Exercise 4.2 (Solutions)
Mathematics 12 (NBF)
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Exc 4.2

Solve the differential eqs by Seperating the Variables.

i) $\frac{dy}{dx} = -\frac{1}{e^{3x}}$

Sol

$$\frac{dy}{dx} = -\frac{1}{e^{3x}}$$

$$dy = -\frac{1}{e^{3x}}dx$$

$$dy = -e^{-3x}dx$$

Apply Integral.

$$\int 1\cdot dy = -\int e^{-3x}dx$$

$$y = -\frac{e^{-3x}}{-3} + c$$

$$y = \frac{e^{-3x}}{3} + c$$

$$3y - e^{-3x} = c \quad \text{Ans.}$$

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② $x\frac{dy}{dx} = 4y$

Sol

$x \, dy = 4y \, dx$

$\frac{dy}{y} = \frac{4}{x} dx$

Integrate b·sides

$\int \frac{1}{y} dy = 4 \int \frac{1}{x} dx$

$Ln y = 4 Ln x + C$

$Ln y = Ln x^4 + C$

$Ln y = Ln x^4 + Ln c \qquad \because C = Ln c$

$Ln y = Ln c x^4$

Apply exponential on b·side

$e^{Ln y} = e^{Ln c x^4}$

$y = c x^4$ Ans.

③ $\frac{dy}{dx} = \frac{y^3}{x^2}$

Sol $\frac{1}{y^3} dy = \frac{1}{x^2} dx$

Integrate b.sides

$$\int \frac{1}{y^3} dy = \int \frac{1}{x^2} dx$$

$$\int y^{-3} dy = \int x^{-2} dx$$

$$\int \frac{y^{-3+1}}{-3+1} = \frac{x^{-2+1}}{-2+1} + C$$

$$\frac{y^{-2}}{-2} = \frac{x^{-1}}{-1} + C$$

$$\frac{-1}{2y^2} = -\frac{1}{x} + C$$

$$\frac{1}{2y^2} - \frac{1}{x} + C = 0$$

Q#4 $\frac{dy}{dx} = e^{2x+3y}$

Sol

$$\frac{dy}{dx} = e^{2x} \cdot e^{3y}$$

$$\frac{1}{e^{3y}} dy = e^{2x} dx$$

$$e^{-3y} dy = e^{2x} dx$$

Integrate b.sides.

$$\int e^{-3y}\,dy = \int e^{2x}\,dx$$

$$\frac{e^{-3y}}{-3} = \frac{e^{2x}}{2} + c$$

$$\frac{1}{2}e^{2x} + \frac{1}{3}e^{-3y} + C = 0$$

$$\frac{1}{2}e^{2x}\ \frac{1}{3e^{3y}} + c = 0$$

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⑤ $$\frac{dy}{dx} = \frac{x^2y^2}{1+x}$$

$$\begin{array}{r|l} & x - 1 \\ \hline 1+x & \cancel{x^2} \\ & \underline{\underset{-}{+}\cancel{x^2}\ \underset{-}{+}x} \\ & -\cancel{x} \\ & \underline{\underset{+}{-}\cancel{x}\ \underset{+}{-}1} \\ & 1 \end{array}$$

Sol

$$\frac{1}{y^2}\,dy = \frac{x^2}{1+x}\,dx$$

$$\frac{1}{y^2}\,dy = (x-1) + \frac{1}{1+x}\,dx$$

Apply Integrate

$$\int \frac{1}{y^2}\,dy = \int (x-1) + \frac{1}{1+x}\,dx$$

$$\int \frac{1}{y^2}\,dy = \int x\,dx - \int 1\,dx + \int \frac{1}{1+x}\,dx$$

$$\int y^{-2}\,dy = \int x\,dx - \int 1\,dx + \int \frac{1}{1+x}\,dx$$

$$\frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} - x + \ln|1+x| + c$$

$$\frac{y^{-1}}{-1} = \frac{x^2}{2} - x + \ln|1+x| + C$$

$$\frac{x^2}{2} + \frac{1}{y} - x + \ln|1+x| + C = 0$$

⑥ $$2y(x+1)\,dy = x\,dx$$

Sol $$2y\,dy = \frac{x}{x+1}\,dx$$

$$\begin{array}{r l} & 1 \\ x+1 \big) & \cancel{x} \\ & \underline{-\cancel{x}+1} \\ & -1 \end{array}$$

$$2y\,dy = 1 - \frac{1}{x+1}\,dx$$

Apply Integral

$$2\int y\,dy = \int 1\,dx - \int \frac{1}{x+1}\,dx$$

$$\cancel{2}\cdot\frac{y^2}{\cancel{2}} = x - \ln|x+1| + C$$

$$y^2 = x - \ln|1+x| + C$$

⑦ $$\frac{dy}{dx} + y^2 \sin x = 0$$

Sol $$\frac{dy}{dx} = -y^2 \sin x$$

$$\frac{1}{y^2} dy = -\sin x \, dx$$

$$y^{-2} dy = -\sin x \, dx$$

Apply Integral

$$\int y^{-2} dy = -\int \sin x \, dx$$

$$\frac{y^{-2+1}}{-2+1} = -\cos x + c$$

$$\frac{y^{-1}}{-1} = -\cos x + c$$

$$-\frac{1}{y} = -\cos x + c \quad \text{Ans.}$$

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⑧ $(\sin x + \cos x) dx = \cot y \cos x \, dy$

Sol

$$\frac{\sin x + \cos x}{\cos x} dx = \frac{\cos y}{\sin y} dy$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x} dx = \frac{\cos y}{\sin y} dy$$

$$(\tan x + 1) dx = \frac{\cos y}{\sin y} dy$$

Apply Integral on b. sides

$$\int (\tan x + 1)\,dx = \int \frac{\cos y}{\sin y}\,dy$$

$$\int \tan x\,dx + \int 1\,dx = \int \frac{\cos y}{\sin y}\,dy$$

$$\ln|\sec x| + x = \ln|\sin y| + C$$ Ans.

Solve the initial value Problems.

⑨

$$\frac{dy}{dx} = \cos x \qquad y(0) = 1$$

Sol

$$dy = \cos x\,dx$$

Integrate

$$\int 1\,dy = \int \cos x\,dx$$

$$y = \sin x + C \quad \text{—(i)}$$

Given $y(0) = 1$

$x = 0, \quad y = 1$

$$1 = \sin(0) + C$$

$$1 = 0 + C \qquad C = 1$$

Put in (i)

$$y = \sin x + 1$$ Ans.

⑩ $2\frac{dy}{dx} = 4x e^{-x}$; $y(0)=2$

Sol

$$\frac{dy}{dx} = 2x e^{-x}$$

$$dy = 2x e^{-x} dx$$

Integrate b. sides

$$\int 1\, dy = 2\int \underset{I}{x}\,\underset{II}{e^{-x}}\, dx$$

$$y = 2\left[x\int e^{-x}dx - \int\left(\int e^{-x}dx\, \frac{d}{dx}x\right)dx\right.$$

$$y = 2\left[x\frac{e^{-x}}{-1} - \int \frac{e^{-x}}{-1}\cdot 1\, dx\right]$$

$$y = 2\left[-\frac{x}{e^{x}} + \frac{e^{-x}}{-1}\,\cancel{d}\right] + c$$

$$y = -2xe^{-x} - 2e^{-x} + c \quad — ①$$

Given $y(0) = 2$

$x=0,\ y=2$

$$2 = -2\,0\,e^{0} - 2e^{0} + c$$

$$2 = 0 - 2 + c$$

$c = 4$ Put In (1)

$$y + 2xe^{-x} + 2e^{-x} - 4 = 0$$

⑪ $\frac{dy}{dx} + \left(\frac{1+x}{x}\right) y = 0$ $y(1) = 1$

Sol $\frac{dy}{dx} = -\left(\frac{1+x}{x}\right) y$

$$\frac{1}{y} dy = -\left(\frac{1+x}{x}\right) dx$$

Integrate b.sides

$$\int \frac{1}{y} dy = -\int \frac{1}{x} dx - \int 1\, dx$$

$$\ln|y| = -\ln|x| - x + C$$

$$\ln|y| + \ln|x| = -x + C$$

$$\ln(xy) = -x + C \quad -(1)$$

Given

$$y(1) = 1$$

$$x = 1, \; y = 1$$

$$\ln(1 \cdot 1) = -1 + C$$

$$0 = -1 + C$$

$$\boxed{C = 1}$$ put in (i)

$$\ln(xy) = -x + 1$$

$$\ln xy + x - 1 = 0$$

⑫ $\frac{dy}{dx} + y \tan 2x = 0 \qquad y(0) = 2$

Sol

$$\frac{dy}{dx} = -y \tan 2x$$

$$\frac{1}{y} dy = -\tan 2x \, dx$$

Apply Integral

$$\int \frac{1}{y} dy = -\int \tan 2x \, dx$$

$$\ln y = -\frac{-\ln|\cos 2x|}{2} + C$$

$$\ln y = \frac{\ln|\cos 2x|}{2} + C$$

$$\ln y = \frac{1}{2} \ln|\cos 2x| + \ln C$$

$$\ln y = \ln\sqrt{\cos 2x} + \ln C$$

$$\ln y = \ln\sqrt{\cos 2x} \cdot C$$

Apply exponential

$$e^{\ln y} = e^{\ln\sqrt{\cos 2x}\; C}$$

$$y = C\sqrt{\cos 2x} \quad \text{—} ①$$

Given $y(0) = 2$

$x = 0, \; y = 2$

$$2 = c\sqrt{\cos 2(0)}$$

$$2 = c\sqrt{1}$$

$$2 = c$$

$$y = 2\sqrt{\cos 2x} \quad \text{Ans.}$$

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⑬

$$\frac{dy}{dx} = y^2+4 \qquad y(0) = -2$$

Sol/

$$\frac{dy}{y^2+4} = dx$$

$$\int \frac{1}{y^2+2^2}\,dy = \int 1\,dx$$

$$\cancel{\frac{d}{dx}\left(\tan^{-1}\frac{x}{a}\right)} = \cancel{\frac{1}{x^2+a^2}} \times \cancel{a}$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\frac{1}{2}\tan^{-1}\frac{y}{2} = x + c \quad \text{—— ①}$$

Given $y(0) = 2$

$x = 0,\ y = -2$

$$\frac{1}{2}\tan^{-1}\frac{-2}{2} = 0 + C$$

$$C = \frac{1}{2}\tan^{-1}(-1)$$

$$C = \frac{1}{2}\left(\frac{-\pi}{4}\right)$$

$$\boxed{C = \frac{-\pi}{8}}$$

$$C = \frac{1}{2}\left(\frac{-\pi}{4} + \pi\right)$$

$$C = \frac{1}{2}\left(\frac{3\pi}{4}\right)$$

$$\boxed{C = \frac{3\pi}{8}}$$

both are Correct

Put in (i)

$$\frac{1}{2}\tan^{-1}\frac{y}{2} = x + \frac{3\pi}{8}$$

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⑭ $(1-x)\,dy + y^{-1}\,dx = 0$ $\quad y(0) = 2$

Sol

$$(1-x)\,dy = -y^{-1}\,dx$$

$$\frac{1}{y^{-1}}\,dy = \frac{-1}{1-x}\,dx$$

Apply Integrl

$$\int y\,dy = \int \frac{-1}{1-x}\,dx$$

$$\int y\,dy = \ln|1-x| + C$$

$$\frac{y^2}{2} = Ln|1-x| + c \quad —①$$

Given $y(0) = 2$

$x = 0, \quad y = 2$

$$\frac{4}{2} = Ln|1-0| + c$$

$$2 = c$$

Put In (1)

$$\frac{y^2}{2} = Ln|1-x| + 2 \qquad Ans$$

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⑮ $2(y-1)\,dy = (3x^2+4x+2)\,dx \qquad y(0)=-1$

Sol Integrate

$$2\int(y-1)\,dy = 3\int x^2\,dx + 4\int x\,dx + 2\int 1\,dx$$

$$2\left[\frac{y^2}{2} - y\right] = 3\frac{x^3}{3} + 4\frac{x^2}{2} + 2x + c$$

$$y^2 - 2y = x^3 + 2x^2 + 2x + C \quad —(1)$$

Given $y(0) = -1$

$x = 0, \quad y = -1$

$(-1)^2 - 2(-1) = (0)^3 + 2(0)^2 + 2(0) + C$

$1 + 2 = C$

$\boxed{C = 3}$

Put in (1)

$y^2 - 2y = x^3 + 2x^2 + 2x + 3$ An.