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Exc 4.3

Homogeneous Function:- A function f has the property that
$$f(tx, ty) = t^n f(x,y)$$
where $t \in R^+$, $n \in R$. Then f is said to be a homogeneous function of degree n.

Q#1 Check whether the function are homogeneous or not. If homogeneous find degree.

i) $f(x,y) = 6xy^3 - x^2y^2$

Sol Put $x = \lambda x$, $y = \lambda y$

$$f(\lambda x, \lambda y) = 6(\lambda x)(\lambda y)^3 - (\lambda x)^2(\lambda y)^2$$
$$= 6\lambda x \lambda^3 y^3 - \lambda^2 x^2 \lambda^2 y^2$$
$$= 6\lambda^4 xy^3 - \lambda^4 x^2 y^2$$
$$= \lambda^4 (6xy^3 - x^2y^2)$$
$$f(\lambda x, \lambda y) = \lambda^4 f(x,y)$$

Yes It is a Homogeneous eq of degree 4.

(ii)

(ii) $F(x,y) = x^2 - y$

Put $x = \lambda x \qquad y = \lambda y$

$F(\lambda x, \lambda y) = (\lambda x)^2 - \lambda y$

$= \lambda^2 x^2 - \lambda y$

$F(\lambda x, \lambda y) = \lambda(\lambda x^2 - y)$

$F(\lambda x, \lambda y) \neq \lambda F(x,y)$

It is non-homogeneous function.

(iii) $F(x,y) = \dfrac{2y^3}{x^2 y} - 7$

Sol Put $x = \lambda x, \qquad y = \lambda y$

$F(\lambda x, \lambda y) = \dfrac{2(\lambda y)^3}{(\lambda x)^2(\lambda y)} - 7$

$= \dfrac{2\cancel{\lambda^3} y^3}{\cancel{\lambda^2} x^2 \cancel{\lambda} y} - 7$

$= \dfrac{2y^3}{x^2 y} - 7$

$F(\lambda x, \lambda y) = \dfrac{2y^3}{x^2 y} - 7$

$F(\lambda x, \lambda y) = \lambda^0 F(x,y)$

It is Homogeneous Function of degree 0.

Homogeneous Differential Eq's

A differential equation of the form

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

is said to be homogeneous if both M and N are homogeneous functions of the same degree.

- The differential equation can be reduced to separable variables by putting

$$y = ux \quad \text{or} \quad x = vy$$

Q#4 $(x-y)\,dx + x\,dy = 0$

Sol

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$x\,dy = -(x-y)\,dx$$

$$\frac{dy}{dx} = \frac{-(x-y)}{x}$$

$$\frac{dy}{dx} = \frac{y-x}{x} \quad \text{--- (1)}$$

Put $y = ux$

Diff w.r.t x

$$\frac{dy}{dx} = u\frac{d}{dx}x + x\frac{du}{dx}$$

$$\boxed{\frac{dy}{dx} = u + x\frac{du}{dx}}$$ Put in (1)

$$u + x\frac{du}{dx} = \frac{y-x}{x}$$

$$u + x\frac{du}{dx} = \frac{ux - x}{x}$$

$$u + x\frac{du}{dx} = \frac{\cancel{x}(u-1)}{\cancel{x}}$$

$$x\frac{du}{dx} = \cancel{u} - 1 - \cancel{u}$$

$$x\frac{du}{dx} = -1$$

Seperating Variable

$$x\,du = -dx$$

$$du = -\frac{1}{x}dx$$

Integr

$$\int du = -\int \frac{1}{x}dx$$

$$u = -\ln|x| + c$$

As $y = Ux$

$U = \frac{y}{x}$

$\frac{y}{x} = -\ln x + c$

$\frac{y}{x} + \ln x = c$ Ans.

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$y\,dx - (y-x)\,dy = 0$

Sol $M(x,y)\,dx + N(x,y)\,dy = 0$

$-(y-x)\,dy = -y\,dx$

$\frac{dy}{dx} = \frac{\cancel{+}y}{\cancel{+}(y-x)}$

$\frac{dy}{dx} = \frac{y}{y-x}$ —(i)

Put $y = Ux$

$\frac{dy}{dx} = U + x\frac{dU}{dx}$ Put in (i)

$U + x\frac{dU}{dx} = \frac{Ux}{Ux - x}$

$$x \frac{du}{dx} = \frac{u}{u-1} - u$$

$$x \frac{du}{dx} = \frac{u - u(u-1)}{u-1}$$

$$x \frac{du}{dx} = \frac{u - u^2 + u}{u-1}$$

$$x \frac{du}{dx} = \frac{2u - u^2}{u-1}$$

$$\frac{(u-1)}{2u-u^2} du = \frac{1}{x} dx$$

$$\frac{(u-1)}{u(2-u)} = \frac{1}{x} dx$$

Apply Integral

$$\int \frac{u-1}{u(2-u)} du = \int \frac{1}{x} dx$$

By Partial fraction.

$$\frac{u-1}{u(2-u)} = \frac{A}{u} + \frac{B}{2-u} \quad — (1)$$

xing b. side by $u(2-u)$

$$u-1 = A(2-u) + Bu \quad — (2)$$

Put $u=0$

$$-1 = A(2) \qquad \boxed{A = -\frac{1}{2}}$$

Put $2-U=0$

$U=2$

$1 = B \cdot 2$

$\boxed{B = \frac{1}{2}}$

$= \frac{-1}{2U} + \frac{1}{2(2-U)}$

$$\int \frac{-1}{2U} + \frac{1}{2(2-U)} \, dU = \int \frac{1}{x} \, dx$$

$$= \frac{-1}{2} \int \frac{1}{U} dU + \frac{1}{2} \int \frac{1}{2-U} dU = \int \frac{1}{x} dx$$

$$\frac{-1}{2} \int \frac{1}{U} dU - \frac{1}{2} \int \frac{-1}{2-U} dU = \int \frac{1}{x} dx$$

$$\frac{-1}{2} \ln|U| - \frac{1}{2} \ln|2-U| = \ln|x| + C$$

$$\frac{-1}{2} \left[\ln|U| + \ln|2-U| \right] = \ln|x| + \ln|c|$$

$$-\frac{1}{2} \left[\ln|2U - U^2| \right] = \ln xc$$

$$\ln|2U - U^2|^{-1/2} = \ln xc$$

Antilog

$$(2U - U^2)^{-1/2} = xc$$

replace $y = ux$

$u = \frac{y}{x}$

$$\left(2\frac{y}{x} - \frac{y^2}{x^2}\right)^{-1/2} = xc$$

$$\left(\frac{2xy - y^2}{x^2}\right)^{-1/2} = xc$$

$$\left(\frac{x^2}{2xy - y^2}\right)^{+1/2} = xc$$

$$\frac{(x^{\cancel{2}})^{\cancel{1}/\cancel{2}}}{(2xy - y^2)^{1/2}} = xc$$

$$\frac{\cancel{x}}{\sqrt{2xy - y^2}} = \cancel{x}\, c$$

$$c = \frac{1}{\sqrt{2xy - y^2}}$$

$$\sqrt{2xy - y^2} = \frac{1}{c} \quad \text{Ans:}$$

Q#6

$$\frac{dy}{dx} = \frac{y - x}{x + y} \quad \text{——(i)}$$

Sol

Put $y = ux$

Diff w.r.t. to x

$$\frac{dy}{dx} = U + x\frac{dU}{dx} \quad \text{put in (i)}$$

$$U + x\frac{dU}{dx} = \frac{Ux - x}{x + Ux}$$

$$U + x\frac{dU}{dx} = \frac{\cancel{x}(U-1)}{\cancel{x}(1+U)}$$

$$U + x\frac{dU}{dx} = \frac{U-1}{1+U}$$

$$x\frac{dU}{dx} = \frac{U-1}{1+U} - U$$

$$x\frac{dU}{dx} = \frac{U - 1 - U(1+U)}{1+U}$$

$$x\frac{dU}{dx} = \frac{\cancel{U} - 1 - \cancel{U} - U^2}{1+U}$$

$$x\frac{dU}{dx} = \frac{-(U^2+1)}{(U+1)}$$

$$\frac{(U+1)}{U^2+1}\,dU = -\frac{1}{x}\,dx$$

$$\frac{U}{U^2+1} + \frac{1}{U^2+1}\,dU = -\frac{1}{x}\,dx$$

Apply Integr1

$$\int \frac{U}{U^2+1}\,dU + \int \frac{1}{U^2+1^2}\,dU = -\int \frac{1}{x}\,dx$$

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\frac{1}{2}\int \frac{2U}{U^2+1}\,dU + \int \frac{1}{U^2+1^2}\,dU = -\int \frac{1}{x}\,dx$$

$$\frac{1}{2}\cdot \ln|U^2+1| + \frac{1}{1}\tan^{-1}\left(\frac{U}{1}\right) = -\ln|x| + C$$

$$\frac{1}{2}\ln|U^2+1| + \tan^{-1}U = -\ln|x| + C$$

$$\ln|U^2+1|^{1/2} + \tan^{-1}U + \ln|x| = C$$

$$\ln\sqrt{U^2+1} + \ln x + \tan^{-1}U = C$$

$$\ln x\sqrt{U^2+1} + \tan^{-1}U = C$$

replace $y = Ux$

$U = \frac{y}{x}$

$$\ln x\cdot\sqrt{\frac{y^2+x^2}{x^2}} + \tan^{-1}\left(\frac{y}{x}\right) = C \quad \text{Ans.}$$

Q#7 $\quad \dfrac{dy}{dx} = \dfrac{y^2+yx}{x^2} \quad \text{---(i)}$

Sol

Put $y = Ux$

$$\frac{dy}{dx} = U + x\frac{dU}{dx} \quad \text{Put in (i)}$$

$$U + x\frac{dU}{dx} = \frac{(Ux)^2 + (Ux)x}{x^2}$$

$$U + x\frac{dU}{dx} = \frac{U^2x^2 + Ux^2}{x^2}$$

$$U + x\frac{dU}{dx} = \frac{\cancel{x^2}(U^2+U)}{\cancel{x^2}}$$

$$x\frac{dU}{dx} = U^2 + \cancel{U} - \cancel{U}$$

$$x\frac{dU}{dx} = U^2$$

$$\frac{1}{U^2}dU = \frac{1}{x}dx$$

$$U^{-2}\,dU = \frac{1}{x}dx$$

Apply Integral

$$\int U^{-2}\,dU = \int \frac{1}{x}dx$$

$$\frac{U^{-2+1}}{-2+1} = \ln|x| + C$$

$$\frac{U^{-1}}{-1} = \ln|x| + C$$

$$= \frac{-1}{u} = \ln|x| + c$$

As $y = ux$

$u = \frac{y}{x}$

$$= \frac{-1}{y/x} = \ln|x| + c$$

$$= \frac{-x}{y} = \ln|x| + c \quad \text{Ans.}$$

Q#8 $\frac{dy}{dx} = \frac{y}{x + \sqrt{xy}}$ —(i)

Sol

Put $y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$ put in (i)

$$u + x\frac{du}{dx} = \frac{ux}{x + \sqrt{x \cdot ux}}$$

$$u + x\frac{du}{dx} = \frac{ux}{x + \sqrt{ux^2}}$$

$$u + x\frac{du}{dx} = \frac{ux}{x + x\sqrt{u}}$$

$$u + x\frac{du}{dx} = \frac{u\cancel{x}}{\cancel{x}(1 + \sqrt{u})}$$

$$U + x\frac{dU}{dx} = \frac{U}{1+\sqrt{U}}$$

$$x\frac{dU}{dx} = \frac{U}{1+\sqrt{U}} - \frac{U}{1}$$

$$x\frac{dU}{dx} = \frac{U - U(1+\sqrt{U})}{1+\sqrt{U}}$$

$$x\frac{dU}{dx} = \frac{\cancel{U} - \cancel{U} - U\sqrt{U}}{1+\sqrt{U}}$$

$$x\frac{dU}{dx} = \frac{-U^{3/2}}{1+\sqrt{U}}$$

$$\frac{1+\sqrt{U}}{U^{3/2}}\,dU = -\frac{1}{x}\,dx$$

$$\frac{1}{U^{3/2}} + \frac{\cancel{U^{1/2}}}{U\cdot\cancel{U^{1/2}}}\,dU = -\frac{1}{x}\,dx$$

$$\frac{1}{U^{3/2}} + \frac{1}{U}\,dU = \frac{-1}{x}\,dx$$

$$U^{-3/2} + \frac{1}{U}\,dU = \frac{-1}{x}\,dx$$

Apply Integral

$$\int U^{-3/2}\,dU + \int \frac{1}{U}\,dU = -\int \frac{1}{x}\,dx$$

$$\frac{U^{-3/2+1}}{-3/2+1} + \ln|U| = -\ln|x| + C$$

$$\frac{U^{-1/2}}{-1/2} + \ln|U| = -\ln|x| + C$$

$$-2U^{-1/2} + \ln|U| + \ln|x| = C$$

As $y = Ux$

$U = \frac{y}{x}$

$$-2\left(\frac{y}{x}\right)^{-1/2} + \ln|Ux| = C$$

$$-2\left(\frac{x}{y}\right)^{1/2} + \ln\left|\frac{y}{\cancel{x}} \cdot \cancel{x}\right| = C$$

$$-2\sqrt{\frac{x}{y}} + \ln|y| = C$$

$$\ln|y| - 2\sqrt{\frac{x}{y}} = C$$

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(9)

$$\frac{dy}{dx} = \frac{3x^3 + y^3}{xy^2} \quad \text{—(1)}$$

Sol

Put $y = Ux$

$$\frac{dy}{dx} = U + x\frac{dU}{dx} \quad \text{Put in (i)}$$

$$U + x\frac{dU}{dx} = \frac{3x^3 + (Ux)^3}{x(Ux)^2}$$

$$U + x\frac{dU}{dx} = \frac{3x^3 + U^3x^3}{U^2x^3}$$

$$U + x\frac{dU}{dx} = \frac{\cancel{x^3}(3 + U^3)}{U^2\cancel{x^3}}$$

$$x\frac{dU}{dx} = \frac{3 + U^3}{U^2} - \frac{U}{1}$$

$$x\frac{dU}{dx} = \frac{3 + \cancel{U^3} - \cancel{U^3}}{U^2}$$

$$x\frac{dU}{dx} = \frac{3}{U^2}$$

$$U^2\,dU = \frac{3}{x}\,dx$$

Apply Integral

$$\int U^2\,dU = 3\int \frac{1}{x}\,dx$$

$$\frac{U^3}{3} = 3\ln|x| + C$$

$$U^3 = 9\ln|x| + C$$

As $y = Ux$

$$U = \frac{y}{x}$$

$$\frac{y^3}{x^3} = 9\ln|x| + C \quad \text{Ans}$$

(10) $$\frac{dy}{dx} = \frac{x+3y}{3x+y} \quad \text{—(i)}$$

Sol

Put $y = ux$

$$\frac{dy}{dx} = u + x\frac{du}{dx} \quad \text{Put in (i)}$$

$$u + x\frac{du}{dx} = \frac{x + 3ux}{3x + ux}$$

$$u + x\frac{du}{dx} = \frac{\cancel{x}(1+3u)}{\cancel{x}(3+u)}$$

$$u + x\frac{du}{dx} = \frac{1+3u}{3+u}$$

$$x\frac{du}{dx} = \frac{1+3u}{3+u} - \frac{u}{1}$$

$$x\frac{du}{dx} = \frac{1+3u - u(3+u)}{(3+u)}$$

$$x\frac{du}{dx} = \frac{1+\cancel{3u}-\cancel{3u}-u^2}{(3+u)}$$

$$x\frac{du}{dx} = \frac{1-u^2}{3+u}$$

$$\frac{3+u}{1-u^2}\,du = \frac{1}{x}\,dx$$

$$\frac{3+u}{(1+u)(1-u)}\, du = \frac{1}{x}\, dx$$

First we solve

$$\frac{3+u}{(1+u)(1-u)} = \frac{A}{1+u} + \frac{B}{1-u} \quad \text{— i)}$$

$$3+u = A(1-u) + B(1+u) \quad \text{— ii)}$$

Put $1+u = 0$

$u = -1$

$$3 - 1 = A(1-(-1)) + 0$$

$$2 = A(2)$$

$$\boxed{A = 1}$$

Put $1-u = 0$

$\cancel{-}u = \cancel{-}1$

$u = 1$

$$3+1 = 0 + B(1+1)$$

$$\frac{4}{2} = B \qquad \boxed{B = 2}$$

$$\frac{3+u}{(1+u)(1-u)} = \frac{1}{1+u} + \frac{2}{1-u}$$

$$\left(\frac{1}{1+u}+\frac{2}{1-u}\right)du = \frac{1}{x}\,dx$$

Apply Integral

$$\int \frac{1}{1+u}\,du + 2\int \frac{1}{1-u}\,du = \int \frac{1}{x}\,dx$$

$$\int \frac{1}{1+u} - 2\int \frac{-1}{1-u}\,du = \int \frac{1}{x}\,dx$$

$$\ln|1+u| - 2\ln|1-u| = \ln|x| + c$$

$$\ln|1+u| - \ln|1-u|^2 = \ln|x| + \ln|c|$$

$$\ln\frac{(1+u)}{(1-u)^2} = \ln(xc)$$

Apply Antilog

$$e^{\ln\left(\frac{(1+u)}{(1-u)^2}\right)} = e^{\ln(xc)}$$

$$\frac{1+u}{(1-u)^2} = xc$$

As $y = ux$

$u = \frac{y}{x}$

$$\frac{\frac{x+y}{x}}{\left(\frac{x-y}{x}\right)^2} = xc$$

$$\frac{x+y}{\cancel{x}} \times \frac{x^2}{(x-y)^2} = xc$$

$$\frac{x(x+y)}{(x-y)^2} = xc$$

$$(x+y) = c(x-y)^2$$

(11)

$$\left[y + x\cot\left(\frac{y}{x}\right)\right]dx - x\,dy = 0$$

Sol

$$\cancel{-}x\,dy = \cancel{-}\left[y + x\cot\left(\frac{y}{x}\right)\right]dx$$

$$\frac{dy}{dx} = \frac{y + x\cot y/x}{x}$$

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\cancel{x}\cot(y/x)}{\cancel{x}}$$

$$\frac{dy}{dx} = \frac{y}{x} + \cot\left(y/x\right)$$

Put $y = ux \Rightarrow \frac{y}{x} = u$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$u + x\frac{du}{dx} = u + \cot u$$

$$x\frac{du}{dx} = \cancel{u} + \cot u - \cancel{u}$$

$$x\frac{du}{dx} = \cot u$$

$$\frac{1}{\cot u}du = \frac{1}{x}dx$$

$$\tan u \, du = \frac{1}{x}dx$$

Apply Integral

$$\int \tan u \, du = \int \frac{1}{x}dx$$

$$\ln \sin u = \ln|x| + c$$

$$\ln \sin u = \ln|x| + \ln|c|$$

$$\ln \sin u = \ln(xc)$$

Apply Antilog

$$e^{\ln \sin u} = e^{\ln xc}$$

$$\sin u = xc$$

$$\sin\frac{y}{x} = xc \quad \text{Ans.}$$

(12) $xy^2 \frac{dy}{dx} = y^3 - x^3 \qquad y(1) = 2$

Sol

$$\frac{dy}{dx} = \frac{y^3 - x^3}{xy^2}$$

$$\frac{dy}{dx} = \frac{y^3}{xy^2} - \frac{x^3}{xy^2}$$

$$\frac{dy}{dx} = \frac{y}{x} - \frac{x^2}{y^2} \quad \text{---(i)}$$

Put $y = ux \qquad u = \frac{y}{x}$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$u + x\frac{du}{dx} = \frac{u\cancel{x}}{\cancel{x}} - \frac{\cancel{x^2}}{u^2\cancel{x^2}}$$

$$u + x\frac{du}{dx} = u - \frac{1}{u^2}$$

$$x\frac{du}{dx} = \cancel{u} - \frac{1}{u^2} - \cancel{u}$$

$$x\frac{du}{dx} = \frac{-1}{u^2}$$

$$\cancel{du} = \cancel{x^3}\, dx$$

$$x \frac{du}{dx} = \frac{-1}{u^2}$$

$$u^2 du = \frac{-1}{x} dx$$

Apply Integral

$$\int u^2 du = -\int \frac{1}{x} dx$$

$$\frac{u^3}{3} = -\ln|x| + C \Rightarrow \frac{y^3}{x^3} = -3\ln(x) + C$$

As $y(1) = 2$ $\quad \frac{y^3}{x^3} + 3\ln x = C$

$x = 1, y = 2$

$$\frac{8}{1} + 3\ln(1) = C$$

$$C = 8$$

$$\frac{y^3}{x^3} + 3\ln x = 8 \quad \text{Ans:}$$

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13 $\quad (x^2 + 2y^2) dx = xy\, dy \qquad y(1) = 1$

Sol/

$$\frac{x^2 + 2y^2}{xy} = \frac{dy}{dx}$$

Put $y = ux$

$$\frac{dy}{dx} = u + x\frac{du}{dx} \quad \text{Put In (1)}$$

$$\frac{dy}{dx} = \frac{x^2 + 2y^2}{xy}$$

$$u + x\frac{du}{dx} = \frac{x^2 + 2u^2x^2}{x \cdot ux}$$

$$u + x\frac{du}{dx} = \frac{\cancel{x^2}(1 + 2u^2)}{u\cancel{x^2}}$$

$$x\frac{du}{dx} = \frac{1 + 2u^2}{u} - \frac{u}{1}$$

$$x\frac{du}{dx} = \frac{1 + 2u^2 - u^2}{u}$$

$$x\frac{du}{dx} = \frac{1 + u^2}{u}$$

$$\frac{u}{1+u^2}\,du = \frac{1}{x}\,dx$$

Apply Integral

$$\int \frac{u}{1+u^2}\,du = \int \frac{1}{x}\,dx$$

Xing and ÷ing by 2

$$= \frac{1}{2}\int \frac{2u}{1+u^2}\,du = \int \frac{1}{x}\,dx$$

$$= \frac{1}{2} \cdot \ln|1+u^2| = \ln|x| + c$$

$$\ln\sqrt{1+u^2} = \ln|x| + c$$

$$\ln\sqrt{1+u^2} = \ln|x| + \ln|c|$$

$$\ln\sqrt{1+u^2} = \ln(x\,c)$$

Apply exponential

$$e^{\ln\sqrt{1+u^2}} = e^{\ln(xc)}$$

$$\sqrt{1+u^2} = x\,c$$

As $y = ux$

$u = \frac{y}{x}$

$$\sqrt{\frac{x^2+y^2}{x^2}} = x\,c$$

$$\frac{\sqrt{x^2+y^2}}{x} = x\,c$$

$$\sqrt{x^2+y^2} = x^2\,c \quad \text{(ii)}$$

$y(1) = 1$

$x = 1,\ y = 1$

$$\sqrt{1+1} = 1\,c$$

$$\sqrt{2} = c$$

Put in (ii)

$$\sqrt{x^2+y^2} = \sqrt{2}\,x^2 \quad \text{Ans}$$

14) $2x^2 \frac{dy}{dx} = 3xy + y^2$; $y(1) = -2$

Sol:

$$\frac{dy}{dx} = \frac{3xy + y^2}{2x^2} \quad \text{--- (1)}$$

$$\boxed{\begin{aligned} &\text{Put } y = ux \\ &\frac{dy}{dx} = u + x\frac{du}{dx} \end{aligned}}$$ Put in (1)

$$u + x\frac{du}{dx} = \frac{3x(ux) + (ux)^2}{2x^2}$$

$$u + x\frac{du}{dx} = \frac{3ux^2 + u^2x^2}{2x^2}$$

$$u + x\frac{du}{dx} = \frac{\cancel{x^2}(3u + u^2)}{2\cancel{x^2}}$$

$$x\frac{du}{dx} = \frac{3u + u^2}{2} - u$$

$$x\frac{du}{dx} = \frac{3u + u^2 - 2u}{2}$$

$$x\frac{du}{dx} = \frac{u + u^2}{2}$$

$$\frac{1}{u + u^2}\, du = \frac{1}{2x}\, dx$$

$$\frac{2}{U+U^2}\,dU = \frac{1}{x}\,dx$$

Apply Partial Fraction.

$$\frac{2}{U(1+U)} = \frac{A}{U} + \frac{B}{1+U} \quad \text{--- (i)}$$

Xing b.side by $U(1+U)$

$$2 = A(1+U) + B \cdot U \quad \text{--- (ii)}$$

Put $U = 0$

$$2 = A(1+0) + 0$$

$$\boxed{A = 2}$$

Put $U+1 = 0$

$$U = -1$$

$$2 = 0 + B(-1)$$

$$2 = -B$$

$$\boxed{B = -2}$$

$$\frac{2}{U(1+U)} = \frac{2}{U} - \frac{2}{1+U}$$

$$\frac{2}{U} - \frac{2}{1+U}\,dU = \frac{1}{x}\,dx$$

Apply Integral

$$\int \frac{2}{u}\,du - \int \frac{2}{1+u}\,du = \int \frac{1}{x}\,dx$$

$$2\ln|u| - 2\ln|1+u| = \ln|x| + c$$

$$\ln u^2 - \ln(1+u)^2 = \ln|x| + c$$

$$\ln u^2 - \ln(1+u)^2 = \ln|x| + \ln|c|$$

$$\ln\left(\frac{u^2}{(1+u)^2}\right) = \ln xc$$

Apply exponential

$$e^{\cancel{\ln}\left(\frac{u^2}{(1+u)^2}\right)} = \cancel{e}^{\cancel{\ln}\, xc}$$

$$\frac{u^2}{(1+u)^2} = xc$$

As $y = ux$

$$u = \frac{y}{x}$$

$$\frac{y^2/x^2}{(1+y/x)^2} = xc$$

$$\frac{y^2/x^2}{\left(\frac{x+y}{x}\right)^2} = xc$$

$$= \frac{\frac{y^2}{x^2}}{\frac{(x+y)^2}{x^2}} = xC$$

$$= \frac{y^2}{\cancel{x^2}} \times \frac{\cancel{x^2}}{(x+y)^2} = xC$$

$$\frac{y^2}{(x+y)^2} = xC \quad \text{---} (3)$$

As $y(1) = -2$

$x = 1, \; y = -2$

$$\frac{(-2)^2}{(1-2)^2} = 1C$$

$$\frac{4}{1} = C \qquad \boxed{C = 4}$$

eq (3) becomes

$$\left(\frac{y}{x+y}\right)^2 = 4x \quad \text{Ans.}$$

---

15 $\left(x + y e^{y/x}\right) dx - x e^{y/x} dy = 0$

$y(1) = 0$

Sol $\left(x+y e^{y/x}\right) dx = x e^{y/x} dy$

$$\frac{x+y e^{y/x}}{x e^{y/x}} = \frac{dy}{dx}$$

$$\frac{\cancel{x}}{\cancel{x} e^{y/x}} + \frac{y \cancel{e^{y/x}}}{x \cancel{e^{y/x}}} = \frac{dy}{dx}$$

$$\frac{1}{e^{y/x}} + \frac{y}{x} = \frac{dy}{dx} \quad \text{— (i)}$$

Put $y = ux \Rightarrow \frac{y}{x} = u$

$$\frac{dy}{dx} = u + x\frac{du}{dx} \quad \text{Put in (i)}$$

$$u + x\frac{du}{dx} = \frac{1}{e^{u}} + u$$

$$x\frac{du}{dx} = e^{-u} + \cancel{u} - \cancel{u}$$

$$x\frac{du}{dx} = e^{-u}$$

$$\frac{1}{e^{-u}} du = \frac{1}{x} dx$$

$$e^{u} du = \frac{1}{x} dx$$

App'g Integral.

$$\int e^{u}\,du = \int \frac{1}{x}\,dx$$

$$e^{u} = \ln|x| + c$$

$$e^{y/x} = \ln|x| + c \quad \text{---} (2)$$

As $y(1) = 0$

$x = 1, \; y = 0$

$$e^{0/1} = \ln|1| + c$$

$$e^{0} = 0 + c$$

$$c = 1$$

Put in (ii)

$$e^{y/x} = \ln|x| + 1$$

$$e^{y/x} - 1 = \ln|x| \qquad \text{Ans.}$$

Complete.