Exercise 3.8 (Solutions)
Mathematics 12 (NBF)
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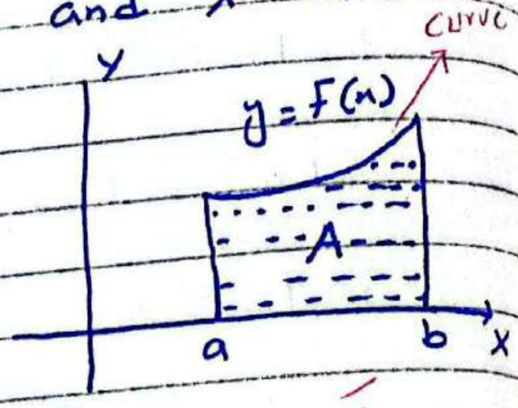
Exc 3.8

Area between a curve and X-axis



$$\text{Area} = \int_a^b y\,dx$$

a = Lower Limit
b = Upper Limit

جب ایک curve ہو گی تو یہ formula اپلائی ہو گا۔

Area between Curves

Two Functions = $F(x)$, $g(x)$

- $F(x)$ is greater than $g(x)$ on $[a, b]$

$$A = \int_a^b [F(x) - g(x)]\,dx$$

upper curve − Lower curve

جب دو curves ہوں گی تو یہ والا فارمولا اپلائی ہو گا۔

Volume of Solids of Revolution
or
Disc Method

Volume of Disc = $V = \pi \int_a^b y^2\,dx$ OR

$$V = \pi \int_a^b [F(x)]^2\,dx$$

Q#1: Find the area of region bounded by the curve $y = x^2$, the x-axis, Lines $x = 1$ and $x = 3$.

Sol Given

$y = x^2$, $x = 1$ and $x = 2$

We Know

$$\boxed{\text{Area} = \int_a^b y \, dx}$$

$$= \int_1^2 x^2 \, dx \Rightarrow \frac{x^3}{3} \Big|_1^2$$

$$= \frac{1}{3}\left[(2)^3 - (1)^3\right]$$

$$= \frac{1}{3}\left[8 - 1\right] = \frac{7}{3} \text{ square Unit}$$

Q #2

Given $y = \sqrt{6x+4}$, $x = 0$ and $x = 2$

We Know $\boxed{\text{Area} = \int_a^b y \, dx}$

$$= \int_0^2 (6x+4)^{1/2} \, dx$$

Xing and ÷ ing by 6

$$= \frac{1}{6} \int_0^2 (6x+4)^{1/2} \cdot 6 \, dx$$

(Rule I)

$$= \frac{1}{6} \left. \frac{(6x+4)^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right|_0^2$$

$$= \frac{1}{6} \left. \frac{(6x+4)^{3/2}}{3/2} \right|_0^2$$

$$= \frac{1}{6} \cdot \frac{2}{3} \left. (6x+4)^{3/2} \right|_0^2$$

$$= \frac{1}{9} \left[(6(2)+4)^{3/2} - (6(0)+4)^{3/2} \right]$$

$$= \frac{1}{9} \left[(4^{\cancel{2}})^{3/\cancel{2}} - (2^{\cancel{2}})^{3/\cancel{2}} \right]$$

$$= \frac{1}{9} \left[4^3 - 2^3 \right]$$

$$= \frac{1}{9} \left[64 - 8 \right] = \frac{56}{9} \text{ Square Unit.}$$

Q#3 Find the area of region bounded by the curve $y^2 = 4x$ and Line $x = 3$.

Sol Note: IF Limit is not given then Put $y = 0$ to Find 2nd Limit.

Given $y^2 = 4x$ —(1), $x = 3$

$$\sqrt{y^2} = \sqrt{4x}$$

$$y = \pm 2\sqrt{x}$$

$y = 2\sqrt{x}$ $\qquad$ $y = -2\sqrt{x}$

(Here two curves)

To find Limit Put $y = 0$ in (1)

$$0 = 4x$$

$$x = 0$$

So $x = 0$, $x = 3$ are Limits.

$$\text{Area} = A = \int_a^b (Y_{\text{upper curve}} - Y_{\text{Lower curve}})\, dx$$

$y = 2\sqrt{x}$
$x = 0, 3$
$y = 2\sqrt{1} = 2$ (Upper)
$y = -2\sqrt{x}$
$x = 0, 3$
$y = -2\sqrt{1} = -2$ Lower

$$A = \int_0^3 [2\sqrt{x} - (-2\sqrt{x})]\, dx$$

$$A = \int_0^3 (2\sqrt{x} + 2\sqrt{x})\, dx$$

$$A = 4\int_0^3 x^{1/2}\, dx = 4\left.\frac{x^{1/2+1}}{1/2+1}\right|_0^3$$

$$= 4 \cdot \left.\frac{x^{3/2}}{3/2}\right|_0^3 = \frac{4}{1}\cdot\frac{2}{3}\left. x^{3/2}\right|_0^3$$

$$= \frac{8}{3} x^{3/2} \Big|_0^3$$

$$= \frac{8}{3} \left[(3)^{3/2} - (0)^{3/2} \right]$$

$$= \frac{8}{3} \cdot 3^{3/2}$$

$$= 8 \cdot 3^{\frac{3}{2}-1}$$

$$= 8\sqrt{3} \text{ square unit.}$$

Q #4 In the Figure, a sketch of the Function $y = \frac{1}{2}(0.2x^2 + x)$ is shown Find

Sol Given $y = \frac{1}{2}(0.2x^2 + x)$

Find (i) The area of region A

$x = 1, \quad x = 3$

$$\text{Area} = \int_a^b y \, dx$$

$$A = \int_1^3 \frac{1}{2}(0.2x^2 + x)\, dx$$

$$A = \frac{1}{2}\int_1^3 (0.2x^2 + x)\, dx$$

$$A = \frac{1}{2}\left[0.2\int_1^3 x^2\,dx + \int_1^3 x\,dx\right]$$

$$A = \frac{1}{2}\left[0.2 \cdot \frac{x^3}{3}\Big|_1^3 + \frac{x^2}{2}\Big|_1^3\right]$$

$$A = \frac{1}{10} \cdot \frac{x^3}{3}\Big|_1^3 + \frac{x^2}{4}\Big|_1^3$$

$$A = \frac{x^3}{30}\Big|_1^3 + \frac{x^2}{4}\Big|_1^3$$

$$A = \frac{1}{30}\left[3^3 - 1^3\right] + \frac{1}{4}\left[3^2 - 1^2\right]$$

$$A = \frac{1}{30}(26) + \frac{1}{4}(8)$$

$$A = \frac{26}{30} + \frac{8}{4} = \frac{13}{15} + \frac{2}{1}$$

$$A = \frac{43}{15} \text{ Square unit.}$$

(ii) The area of region B.

$x = 3 \quad , \quad x = 4$

$$\boxed{\text{Area} = \int_a^b y\,dx}$$

$$= \int_3^4 \frac{1}{2}(0.2x^2 + x)\,dx$$

• Solve in Part i

& we write

direct answer.

$$A = \frac{x^3}{30}\Big|_3^4 + \frac{x^2}{4}\Big|_3^4$$

$$A = \frac{1}{30}\left[4^3 - 3^3\right] + \frac{1}{4}\left[4^2 - 3^2\right]$$

$$A = \frac{1}{30}\left[64 - 27\right] + \frac{1}{4}\left[16 - 9\right]$$

$$A = \frac{1}{30}(37) + \frac{1}{4}(7)$$

$$A = \frac{179}{60} \text{ Square unit.}$$

(iii) Area of the region from $x=1$ to $x=4$

$$\boxed{\text{Area} = \int_a^b y \, dx}$$

$$= \int_1^4 \frac{1}{2}(0.2x^2 + x)\, dx$$

• Solve in Part (i) we write direct answer

$$A = \left.\frac{x^3}{30}\right|_1^4 + \left.\frac{x^2}{4}\right|_1^4$$

$$A = \frac{1}{30}\left[4^3 - 1^3\right] + \frac{1}{4}\left[4^2 - 1^2\right]$$

$$A = \frac{1}{30}\left[64 - 1\right] + \frac{1}{4}\left[16 - 1\right]$$

$A = \frac{63}{30} + \frac{15}{4}$

$A = \frac{117}{20}$ square unit.

(iv) Area of the region from $x=-1$ to $x=-4$.

$$\boxed{Area = \int_a^b y\,dx}$$

$$= \int_{-1}^{-4} \frac{1}{2}(0.2x^2+x)\,dx$$

(Change the Limit Apply (-) sign)

$$= -\int_{-4}^{-1} \frac{1}{2}(0.2x^2+x)\,dx$$

Solve in Part (i) we write direct answer.

$$A = -\left[\frac{x^3}{30}\Big|_{-4}^{-1} + \frac{x^2}{4}\Big|_{-4}^{-1}\right]$$

$$A = -\left(\left[\frac{(-1)^3}{30} - \frac{(-4)^3}{30}\right] + \left[\frac{(-1)^2}{4} - \frac{(-4)^2}{4}\right]\right)$$

$$A = -\left(\left[\frac{-1}{30} + \frac{64}{30}\right] + \left[\frac{1}{4} - \frac{16}{4}\right]\right)$$

$$A = -\left(\frac{63}{30} - \frac{15}{4}\right) = \frac{-63}{30} + \frac{15}{4}$$

$A = \frac{33}{20}$ Square unit.

Q #5 Find the area bounded by the graph

i) $y = 1 + \cos x \quad , \quad [0, 3\pi]$.

Sol

As we know

$$\boxed{\text{Area} = \int_a^b y \, dx}$$

$$= \int_0^{3\pi} (1 + \cos x) \, dx$$

$$= \int_0^{3\pi} 1 \, dx + \int_0^{3\pi} \cos x \, dx$$

$$= x \Big|_0^{3\pi} + \sin x \Big|_0^{3\pi}$$

$$= (3\pi - 0) + (\sin 3\pi - \sin 0)$$

$$= 3\pi + (0 - 0)$$

$$= 3\pi$$

(ii) $y = -1 + \sin x$ $\left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$

Sol As we know

$$\boxed{Area = \int_a^b y\,dx}$$

$$= \int_{-3\pi/2}^{3\pi/2} (-1 + \sin x)\,dx$$

$$= -\int_{-3\pi/2}^{3\pi/2} 1\,dx + \int_{-3\pi/2}^{3\pi/2} \sin x\,dx$$

$$= -x\Big|_{-3\pi/2}^{3\pi/2} - \cos x\Big|_{-3\pi/2}^{3\pi/2}$$

$$= -\left[\frac{3\pi}{2} - \left(-\frac{3\pi}{2}\right)\right] - \left[\cos\frac{3\pi}{2} - \cos -\frac{3\pi}{2}\right]$$

$$= -\left[\frac{3\pi}{2} + \frac{3\pi}{2}\right] - \left[0 - 0\right]$$

$$\boxed{A = -3\pi}$$

Area cannot be negative However negative sign shows that graph is below the X-axis.

Q#6 Find the area of the region bounded by the graphs of $y = x$, $y = -2x$ and $x = 3$.

Sol

Given

$y = x$ —(i) $\qquad$ $y = -2x$ —(ii)

To find Limit Put $y = 0$ in any eq (i) or (ii)

$\boxed{x = 0}$ $\qquad$ $-2x = 0$

$\boxed{x = 0}$

So Limits are 0 and 3.

Now

$$\text{Area} = \int_a^b (y_{\text{upper curve}} - y_{\text{Lower curve}})\, dx$$

$$\text{Area} = \int_0^3 [x - (-2x)]\, dx$$

$$= \int_0^3 (x + 2x)\, dx$$

$$= \int_0^3 x\, dx + 2\int_0^3 x\, dx$$

$$= \frac{x^2}{2}\Big|_0^3 + \cancel{2}\,\frac{x^2}{\cancel{2}}\Big|_0^3$$

Checking
- Put $x = 1$
  $y = 1$ +ve
  $y = x$ upper curve
- Put $x = 1$
  $y = -2$ -ve
  $y = -2x$ Lower curve

$$= \left[\frac{(3)^2}{2} - \frac{(0)^2}{2}\right] + \left[(3)^2 - (0)^2\right]$$

$$= \frac{9}{2} + \frac{9}{1}$$

$$= \frac{27}{2} \text{ Sq unit.}$$

Q#7 Find the area of the region bounded above by $y = x+6$, bounded below by $y = x^2$ bounded on the sides by the Lines $x = 0$ and $x = 2$.

Sol Given

$y = x + 6$ $\quad\quad$ $y = x^2$

$$\text{Area} = \int_a^b \left(Y_{\text{upper curve}} - Y_{\text{Lower curve}}\right) dx$$

Checking:

Put $x = 1$

$y = 1+6$

$y = 7$ true upper curve

$y = x^2$

$y = (1)^2$

$y = 1$ Lower curve

$$\text{Area} = \int_0^2 (x + 6 - x^2)\, dx$$

$$= \int_0^2 x\, dx + 6\int_0^2 1\, dx - \int_0^2 x^2\, dx$$

$$= \frac{x^2}{2}\Big|_0^2 + 6x\Big|_0^2 - \frac{x^3}{3}\Big|_0^2$$

$$= \left[\left(\frac{4}{2} - \frac{0}{2}\right) + (12 - 0) - \left(\frac{8}{3} - \frac{0}{3}\right)\right]$$

$$= \left[2 + 12 - \frac{8}{3}\right] = \left[14 - \frac{8}{3}\right] = \left[\frac{34}{3}\right] = \frac{34}{3} \text{ Sq unit}$$

8. Find the area bounded by the curve $y = x^3 + 1$, the x-axis and the line $x = 1$

Sol Given $y = x^3 + 1$ $\qquad$ $x = 1$

For 2nd Limit put $y = 0$

$$x^3 + 1 = 0 \implies x^3 + 1^3 = 0$$

$$(x+1)(x^2 - x + 1) = 0$$

$x + 1 = 0$	$x^2 - x + 1 = 0$
$x = -1$	This eq gives imaginary roots so these are neglected.

So $x = -1$, $x = 1$ are Limits.

$$\text{Area} = A = \int_a^b y\,dx$$

$$A = \int_{-1}^{1} (x^3 + 1)\,dx$$

$$= \int_{-1}^{1} x^3\,dx + \int_{-1}^{1} 1\,dx$$

$$= \frac{x^4}{4}\Big|_{-1}^{1} + x\Big|_{-1}^{1}$$

$$= \left[\frac{1}{4} - \frac{1}{4}\right] + \left[1 - (-1)\right]$$

$$= 0 + 2$$

$$= 2 \text{ sq unit.}$$

Q#9

Given

$x = y^2$ $\quad$ $y = x - 2$

$\boxed{\text{integrating w.r.to } y}$

$x = y^2$ —① , $\quad x = y + 2$ —②

for Limits Find Point of intersection

Solving (i) and (ii)

$$y^2 = y + 2$$

$$y^2 - y - 2 = 0$$

$$y^2 - 2y + y - 2 = 0$$

$$y(y-2) + 1(y-2) = 0$$

$$(y+1)(y-2) = 0$$

$y + 1 = 0$ $\quad$ $y - 2 = 0$

$y = -1$ $\quad$ $y = 2$

So Limits are $y = -1, 2$

Checking upper and Lower

$y = -1, 2$

- $x = y^2$

Put $y = 0$

$\boxed{x = 0}$ Lower

- $x = y + 2$

$x = 0 + 2$

$\boxed{x = 2}$ upper

$$\text{Area} = A = \int_a^b (X_{\text{upper Curve}} - X_{\text{Lower Curve}})\, dy$$

$$= \int_{-1}^{2} (y + 2 - y^2)\, dy$$

$$= \int_{-1}^{2} y\, dy + 2\int_{-1}^{2} 1\, dy - \int_{-1}^{2} y^2\, dy$$

$$= \frac{y^2}{2}\Big|_{-1}^{2} + 2y\Big|_{-1}^{2} - \frac{y^3}{3}\Big|_{-1}^{2}$$

$$= \left[\frac{4}{2} - \frac{1}{2}\right] + \left[4 + 2\right] - \left[\frac{8}{3} + \frac{1}{3}\right]$$

$$= \left[\frac{3}{2}\right] + \left[6\right] - \left[\frac{9}{3}\right]$$

$$= \frac{3}{2} + 6 - 3$$

$$= \frac{3}{2} + \frac{3}{1}$$

$$= \frac{9}{2}$$

$A = 9/2$ sq unit.

(10) Find the volume of the solid that is obtained — — —
— — — — — —

Sol. Given $y = \sqrt{x}$, $[1, 4]$

Volume = $V = \pi \int_a^b y^2 \, dx$

$V = \pi \int_1^4 (\sqrt{x})^2 \, dx$

$= \pi \int_1^4 x \, dx$

$= \pi \cdot \frac{x^2}{2} \Big|_1^4$

$= \frac{\pi}{2} \left[(4)^2 - (1)^2 \right]$

$= \frac{\pi}{2} \left[16 - 1 \right]$

$V = \frac{15\pi}{2}$ cubic unit.

(11) Find the volume of the solid that results when the shaded region is revolved — — —

Sol

(i) Given $y = \sqrt{3-x}$ $x = -1$, $x = 3$
(About X-axis)

$$\text{Volume} = V = \pi \int_a^b y^2 \, dx$$

$$= \pi \int_{-1}^{3} \left(\sqrt{3-x}\right)^2 dx$$

$$= \pi \int_{-1}^{3} (3-x) \, dx$$

$$= \pi \left[\int_{-1}^{3} 3 \, dx - \int_{-1}^{3} x \, dx \right]$$

$$= \pi \left[3 \int_{-1}^{3} 1 \, dx - \int_{-1}^{3} x \, dx \right]$$

$$= \pi \left[3x \Big|_{-1}^{3} - \frac{x^2}{2} \Big|_{-1}^{3} \right]$$

$$= \pi \left[(9+3) - \left(\frac{9}{2} - \frac{1}{2} \right) \right]$$

$$= \pi [12 - 4]$$

$$= 8\pi \text{ cubic unit.}$$

(ii) Given

$y = 3 - 2x$ about y-axis

For y-axis put $x = 0$

$y = 3 - 2(0)$

$\boxed{y = 3}$

So Limits are $y = 0$, $y = 3$

Now $y = 3 - 2x$

$\frac{y - 3}{-2} = x$ $\boxed{x = \frac{3 - y}{2}}$

Volume = $V = \pi \int_a^b x^2 \, dy$

$= \pi \int_0^3 \left(\frac{3 - y}{2}\right)^2 dy$

$= \frac{\pi}{4} \int_0^3 (3 - y)^2 \, dy$

x ing and ÷ ing by -1

$= \frac{\pi}{-4} \int_0^3 (3 - y)^2 \cdot -1 \, dy$

$= \frac{-\pi}{4} \cdot \left. \frac{(3 - y)^{2+1}}{2+1} \right|_0^3$

$$= \frac{-\pi}{12} (3-y)^3 \Big|_0^3$$

$$= \frac{-\pi}{12}\left[(3-3)^3 - (3-0)^3\right]$$

$$= \frac{-\pi}{12}\left[-27\right]$$

$$= \frac{27\pi}{12}$$

$$V = \frac{9\pi}{4} \text{ Cubic unit.}$$

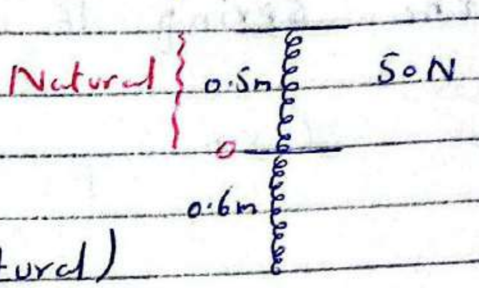
Work:

Let $f(x)$ be a continuous function or continuous force acting at a point in the interval $[a,b]$ then the work W by the force on moving object from 'a' to 'b' is

$$W = F \cdot d$$

$$W = \int_a^b f(x)\,dx \qquad [a,b]$$

(13) It takes a force of 50N to stretch a spring of 0.5m. Find the work done in stretching spring 0.6m beyond its natural Length.



Sol Given $F = 50\,N$

distance $= x = 0.5\,m$ (Natural)

According to Hook's Law

$$F \propto x$$

$$F = Kx \quad \text{---(1)}, \qquad K = \text{Spring Constant}$$

$$50 = K\, 0.5$$

$$K = \frac{50}{0.5} = 100 \quad \text{Put in (1)}$$

$$F = 100x \quad , \qquad x = 0, \qquad x = 0.6$$

Now

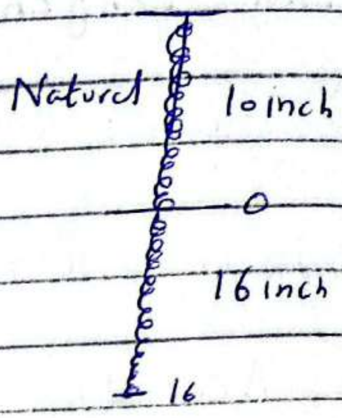
$$W = \int_a^b F(x)\,dx$$

$$= \int_0^{0.6} 100x\,dx$$

$$= 100 \cdot \frac{x^2}{2}\Big|_0^{0.6} \qquad = 50\,(0.36)$$

$$= 50\left[(0.6)^2 - (0)^2\right] \qquad = 18\,J \quad \text{Ans.}$$

(14) A force $F = \frac{3}{2}x$ lb is needed to stretch a 10 inch spring an additional x inch. Find the work done in stretching the spring 16 inch.



Sol:

Given

$$F = \frac{3}{2} x$$

$$x = 0 \text{ to } 16$$

We know

$$W = \int_a^b F(x)\,dx$$

$$W = \int_0^{16} \frac{3x}{2}\,dx$$

$$W = \frac{3}{2}\int_0^{16} x\,dx$$

$$= \frac{3}{2}\left.\frac{x^2}{2}\right|_0^{16}$$

$$= \frac{3}{4}\left[(16)^2 - (0)^2\right]$$

$$= \frac{3}{4}\cdot 256$$

$$W = 192 \text{ J} \quad \text{Ans.}$$

Consumer Surplus

$$CS = \int_0^{x_0} [D(x) - P_0] \, dx$$

Producer Surplus

$$PS = \int_0^{x_0} [P_0 - S(x)] \, dx$$

Where

$D(x)$ = Demand Function.
$S(x)$ = Supply Function.
x_0 = equilibrium point
P_0 = equilibrium Price.

Q#15 Find the Consumer and producer surpluses, when

i) $S(x) = 24$ (Supply Function), $D(x) = 100 - 2x$ (Demand function)

$CS = ?$ $PS = ?$

• To find equilibrium point (جہاں پہ دونوں کی values equal ہو جائیں۔)

$$S(x) = D(x)$$
$$24 = 100 - 2x$$
$$-100 + 24 = -2x$$
$$-76 = -2x$$
$$x = 38 \quad \text{or} \quad \boxed{x_0 = 38}$$

equilibrium Point

Now we find equilibrium Price:

$S(x) = 24$

$\boxed{P_0 = 24}$

$D(x) = 100 - 2(38)$

$P_0 = 100 - 76$

$\boxed{P_0 = 24}$

$x_0 = 38,\quad P_0 = 24$

Now

$$CS = \int_0^{x_0} [D(x) - P_0]\,dx$$

$$= \int_0^{38} (100 - 2x - 24)\,dx$$

$$= \int_0^{38} (76 - 2x)\,dx$$

$$= 76\int_0^{38} 1\,dx - 2\int_0^{38} x\,dx$$

$$= 76x\Big|_0^{38} - 2\cdot\frac{x^2}{2}\Big|_0^{38}$$

$$= 76(38) - (38)^2$$

$$= 2888 - 1444$$

$$\boxed{CS = 1444}$$

Now $PS = \int_0^{x_0} [P_0 - S(x)]\, dx$

$$PS = \int_0^{38} (24 - 24)\, dx$$

$$\boxed{PS = 0}$$

(ii) $S(x) = x^2 - 4$ $D(x) = -x + 8$

- First we find equilibrium point.

$$S(x) = D(x)$$
$$x^2 - 4 = -x + 8$$
$$x^2 - 4 + x - 8 = 0$$
$$x^2 + x - 12 = 0$$
$$x^2 + 4x - 3x - 12 = 0$$
$$x(x+4) - 3(x+4) = 0$$
$$(x-3)(x+4) = 0$$

$x = 3$ $x = -4$ (Cannot be −ve) (Ignore −ve)

$\boxed{x_0 = 3}$

- Now we find equilibrium price

$S(x) = x^2 - 4$	$D(x) = -x + 8$
$P_0 = (3)^2 - 4$	$P_0 = -3 + 8$
$P_0 = 5$	$P_0 = 5$

$x_0 = 3 \qquad P_0 = 5$

Now

$$CS = \int_0^3 (8 - x - 5)\, dx$$

$$= \int_0^3 (3 - x)\, dx$$

$$= 3\int_0^3 1\, dx - \int_0^3 x\, dx$$

$$= 3x \Big|_0^3 - \frac{x^2}{2}\Big|_0^3$$

$$= [3(3) - 0] - \frac{1}{2}\left[(3)^2 - (0)^2\right]$$

$$= 9 - \frac{9}{2}$$

$$\boxed{CS = \frac{9}{2}}$$

Now

$$PS = \int_0^3 (5 - x^2 + 4)\, dx$$

$$= \int_0^3 9 - x^2\, dx$$

$$= 9\int_0^3 1\, dx - \int_0^3 x^2\, dx$$

$$= 9x \Big|_0^3 - \frac{x^3}{3} \Big|_0^3$$

$$= [27 - 0] - \frac{1}{3}[27 - 0]$$

$$= 27 - \frac{27}{3}$$

$$= 27 - 9$$

$$\boxed{PS = 18}$$

(iii)

$S(x) = 2x^2 + 3x \qquad D(x) = 36 - x^2$

Sol First we find equilibrium point.

$$S(x) = D(x)$$

$$2x^2 + 3x = 36 - x^2$$

$$2x^2 + x^2 + 3x - 36 = 0$$

$$3x^2 + 3x - 36 = 0$$

$$x^2 + x - 12 = 0$$

$$x^2 + 4x - 3x - 12 = 0$$

$$x(x+4) - 3(x+4) = 0$$

$$(x-3)(x+4) = 0$$

$x = 3 \qquad x = -4$ (Ignore.

$$\boxed{x_0 = 3}$$

Now we find equilibrium price

$S(x) = 2x^2 + 3x$ $\quad\quad$ $D(x) = 36 - x^2$

$P_0 = 2(3)^2 + 3(3)$ $\quad\quad$ $D(x) = 36 - (3)^2$

$= 18 + 9$

$P_0 = 27$ $\quad\quad$ $P_0 = 36 - 9$

$P_0 = 27$

$x_0 = 3 \quad\quad P_0 = 27$

$$CS = \int_0^3 (36 - x^2 - 27)\,dx$$

$$= \int_0^3 (9 - x^2)\,dx$$

$$= 9\int_0^3 1\,dx - \int_0^3 x^2\,dx$$

$$= 9x\Big|_0^3 - \frac{x^3}{3}\Big|_0^3$$

$$= [27 - 0] - [27/3 - 0/3]$$

$$= 27 - 9$$

$$\boxed{CS = 18}$$

Now

$$PS = \int_0^{x_0} [P_0 - S(x)]\,dx$$

$$= \int_0^3 [27 - 2x^2 - 3x]\,dx$$

$$= 27\int_0^3 1\,dx - 2\int_0^3 x^2\,dx - 3\int_0^3 x\,dx$$

$$= 27x\Big|_0^3 - 2\frac{x^3}{3}\Big|_0^3 - 3\frac{x^2}{2}\Big|_0^3$$

$$= 27(3) - 2\frac{(3)^3}{3} - 3\frac{(3)^2}{2}$$

$$= 81 - 18 - \frac{27}{2}$$

$$\boxed{PS = \frac{99}{2}}$$

Q#16 Find the total revenue obtained in 4 years if the rate of increase in dollars per year is $f(t) = 200(t-5)^2$

Sol

$$T.R = \int_0^4 200(t-5)^2\,dt$$

$$= 200\int_0^4 (t-5)^2 \cdot 1\,dt$$

$$= 200 \left. \frac{(t-5)^{2+1}}{2+1} \right|_0^4$$

$$= \frac{200}{3} \left. (t-5)^3 \right|_0^4$$

$$= \frac{200}{3} \left[(4-5)^3 - (0-5)^3 \right]$$

$$= \frac{200}{3} \left[-1 + 125 \right]$$

$$= \frac{200}{3} \cdot 124$$

$$T.R = \frac{24800}{3} = 8266.67\$ \text{ Ans.}$$

17

$$f(t) = 600\sqrt{1+3t} \qquad t=0 \text{ to } t=8$$

$$T.R = \int_0^8 600 (1+3t)^{1/2} dt$$

$$= 600 \int_0^8 (1+3t)^{1/2} dt$$

xing and ÷ing by 3

$$= \frac{600}{3}\int_0^8 (1+3t)^{1/2} \cdot 3 \, dt$$

$$= 200 \cdot \frac{(1+3t)^{3/2}}{3/2} \Big|_0^8$$

$$= 200 \cdot \frac{2}{3} (1+3t)^{3/2} \Big|_0^8$$

$$= \frac{400}{3}\left[(1+3(8))^{3/2} - (1+0)^{3/2} \right]$$

$$= \frac{400}{3}\left[(25)^{3/2} - 1 \right]$$

$$= \frac{400}{3}\left[125-1 \right]$$

$$= \frac{400}{3}(124)$$

$T.R = 116533.33$

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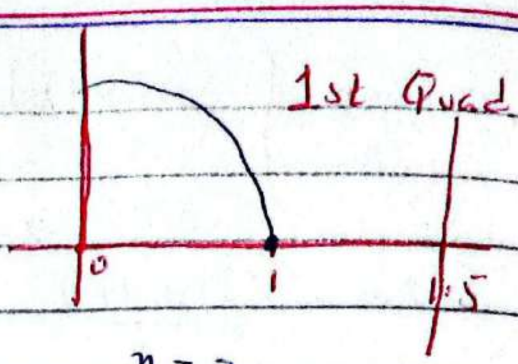
(18) Find the area bounded by the curve $F(x) = x^3 - 2x^2 + 1$ and the x-axis in the first Quadrant by the Line $x = 1.5$

Sol



Sol $y = x^3 - 2x^2 + 1$

For Limit put $y = 0$

$x^3 - 2x^2 + 1 = 0$ $\qquad x = 0$

- Put $x = 1$

$1 - 2 + 1 = 0$

$0 = 0$

$$\begin{array}{c|cccc} 1 & 1 & -2 & 0 & 1 \\ & & 1 & -1 & -1 \\ \hline & 1 & -1 & -1 &)\,0 \end{array}$$

$x^2 - x - 1 = 0$

By using Quadratic Formula.

$x = \frac{1 + \sqrt{5}}{2} = 1.6$ $\qquad x = \frac{1 - \sqrt{5}}{2} = -0.6$

Not include $\qquad$ Not include

$x = 0, 1$

$$A = \int_a^b F(x)\, dx$$

$$= \int_0^1 (x^3 - 2x^2 + 1)\, dx$$

$$= \frac{x^4}{4}\Big|_0^1 - \frac{2x^3}{3}\Big|_0^1 + x\Big|_0^1$$

$$= \frac{1}{4} - \frac{2}{3} + \frac{1}{1}$$

$$= \frac{7}{12} \text{ Square unit.}$$

Q#12 An object moves in a straight line according to the position — — — — — —

i) $S(t) = t^2 - 2t \; ; \; [0, 5]$

Sol First we find velocity

$$V = \frac{d}{dt} S(t)$$

$$V = \frac{d}{dt}(t^2 - 2t)$$

$$V = 2t - 2$$

If we take ~~derivative~~ integral of velocity we find distance

Put $V = 0$

$$2t - 2 = 0$$

$$2t = 2 \qquad t = 1 \in [0, 5]$$

$t = 0, 1, 5$ are the points.

$$S(t) = \int_0^1 -v\,dt + \int_1^5 +v\,dt$$

Checking

$V(t) = 2t - 2$

$t = 0$ to 1

Put $t = 0.5$

$= 2(0.5) - 2$

$= 1 - 2 = -1$

$V = -ve$

$t = 1$ to 5

Put 2

$= 2(2) - 2$

$= 4 - 2$

$= 2$

$$= -\int_0^1 (2t-2)\,dt + \int_1^5 (2t-2)\,dt$$

$$= -\left[2\frac{t^2}{2} - 2t \Big|_0^1\right] + \left[2\frac{t^2}{2} - 2t \Big|_1^5\right]$$

$$= -\left[t^2 - 2t \Big|_0^1\right] + \left[t^2 - 2t \Big|_1^5\right]$$

$$= -\left[(1 - 2(1)) - 0\right] + \left[\left((5)^2 - 2(5)\right) - (1-2)\right]$$

$$= -\left[-1\right] + \left[25 - 10 + 1\right]$$

$$= 1 + 16$$

$$= \boxed{S(t) = 17 \text{ cm}}$$

(ii) $S(t) = t^3 - 3t^2 - 9t$ $[0, 4]$

Sol First we Find velocity

$$V = \frac{d}{dt} S(t)$$

$$V = \frac{d}{dt}(t^3 - 3t^2 - 9t)$$

$$V = 3t^2 - 6t - 9$$

If we take integral of velocity we find distance

Put $V = 0$

$$3t^2 - 6t - 9 = 0$$

$$t^2 - 2t - 3 = 0$$

$$t^2 - 3t + t - 3 = 0$$

$$t(t-3) + 1(t-3) = 0$$

$$(t+1)(t-3) = 0$$

$$t = -1 \qquad t = 3$$

$t = -1 \notin [0, 4]$ $\qquad$ $t = 3 \in [0, 4]$

$t = 0, 3, 4$ are the points.

$$S(t) = \int_0^3 -v\,dt + \int_3^4 +v\,dt$$

$$= -\int_0^3 (3t^2 - 6t - 9)\,dt + \int_3^4 (3t^2 - 6t - 9)\,dt$$

$$= -\left[\frac{3t^3}{3} - \frac{6t^2}{2} - 9t\Big|_0^3\right] + \left[\frac{3t^3}{3} - \frac{6t^2}{2} - 9t\Big|_3^4\right]$$

checking

$V(t) = 3t^2 - 6t - 9$

$t = 0$ to 3

$t = 1$

$3 - 6 - 9 =$ $-ve$

$t = 3.5$

$3(3.5)^2 - 6(3.5) - 9$

$+ve$

$$= -\left[\left(3^3 - 3(3)^2 - 9(3)\right) - 0\right] + \left[\left(4^3 - 3(4)^2 - 9(4)\right) - \left(3^3 - 3(3)^2\right.\right.$$

$$= -\left[\cancel{27} - \cancel{27} - 27\right] + \left[64 - 48 - 36 - \cancel{27} + \cancel{27} + 27\right]$$

$$= 27 + 64 - 48 - 36 + 27$$

$$S(t) = 34\,cm.$$

(iii) $S(t) = 6 \sin \pi t$ $[1, 3]$

Sol First we find velocity

$$V = \frac{d}{dt} S(t)$$

$$V = \frac{d}{dt} 6 \sin \pi t$$

$$V = 6 \cos \pi t \cdot \pi$$

$$V = 6\pi \cos \pi t$$

If we take integral of velocity we find distance

Put $V(t) = 0$

$$6\pi \cos \pi t = 0$$

$$\cos \pi t = 0$$

$$\pi t = \cos^{-1}(0)$$

$$\pi t = (2n+1)\frac{\pi}{2} \qquad n \in Z$$

$$t = \frac{2n+1}{\pi} \cdot \frac{\pi}{2}$$

$$t = \frac{2n+1}{2} = n + \frac{1}{2} \quad , \quad n \in Z$$

- At $n = 0$

$$t = \frac{1}{2} = 0.5 \notin [1, 3]$$

- At $n = 1$

$$t = 1 + \frac{1}{2} = \frac{3}{2} = 1.5 \in [1, 3]$$

- At $n = 2$

$$t = 2 + \frac{1}{2} = \frac{5}{2} = 2.5 \in [1, 3]$$

- At $n = 3$

$$t = 3 + \frac{1}{2} = \frac{7}{2} = 3.5 \notin [1, 3]$$

$t = 1, 1.5, 2.5, 3$ are the points.

$$S(t) = \int_1^{3/2} -V\,dt + \int_{3/2}^{5/2} +V\,dt + \int_{5/2}^{3} -V\,dt$$

$$= -\int_1^{3/2} 6\pi \cos \pi t\,dt + \int_{3/2}^{5/2} 6\pi \cos \pi t\,dt - \int_{5/2}^{3} 6\pi \cos \pi t\,dt$$

$$= -6\pi \int_1^{3/2} \cos \pi t\,dt + 6\pi \int_{3/2}^{5/2} \cos \pi t\,dt - 6\pi \int_{5/2}^{3} \cos \pi t\,dt$$

$V = 6\pi \cos \pi t$

$[1, 3/2]$

Put $t = 1.1$

$V = 6\pi \cos \pi (1.1)$

$-ve$

Put $t = 2$

$+ve$

Put $t = 2.7$

$-ve$

$$= -6\pi \left.\frac{\sin \pi t}{\pi}\right|_1^{3/2} + 6\pi \left.\frac{\sin \pi t}{\pi}\right|_{3/2}^{5/2} - 6\pi \left.\frac{\sin \pi t}{\pi}\right|_{5/2}^{3}$$

$$= -6\left[\sin \frac{3\pi}{2} - \sin \pi\right] + 6\left[\sin \frac{5\pi}{2} - \sin \frac{3\pi}{2}\right] - 6\left[\sin 3\pi - \sin\right.$$

$$= -6\left[-1 - 0\right] + 6\left[1 - (-1)\right] - 6\left[0 - 1\right]$$

$$= 6 + 6(2) - 6(-1)$$

$$= 6 + 12 + 6$$

$$= 24 \text{ cm}$$

Complete