Exc 3.7

Q#1 $\displaystyle\int_{-1}^{2}(2x+3)\,dx$

Sol

$$=\int_{-1}^{2}2x\,dx+\int_{-1}^{2}3\,dx$$

$$=2\int_{-1}^{2}x\,dx+3\int_{-1}^{2}1\,dx$$

$$=2\cdot\frac{x^{1+1}}{1+1}\Bigg|_{-1}^{2}+3\cdot x\Bigg|_{-1}^{2}$$

$$=\cancel{2}\cdot\frac{x^{2}}{\cancel{2}}\Bigg|_{-1}^{2}+3x\Bigg|_{-1}^{2}$$

$$=x^{2}\Bigg|_{-1}^{2}+3x\Bigg|_{-1}^{2}$$

$$=\left[(2)^2-(-1)^2\right]+3\left[(2)-(-1)\right]$$

$$=(4-1)+3(2+1)$$

$$=3+9$$

$$=12$$

Q#2 $\displaystyle\int_{-4}^{12}\sqrt{y+4}\,dy$

Sol

$$\int_{-4}^{12} (y+4)^{1/2} \cdot 1\,dy.$$

Apply Rule I

$$\int [F(x)]^{n} \cdot F'(x)\,dx = \frac{[F(x)]^{n+1}}{n+1} + C$$

$$= \left.\frac{(y+4)^{1/2+1}}{1/2+1}\right|_{-4}^{12} = \left.\frac{(y+4)^{3/2}}{3/2}\right|_{-4}^{12}$$

$$= \left.\frac{2}{3}(y+4)^{3/2}\right|_{-4}^{12}$$

$$= \frac{2}{3}\left[(12+4)^{3/2} - (-4+4)^{3/2}\right]$$

$$= \frac{2}{3}\left[(4^{\cancel{2}})^{3/\cancel{2}} - (0)^{3/2}\right]$$

$$= \frac{2}{3}(4)^{3}$$

$$= \frac{2}{3} \cdot 64 = \frac{128}{3} \quad \text{Ans.}$$

Q # 3 $$\int_{0}^{1/2} (2x+1)^{-1/3}\,dx$$

Sol

xing and ÷ing by 2

$$= \frac{1}{2}\int_0^{1/2} (2x+1)^{-1/3} \cdot 2 \, dx.$$

$$= \frac{1}{2} \left. \frac{(2x+1)^{-1/3+1}}{-1/3+1} \right|_0^{1/2}$$

$$= \frac{1}{2} \left. \frac{(2x+1)^{2/3}}{2/3} \right|_0^{1/2}$$

$$= \frac{1}{2} \cdot \frac{3}{2} \left. (2x+1)^{2/3} \right|_0^{1/2}$$

$$= \frac{3}{4} \left. (2x+1)^{2/3} \right|_0^{1/2}$$

$$= \frac{3}{4}\left[\left(\cancel{2} \cdot \frac{1}{\cancel{2}}+1\right)^{2/3} - \left(2 \cdot 0+1\right)^{2/3}\right]$$

$$= \frac{3}{4}\left[(2)^{2/3} - (1)^{2/3}\right]$$

$$= \frac{3}{4}\left[2^{2/3} - 1\right] \quad \text{Ans.}$$

Q#4 $\int_0^3 (6x^2 - 4x + 5)\, dx$

Sol

$= \int_0^3 6x^2 dx - \int_0^3 4x\, dx + \int_0^3 5\, dx$

$= 6\int_0^3 x^2 dx - 4\int_0^3 x\, dx + 5\int_0^3 1\, dx$

$= 6 \cdot \frac{x^3}{3}\Big|_0^3 - 4 \cdot \frac{x^2}{2}\Big|_0^3 + 5x\Big|_0^3$

$= 2x^3\Big|_0^3 - 2x^2\Big|_0^3 + 5x\Big|_0^3$

$= 2\left[(3)^3 - (0)^3\right] - 2\left[(3)^2 - (0)^2\right] + 5\left[3 - 0\right]$

$= 2\left[27 - 0\right] - 2\left[9 - 0\right] + 5\left[3 - 0\right]$

$= 54 - 18 + 15$

$= 51$ Ans.

Q#5 $\int_{-2}^{1} (12x^5 - 36)\, dx$

Sol

$$= \int_{-2}^{1} 12x^5\, dx - \int_{-2}^{1} 36\, dx$$

$$= 12 \int_{-2}^{1} x^5\, dx - 36 \int_{-2}^{1} 1\, dx$$

$$= 12 \cdot \frac{x^6}{6} \Big|_{-2}^{1} - 36 \cdot x \Big|_{-2}^{1}$$

$$= 2x^6 \Big|_{-2}^{1} - 36x \Big|_{-2}^{1}$$

$$= 2\left[(1)^6 - (-2)^6 \right] - 36 \left[1 - (-2) \right]$$

$$= 2\left[1 - 64 \right] - 36 \left[1 + 2 \right]$$

$$= -126 - 108$$

$$= -234 \quad \text{Ans.}$$

Q#6 $\int_{-\pi/3}^{\pi/4} \cos\theta \, d\theta$

Sol

$$= \sin\theta \Big|_{-\pi/3}^{\pi/4}$$

$$= \left[\sin \pi/4 - \sin -\pi/3 \right]$$

$$= \left[\sin \pi/4 + \sin \pi/3 \right]$$

$$= \left[\sin 45^\circ + \sin 60^\circ \right]$$

$$= \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \quad \text{Ans.}$$

Q#7 $\int_0^{\pi/4} \sec^2 2\theta \, d\theta$ $\int \sec^2\theta \, d\theta = \tan\theta$

Sol $= \frac{\tan 2\theta}{2} \Big|_0^{\pi/4}$

$= \frac{1}{2}\left[\tan 2\frac{\pi}{4} - \tan 2(0)\right]$

$= \frac{1}{2}\left[\tan\frac{\pi}{2} - \tan 0\right]$

$= \frac{1}{2}\left[\infty - 0\right]$

$= \infty$ Ans

Q#8 $\int_2^4 \frac{x^2+8}{x^2} \, dx$

Sol $= \int_2^4 \frac{\cancel{x^2}}{\cancel{x^2}} + \frac{8}{x^2} \, dx$

$= \int_2^4 1 + 8x^{-2} \, dx$

$= \int_2^4 1 \, dx + \int_2^4 8x^{-2} \, dx$

$$= \int_2^4 1\,dx + 8\int_2^4 x^{-2}\,dx$$

$$= x\Big|_2^4 + 8\cdot\frac{x^{-1}}{-1}\Big|_2^4$$

$$= x\Big|_2^4 - 8\cdot\frac{1}{x}\Big|_2^4$$

$$= \left[4-2\right] - 8\left[\frac{1}{4}-\frac{1}{2}\right]$$

$$= 2 - 8\left[\frac{1-2}{4}\right]$$

$$= 2 - \cancel{8}^{2}\left(\frac{-1}{\cancel{4}_1}\right)$$

$$= 2 + 2 = 4$$

Q#9

$$\int_{-1/2}^{3/2} (x - \cos \pi x)\,dx$$

Sol

$$\int_{-1/2}^{3/2} x\,dx - \int_{-1/2}^{3/2} \cos \pi x\,dx$$

$$= \frac{x^2}{2}\Big|_{-1/2}^{3/2} - \frac{\sin \pi x}{\pi}\Big|_{-1/2}^{3/2}$$

$$= \frac{1}{2}\cdot\left[x^2\right]\Big|_{-1/2}^{3/2} - \frac{1}{\pi}\left[\sin \pi x\right]\Big|_{-1/2}^{3/2}$$

$$= \frac{1}{2}\left[\left(\frac{3}{2}\right)^2 - \left(\frac{-1}{2}\right)^2\right] - \frac{1}{\pi}\left[\sin \pi\cdot\frac{3}{2} - \sin\pi\cdot\frac{-1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{9}{4} - \frac{1}{4}\right] - \frac{1}{\pi}\left[\sin\frac{3\pi}{2} + \sin\frac{\pi}{2}\right]$$

$$= \frac{1}{2}\left[\frac{8}{4}\right] - \frac{1}{\pi}\left[\sin 270^\circ + \sin 90^\circ\right]$$

$$= 1 - \frac{1}{\pi}\left[-1+1\right]$$

$$= 1 - \frac{1}{\pi}(0)$$

$$= 1 \quad \text{Ans.}$$

Q#10 $\displaystyle\int_1^4 \frac{\cos\sqrt{x}}{2\sqrt{x}}\,dx$

Sol $\displaystyle\int_1^4 \cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}\,dx$

Put $\sqrt{x} = y$

$$\frac{1}{2}(x)^{\frac{1}{2}-1} = \frac{dy}{dx}$$

$$\frac{1}{2\sqrt{x}}\,dx = dy$$

When $x = 1$ than $y = 1$
When $x = 4$ than $y = 2$

$$\int_1^4 \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}\,dx$$

$$\int_1^2 \cos y \; dy$$

$$= \sin y \Big|_1^2$$

$= \sin 2 - \sin 1$ Ans

Q # 11 $\int_{\pi/6}^{\pi/3} \sin x \cos x \, dx$

Sol. $\int_{\pi/6}^{\pi/3} [\sin x]^1 \cos x \, dx$

$$= \left. \frac{\sin^{1+1} x}{1+1} \right|_{\pi/6}^{\pi/3}$$

$$= \left. \frac{\sin^2 x}{2} \right|_{\pi/6}^{\pi/3}$$

$$= \frac{1}{2}\left[\left(\sin\frac{\pi}{3}\right)^2 - \left(\sin\frac{\pi}{6}\right)^2\right]$$

$$= \frac{1}{2}\left[(\sin 60)^2 - (\sin 30)^2\right]$$

$$= \frac{1}{2}\left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right]$$

$$= \frac{1}{2}\left[\frac{3}{4} - \frac{1}{4}\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{4}\right]$$

$$= \frac{1}{4} \quad \text{Ans}$$

Q#12 $$\int_{\pi/6}^{\pi/2} \frac{1+\cos\theta}{(\theta+\sin\theta)^2}\, d\theta$$

Sol

$$= \int_{\pi/6}^{\pi/2} (\theta+\sin\theta)^{-2} (1+\cos\theta)\, d\theta.$$

$$= \left. \frac{(\theta+\sin\theta)^{-2+1}}{-2+1} \right|_{\pi/6}^{\pi/2}$$

$$= \left. \frac{(\theta+\sin\theta)^{-1}}{-1} \right|_{\pi/6}^{\pi/2}$$

$$= \left. \frac{-1}{(\theta+\sin\theta)} \right|_{\pi/6}^{\pi/2}$$

$$= -\left[\frac{1}{\pi/2+\sin\pi/2} - \frac{1}{\pi/6+\sin\pi/6} \right]$$

$$= -\left[\frac{1}{\pi/2+1} - \frac{1}{\pi/6+\frac{1}{2}} \right]$$

$$= -\left[\frac{1}{\frac{\pi+2}{2}} - \frac{1}{\frac{\pi+3}{6}} \right]$$

$$= -\left[\frac{2}{\pi+2} - \frac{6}{\pi+3} \right]$$

$$= \frac{6}{\pi+3} - \frac{2}{\pi+2} \quad \text{Ans}$$

$$= \frac{6}{3.14+3} - \frac{2}{3.14+2}$$

$$= \frac{6}{6.14} - \frac{2}{5.14}$$

$$= 0.9769 - 0.3889$$

$$= 0.588 \quad \text{Ans.}$$

Q#13 $\int_{-\pi/4}^{\pi/4} (\sec x + \tan x)^2 \, dx$

$(a+b)^2 = a^2 + b^2 + 2ab$

Sol

$$\int_{-\pi/4}^{\pi/4} (\sec^2 x + \tan^2 x + 2 \sec x \tan x) \, dx$$

$\sec^2 x - 1 = \tan^2 x$

$$\int_{-\pi/4}^{\pi/4} (\sec^2 x + \sec^2 x - 1 + 2 \sec x \tan x) \, dx$$

$$\int_{-\pi/4}^{\pi/4} (2 \sec^2 x - 1 + 2 \sec x \tan x) \, dx$$

$$= \int_{-\pi/4}^{\pi/4} 2 \sec^2 x \, dx - \int_{-\pi/4}^{\pi/4} 1 \, dx + \int_{-\pi/4}^{\pi/4} 2 \sec x \tan x \, dx$$

$$= 2 \int_{-\pi/4}^{\pi/4} \sec^2 x \, dx - \int_{-\pi/4}^{\pi/4} 1 \, dx + 2 \int_{-\pi/4}^{\pi/4} \sec x \tan x \, dx$$

$$= 2 \tan x \Big|_{-\pi/4}^{\pi/4} - x \Big|_{-\pi/4}^{\pi/4} + 2 \sec x \Big|_{-\pi/4}^{\pi/4}$$

$$= 2\left[\tan \frac{\pi}{4} - \tan\left(-\frac{\pi}{4}\right)\right] - \left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] + 2\left[\sec \frac{\pi}{4} - \sec\left(-\frac{\pi}{4}\right)\right]$$

$$= 2\left[\tan 45^\circ + \tan 45^\circ\right] - \left[\frac{\pi}{4} + \frac{\pi}{4}\right] + 2\left[\sec 45^\circ - \sec 45^\circ\right]$$

$$= 2\left[1+1\right] - \frac{2\pi}{4} + 2\left(\frac{1}{\cos 45^\circ} - \frac{1}{\cos 45}\right)$$

$$= 4 - \frac{\pi}{2} \quad \text{Ans.}$$

Q#14 $\displaystyle\int_{-\pi/2}^{\pi/2} \cos^2 x \, dx$

Sol

$$\int_{-\pi/2}^{\pi/2} \left(\frac{1+\cos 2x}{2}\right) dx$$

$$\int_{-\pi/2}^{\pi/2} \left(\frac{1}{2} + \frac{\cos 2x}{2}\right) dx$$

$$\int_{-\pi/2}^{\pi/2} \frac{1}{2}\, dx + \int_{-\pi/2}^{\pi/2} \frac{\cos 2x}{2}\, dx$$

$$\frac{1}{2}\int_{-\pi/2}^{\pi/2} 1\, dx + \frac{1}{2}\int_{-\pi/2}^{\pi/2} \cos 2x\, dx$$

$$= \frac{1}{2} \cdot x \Big|_{-\pi/2}^{\pi/2} + \frac{1}{2} \cdot \frac{\sin 2x}{2} \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2}x \Big|_{-\pi/2}^{\pi/2} + \frac{\sin 2x}{4} \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] + \frac{1}{4}\left[\sin 2 \cdot \frac{\pi}{2} - \sin 2 \cdot -\frac{\pi}{2}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} + \frac{\pi}{2}\right] + \frac{1}{4}\left[\sin 180^\circ + \sin 180^\circ\right].$$

$$= \frac{1}{2}\left[\frac{\pi + \pi}{2}\right] + \frac{1}{4}\left[0 + 0\right]$$

$$= \frac{1}{2} \cdot \frac{\cancel{2}\pi}{\cancel{2}} = \frac{\pi}{2} \quad \text{Ans.}$$

Q # 15 $\int_1^3 \ln x \, dx$

Sol First we solve.

$$\int \underset{\text{I}}{\ln x} \cdot \underset{\text{II}}{1} \, dx$$

Integrating by Parts:

$$= \ln x \cdot \int 1 \, dx - \int \left(\int 1 \, dx \; \frac{d}{dx} \ln x\right) dx$$

$$= \ln x \cdot x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} \, dx$$

$= x \ln x - \int 1 \, dx$

$= x \ln x - x$

Integrate

$$\int_1^3 x \ln x \, dx = (x \ln x - x) \Big|_1^3$$

$$= (x \ln x - x) \Big|_1^3$$

$$= (3 \ln 3 - 3) - (1 \ln 1 - 1)$$

$$= 3 \ln 3 - 3 - \ln 1 + 1$$

$$= 3 \ln 3 - 2 - 0$$

$$= 3 \ln 3 - 2 \quad \text{Ans.}$$

Q#16 $\displaystyle\int_2^4 \left(e^{x/2} - e^{x/4}\right) dx$

Sol $\displaystyle\int_2^4 e^{x/2} \, dx - \int_2^4 e^{x/4} \, dx$

$$= \frac{e^{x/2}}{1/2}\Big|_2^4 - \frac{e^{x/4}}{1/4}\Big|_2^4$$

$$= 2e^{x/2}\Big|_2^4 - 4e^{x/4}\Big|_2^4$$

$$= 2\left[e^{4/2} - e^{2/2}\right] - 4\left[e^{4/4} - e^{2/4}\right]$$

$$= 2\left[e^{2} - e\right] - 4\left[e - e^{1/2}\right]$$

$$= 2e^{2} - 2e - 4e + 4e^{1/2}$$

$$= 2e^{2} - 6e + 4e^{1/2} \quad \text{Ans.}$$

Q#17 $\int_0^{\pi/4}\left(\frac{1}{1-\sin x}\right)dx$

Sol

$$= \int_0^{\pi/4}\left(\frac{1}{1-\sin x} \times \frac{1+\sin x}{1+\sin x}\right)dx$$

$$= \int_0^{\pi/4} \frac{1+\sin x}{1-\sin^2 x}\,dx$$

$$\boxed{1-\sin^2 x = \cos^2 x}$$

$$= \int_0^{\pi/4} \frac{1+\sin x}{\cos^2 x}\,dx$$

$$\int_0^{\pi/4} \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \right) dx$$

$$\int_0^{\pi/4} \left(\sec^2 x + \frac{\sin x}{\cos x \cdot \cos x} \right) dx$$

$$\int_0^{\pi/4} \left(\sec^2 x + \tan x \sec x \right) dx.$$

$$\int_0^{\pi/4} \sec^2 x \, dx + \int_0^{\pi/4} \sec x \tan x \, dx.$$

$$\tan x \Big|_0^{\pi/4} + \sec x \Big|_0^{\pi/4}$$

$$\left[\tan 45^\circ - \tan 0 \right] + \left[\sec \frac{\pi}{4} - \sec(0) \right]$$

$$\left[1 - 0 \right] + \left[\frac{1}{\cos 45^\circ} - \frac{1}{\cos 0} \right]$$

$$= 1 + \frac{1}{1/\sqrt{2}} - \frac{1}{1}$$

$$= \cancel{1} + \sqrt{2} - \cancel{1}$$

$$= \sqrt{2} \quad \text{Ans.}$$

Q #18 $\int_0^{\pi/4} \tan^{-1} y \, dy$ — V imp

Sol

$$\int_0^{\pi/4} \underset{\text{II}}{1} \cdot \underset{\text{I}}{\tan^{-1} y} \, dy$$

$$= \tan^{-1} y \int 1 \, dy - \int \left(\int 1 \, dy \, \frac{d}{dy} \tan^{-1} y \right) dy$$

$$= \tan^{-1} y \cdot y - \int y \cdot \frac{1}{1+y^2} \, dy$$

$$= y \tan^{-1} y - \frac{1}{2} \int \frac{2y}{1+y^2} \, dy$$

$$= y \tan^{-1} y - \frac{1}{2} \ln |1+y^2| \Big|_0^{\pi/4}$$

$$= \left[\frac{\pi}{4} \tan^{-1} \frac{\pi}{4} - \frac{1}{2} \ln \left|1 + \left(\frac{\pi}{4}\right)^2\right| \right] - \left[0 \tan^{-1}(0) - \frac{1}{2} \ln |1+0| \right]$$

$$= \left[\frac{\pi}{4} \tan^{-1} \frac{\pi}{4} - \frac{1}{2} \ln \left| 1 + \frac{\pi^2}{16} \right| \right] - 0$$

$$= \left[\frac{\pi}{4} \cdot 1 - \frac{1}{2} \ln \left| \frac{16+\pi^2}{16} \right| \right]$$

$$= \frac{\pi}{4} - \frac{1}{2} \ln \frac{16+\pi^2}{16} \quad \text{Ans.}$$

v.imp

Q #9 $\displaystyle\int_0^{\pi/2} \frac{\sin x}{(2+\cos x)(5+\cos x)}\,dx$

Sol Let $\boxed{\cos x = t}$

$$-\sin x = \frac{dt}{dx}$$

$$-\sin x\,dx = dt$$

$$\boxed{\sin x\,dx = -dt}$$

If $x=0$ than $t = \cos(0) = 1$

If $x = \pi/2$ than $t = \cos(90) = 0$

$$= \int_1^0 \frac{-dt}{(2+t)(5+t)} = -\int_1^0 \frac{dt}{(t+2)(t+5)}$$

$$= \int_0^1 \frac{dt}{(t+2)(t+5)} \quad \text{——— (A)}$$

Consider $\dfrac{1}{(t+2)(t+5)} = \dfrac{A}{t+2} + \dfrac{B}{t+5}$ — (

Xing b. side by $(t+2)(t+5)$

$1 = A(t+5) + B(t+2)$ — (2)

Put $t+2=0$

$t=-2$

$1 = A(-2+5) + 0$

$\boxed{A = \frac{1}{3}}$

Put $t+5=0 \Rightarrow t=-5$

$1 = 0 + B(-5+2)$

$\boxed{B = \frac{-1}{3}}$

$$\frac{1}{(t+2)(t+5)} = \frac{1}{3(t+2)} - \frac{1}{3(t+5)}$$

Apply Integral

$$\int_0^1 \frac{dt}{(t+2)(t+5)} = \int_0^1 \left(\frac{1}{3(t+2)} - \frac{1}{3(t+5)}\right) dt$$

$$= \frac{1}{3}\int_0^1 \frac{1}{t+2}\,dt - \frac{1}{3}\int \frac{1}{t+5}\,dt$$

$$= \frac{1}{3} \cdot \ln|t+2| \Big|_0^1 - \frac{1}{3} \ln|t+5| \Big|_0^1$$

$$= \frac{1}{3}\left[\ln 3 - \ln 2\right] - \frac{1}{3}\left[\ln 6 - \ln 5\right]$$

$$= \frac{1}{3}\left[\ln 3 - \ln 2 - \ln 6 + \ln 5\right]$$

$$= \frac{1}{3}\left[\ln 3 - \ln 2 - \ln(2\times 3) + \ln 5\right]$$

$$= \frac{1}{3}\left[\cancel{\ln 3} - \ln 2 - \ln 2 - \cancel{\ln 3} + \ln 5\right]$$

$$= \frac{1}{3}\left[-2\ln 2 + \ln 5\right]$$

$$= \frac{1}{3}\left[-\ln 2^2 + \ln 5\right]$$

$$= \frac{1}{3}\left[\ln 5 - \ln 4\right]$$

$$= \frac{1}{3}\left[\ln \frac{5}{4}\right] \quad \text{Ans.}$$

v.imp

Q#20

$$\int_2^5 \frac{1}{x(x+1)}\,dx$$

Sol

Consider first we find partial fraction

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} \quad \text{——— (1)}$$

Xing b. sides by $x(x+1)$

$1 = A(x+1) + B(x)$ — (2)

Put $x = 0$

$1 = A(0+1)$

$\boxed{A = 1}$

Put $x+1 = 0$

$x = -1$

$1 = 0 + B(-1)$

$\boxed{B = -1}$

Put in (1)

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$$

Apply Limit

$$\int_2^5 \frac{1}{x(x+1)}\,dx = \int_2^5 \frac{dx}{x} - \int_2^5 \frac{dx}{x+1}$$

$$= \operatorname{Ln} x \Big|_2^5 - \operatorname{Ln}|x+1| \Big|_2^5$$

$$= [\operatorname{Ln} 5 - \operatorname{Ln} 2] - [\operatorname{Ln} 6 - \operatorname{Ln} 3]$$

$= \ln 5 - \ln 2 - \ln 6 + \ln 3$

$= \ln 5 - \ln 2 - \ln(2 \times 3) + \ln 3$

$= \ln 5 - \ln 2 - \ln 2 - \cancel{\ln 3} + \cancel{\ln 3}$

$= \ln 5 - 2\ln 2$

$= \ln 5 - \ln 2^2$

$= \ln 5 - \ln 4$

$= \ln\left(\frac{5}{4}\right)$ Ans.

Complete.