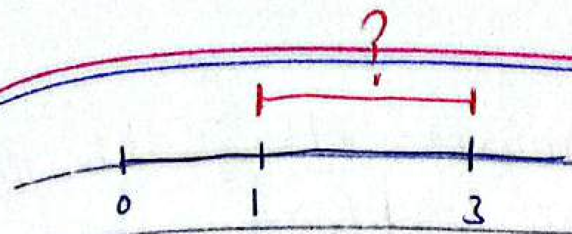
$$t^2 - 4t + 3 = 0$$

$$t^2 - 3t - t + 3 = 0$$

$$t(t-3) - 1(t-3) = 0$$

$$(t-1)(t-3) = 0$$

$t = 1\text{ sec} \quad t = 3\text{ sec}$



$$V = 2t^2 - 8t + 6$$

$$S(t) = \int V \, dt$$

$$S(t) = \int_1^3 (2t^2 - 8t + 6) \, dt$$

$$= 2\frac{t^3}{3} - \overset{4}{\cancel{8}}\frac{t^2}{\cancel{2}\,1} + 6t \Big|_1^3$$

$$= \frac{2t^3}{3} - 4t^2 + 6t \Big|_1^3$$

$$= \left(\frac{2 \cdot 27}{3} - 36 + 18\right) - \left(\frac{2}{3} - 4 + 6\right)$$

$$= (18 - 36 + 18) - \left(\frac{2}{3} + 2\right)$$

$$= \left|0 - \frac{8}{3}\right|$$ Apply Absolute because Distance cannot be negative

$$S(t) = \frac{8}{3} \text{ m } A$$

(11) A particle P moves along the x^+-axis and its acceleration after a given instant 't' is given by $a = (4t - 9)\ m/sec^2$ $t \geq 0$. When $t = 1$, P is moving with velocity of $-3\ m/sec$.

Given $a = (4t - 9)\ m/sec^2$ $\quad t \geq 0$

$t = 1 \quad V = -3\ m/sec$

i) Find the minimum velocity of P.

To find minimum velocity of P, we put $a = 0$

As

$$a = (4t - 9)\ m/sec^2$$

$$V = \int a\, dt$$

$$V = \int (4t - 9)\, dt$$

$$V = \frac{4t^2}{2} - 9t + C$$

$$V = 2t^2 - 9t + C$$

As $V = -3 \quad t = 1$

$$-3 = 2 - 9 + C$$

CamScanner

$-3 = -7 + c$

$7 - 3 = c$

$c = 4$

$$\boxed{v = 2t^2 - 9t + 4}$$

for minimum velocity $\frac{dv}{dt} = a = 0$

$a = 0$

$4t - 9 = 0$

$t = \frac{9}{4}$

$v(t) = 2t^2 - 9t + 4$

$v\left(\frac{9}{4}\right) = 2\left(\frac{9}{4}\right)^2 - 9\left(\frac{9}{4}\right) + 4$

$= \cancel{2} \cdot \frac{81}{\cancel{4}2} - \frac{81}{4} + 4$

$= \frac{81}{2} - \frac{81}{4} + 4$

Minimum velocity $= -\frac{49}{8}$ m/sec

(ii) Determine the time when P is instantaneously at rest:

To determine when P is at rest

جب کوئی چیز rest پر ہوتی ہے وہاں پر اس کی Zero velocity ہو جاتی ہے

$$V(t) = 0$$

$$2t^2 - 9t + 4 = 0$$

$$2t^2 - 8t - t + 4 = 0$$

$$2t(t-4) - 1(t-4) = 0$$

$$(2t-1)(t-4) = 0$$

$$t = \frac{1}{2} \qquad t = 4$$

Hence $t = 4$ sec and $t = \frac{1}{2}$ sec.

(III) Find the distance travelled by P in the First $4\frac{1}{2}$ Second of its motion.
(velocity لما يبقى Zero نقسم عنده)

$$S(t) = \int V \, dt$$

$$S(t) = \int (2t^2 - 9t + 4) \, dt$$

$$S(t) = \left(\frac{2t^3}{3} - \frac{9t^2}{2} + 4t \right) + C$$

initial Condition $t = 0$ $S = 0$

$0 = 0 - 0 + 0 + C$

$C = 0$

$$S(t) = \frac{2}{3}t^3 - \frac{9}{2}t^2 + 4t$$

$S(0) = 0$ $\quad$ $S(\frac{1}{2}) = \frac{23}{24}$

$S(4) = -\frac{40}{3}$ $\quad$ $S(4.5) = -\frac{99}{8}$

<u>Formula of Distance</u>

$$d = \left|S(1/2) - S(0)\right| + \left|S(4) - S(1/2)\right| + \left|S(4\tfrac{1}{2}) - S(4)\right|$$

$$d = \left|\frac{23}{24} - 0\right| + \left|-\frac{40}{3} - \frac{23}{24}\right| + \left|-\frac{99}{8} + \frac{40}{3}\right|$$

$$d = \frac{23}{24} + \frac{343}{24} + \frac{23}{24}$$

$$d = \frac{389}{24} = 16.20\ m \quad \text{Ans.}$$

Q #12 A car moving on a straight road is modelled as a particle moving along the x^+-axis, and its acceleration $a\ m/sec^2$ after a given instant t, is given by

$$a = \begin{cases} 4 - \frac{1}{2}t & 0 \le t \le 8 \\ 0 & t > 8 \end{cases}$$

The car starts from rest.

Sol • For $0 \le t \le 8$

$$V = \int a\, dt$$

$$V = \int \left(4 - \frac{1}{2}t\right) dt$$

$$V(t) = 4t - \frac{1}{2} \cdot \frac{t^2}{2} + C$$

$$V(t) = 4t - \frac{t^2}{4} + C$$

Initial condition

$$t = 0 \qquad V = 0$$

$$0 = 0 + 0 + C$$

$$C = 0$$

$$\boxed{V(t) = 4t - \frac{t^2}{4}}$$

For $t > 8$

$a = 0$ (Constant acceleration)

(velocity Constant)

$V(t) = 4t - \frac{1}{4}t^2$

$V(8) = 4 \cdot 8 - \frac{1}{4} \cdot (8)^2$

$V(8) = 32 - 16$

$\boxed{V(8) = 16}$ m/sec

$$V(t) = \begin{cases} 4t - \frac{t^2}{2} & 0 \le t \le 8 \\ 16 & t > 8 \end{cases}$$

Ans.

(iii) Find the time it takes for the car to reach its maximum speed.

To state the time, it takes for the car to reach its maximum speed, which occurs when $a(t) = 0$

$$a = 4 - \frac{1}{2}t = 0$$

$$4 - \frac{1}{2}t = 0$$

$$+\frac{1}{2}t = +4$$

$$t = 8 \text{ sec.}$$

(iii) Show that the displacement of P from O is given by

$$S = \begin{cases} 2t^2 - \frac{1}{12}t^3 & 0 \le t \le 8 \\ 16t - \frac{128}{3} & t > 8 \end{cases}$$

For $0 \le t \le 8$

$$V = 4t - \frac{1}{4}t^2$$

$$S = \int V\,dt$$

$$S = \int 4t - \frac{1}{4}t^2$$

$$S = \frac{4t^2}{2} - \frac{t^3}{12} + C$$

$$S = 2t^2 - \frac{t^3}{12} + C$$

At rest $t = 0$ $S = 0$

$$\boxed{S = 2t^2 - \frac{t^3}{12}}$$

For $t > 8$

$V(t) = 16$

$S = \int v \, dt$

$S = \int 16 \, dt$

$S = 16t + c$

یہاں سے (c) کا جواب لگانا ہے۔

Continuity Condition

L.H.L = R.H.L = f(8)

$$\lim_{t \to 8^-} \left(2t^2 - \frac{1}{12}t^3\right) = \lim_{n \to 8^+} (16t + c) = f(8)$$

$$2(8)^2 - \frac{1}{12}(8)^3 = 16(8) + c$$

$$\boxed{c = \frac{-128}{3}}$$

$$\boxed{S(t) = 16t - \frac{128}{3}}$$

Hence Proved

(IV) Calculate the time it takes the car to cover the first 1000m.

Sol:

For $t > 8$

$$S(t) = 16t - \frac{128}{3}$$

Put $S(t) = 1000$

$$1000 = 16t - \frac{128}{3}$$

$$16t = 1000 + \frac{128}{3}$$

$$16t = 3128$$

$$t = \frac{3128}{16}$$

$$\boxed{t = 65.17 \text{ sec}}$$

3000
128
3128

(13) A Particle is moving with constant acceleration 'a' with an initial velocity V_i and after time t it covers a distance S.

Prove that

1) $V_F = V_i + at$

$$a = \frac{\text{Change in Velocity}}{\text{Change in time}}$$

$$a = \frac{V_F - V_i}{t - 0}$$

$$a t = V_F - V_i$$

$$V_F = V_i + a t$$

(iii) $S = V_i t + \frac{1}{2} a t^2$

$$S = V t$$

$$\text{Average Velocity} = \frac{V_i + V_F}{2}$$

$$V_{avg} = \frac{V_i + V_F}{2}$$

$$V_{avg} \times t = \frac{V_i + V_F}{2} \times t$$

$$S = \frac{V_i t}{2} + \frac{V_F t}{2}$$

$$2S = V_i t + V_F t$$

$\because V_f = V_i + a t$

$$2S = V_i t + (V_i + a t) t$$

$$2S = V_i t + V_i t + a t^2$$

$$2S = 2 V_i t + a t^2$$

$$S = V_i t + \frac{1}{2} a t^2$$

(iii) $2aS = V_f^2 - V_i^2$

Sol

$$S = Vt$$

$$S = \frac{V_i + V_f}{2} \times t$$

$$2S = (V_f + V_i)t$$

multiply a on b.sides

$$2aS = (V_f + V_i)at$$

As in (i) $at = V_f - V_i$

$$2aS = (V_f + V_i)(V_f - V_i)$$

$$2aS = V_f^2 - V_i^2$$

Complete