Exc 3.6

Definite Integral:-

$$\int_a^b F(x)\,dx$$

a = Lower Limit.
b = upper Limit.

Properties of Definite Integral

i) $\int_a^a F(x)\,dx = 0$

ii) $\int_a^b F(x)\,dx = -\int_b^a F(x)\,dx$

iii) If 'c' is the point between 'a' and 'b' then

$$\int_a^b F(x) = \int_a^c F(x)\,dx + \int_c^b F(x)\,dx$$

Area of ΔABC = $\Delta ABC = \frac{1}{2}(B \times H)$

Area of rectangle = $L \times W$.

Note:-

$$\int_a^b F(x)\,dx \Rightarrow F'(x)\Big|_a^b \Rightarrow F'(b) - F'(a)$$

upper Limit − Lower Limit

Q #1 (i) $\int_0^4 x \, dx$ $\quad \because \int_a^b F(x) \, dx$

Sol

$F(x) = x$

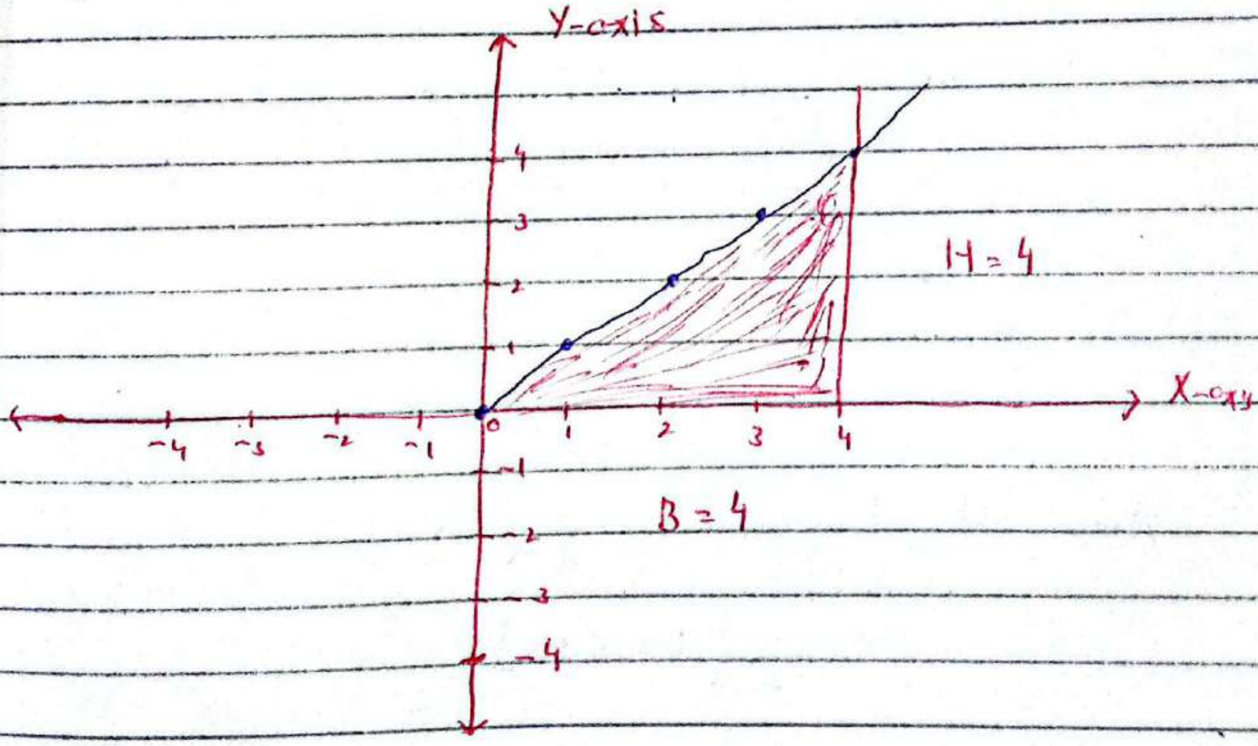
As $y = F(x)$

$y = x$

$x = 0, 4$

x	0	1	2	3	4
y	0	1	2	3	4

(x, y) $(0,0)$ $(1,1)$ $(2,2)$ $(3,3)$ $(4,4)$



We Know

Area of triangle $= \frac{1}{2}(B \times H)$

$= \frac{1}{2}(4 \times 4) = \frac{16}{2} = 8 \text{ unit}^2$.

Q # 1 (iii) $\int_{-3}^{0} x \, dx$

Sol

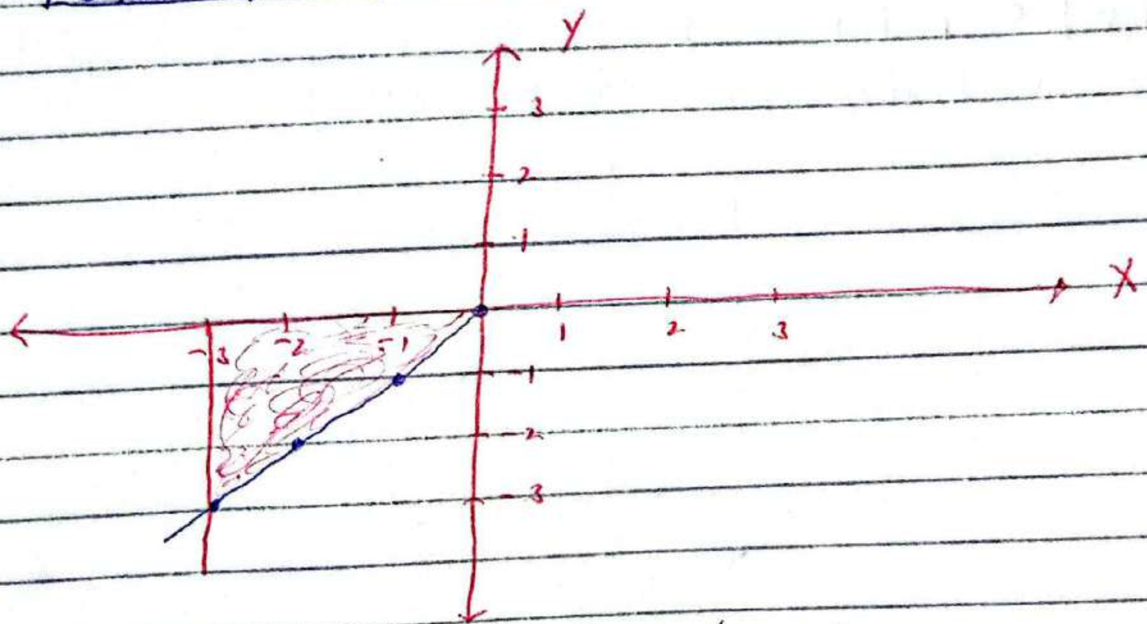
$F(x) = x$

As $y = F(x)$

$y = x$

$x = -3$ to 0

x	-3	-2	-1	0
y	-3	-2	-1	0



Area of triangle $= \frac{1}{2}(B \times H)$

$= \frac{1}{2}(+3 \times +3)$

$= \frac{9}{2}$

$= 4.5 \text{ unit}^2$

Note:

Area of triangle $= -4.5$ unit

- Below X-axis

(iii) $\int_0^2 (x-1)\,dx$

Sol

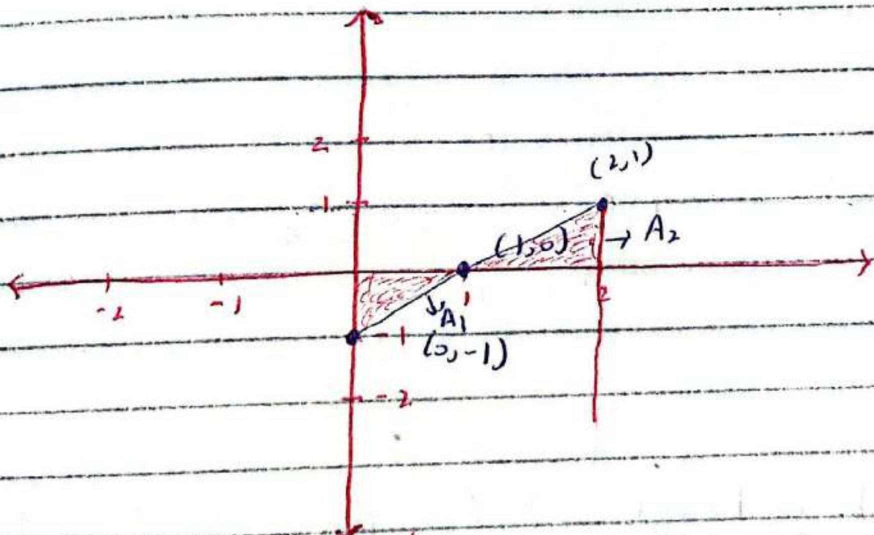
$F(x) = x-1$

As $y = F(x)$

$y = x-1$

$x = 0$ to 2

x	0	1	2
y	-1	0	1



$A_1 = \frac{1}{2}(B \times H)$

$A_1 = \frac{1}{2}(1 \times +1) = \frac{+1}{2}$

$A_1 = \frac{-1}{2}$ (Below X-axis)

$A_2 = \frac{1}{2}(B \times H)$

$= \frac{1}{2}(1 \times 1)$

$= \frac{1}{2}$

Total Area $= A_1 + A_2$

$= \frac{-1}{2} + \frac{1}{2} = 0$ Ans:

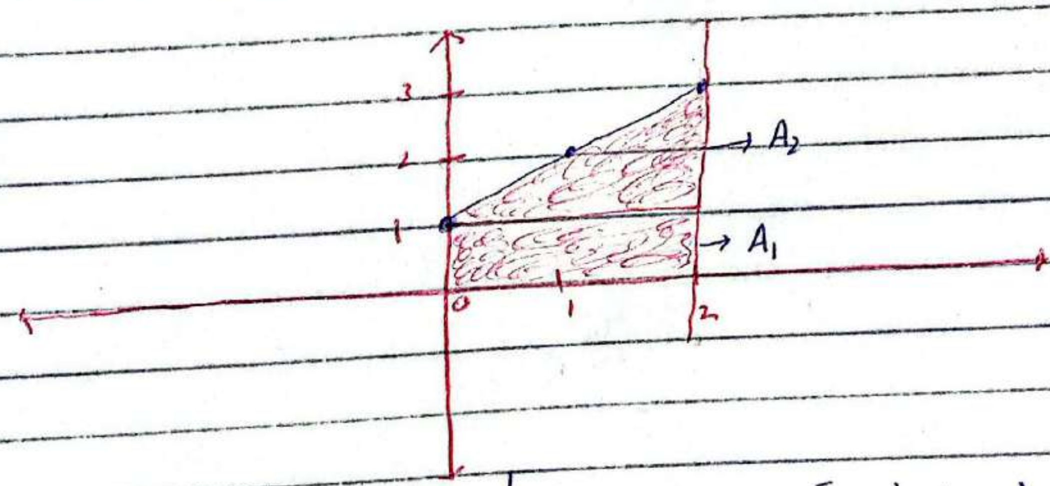
(IV) $\int_0^2 (x+1)\,dx$

Sol $F(x) = x+1$ As $y = f(x)$

$y = x+1$

$x = 0$ to 2

x	0	1	2
y	1	2	3



A_1 = Area of rectangle

$A_1 = L \times W$
$= 2 \times 1$
$A_1 = 2$

A_2 = Area of triangle

$= \frac{1}{2}(B \times H)$
$= \frac{1}{2}(2 \times 2)$
$A_2 = 2$

Now

Total Area $= A_1 + A_2$
$= 2 + 2$
$= 4 \text{ unit}^2$ Ans.

(V) $\int_{-3}^{3} 2 \, dx$

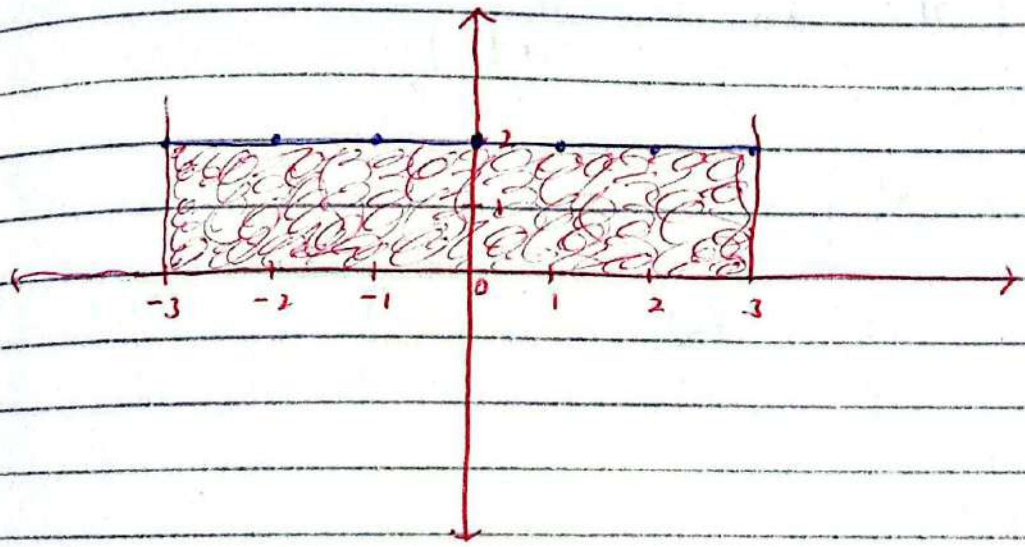
Sol

$F(x) = 2$ As $y = F(x)$

$y = 2$

$x = -3$ to 3

x	-3	-2	-1	0	1	2	3
y	2	2	2	2	2	2	2



Area of rectangle $= L \times W$

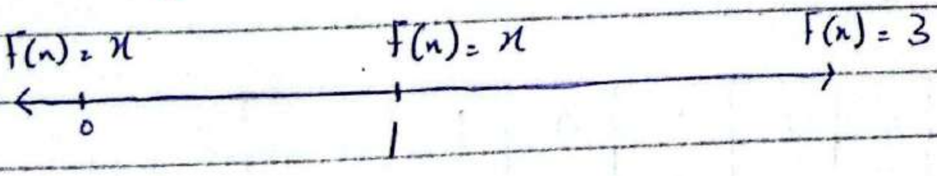
$= 6 \times 2$

$= 12 \text{ unit}^2$

Q#2 $\quad f(x) = \begin{cases} x & ; \quad x \leq 1 \\ 3 & ; \quad x > 1 \end{cases}$

Sol

i) $\displaystyle\int_0^1 f(x)\,dx$



$$\int_0^1 x \, dx = \left.\frac{x^{1+1}}{1+1}\right|_0^1$$

$$= \left.\frac{x^2}{2}\right|_0^1$$

$$= \frac{(1)^2}{2} - \frac{(0)^2}{2} = \frac{1}{2}$$

ii) $\displaystyle\int_{-1}^1 f(x)\,dx$

$$\int_{-1}^1 x \, dx = \left.\frac{x^2}{2}\right|_{-1}^1$$

$$= \frac{(1)^2}{2} - \frac{(-1)^2}{2} =$$

$$= \frac{1}{2} - \frac{1}{2}$$

$$= 0$$

(iii) $\int_1^4 F(x)\,dx$

$$\int_1^4 3\,dx = 3\int_1^4 1\,dx$$

$$= 3x \Big|_1^4$$

$$= 3(4) - 3(1)$$
$$= 12 - 3$$
$$= 9$$

(iv) $\int_{-1}^2 F(x)\,dx$

$$= \int_{-1}^1 F(x)\,dx + \int_1^2 F(x)\,dx$$

$$= \int_{-1}^1 x\,dx + \int_1^2 3\,dx$$

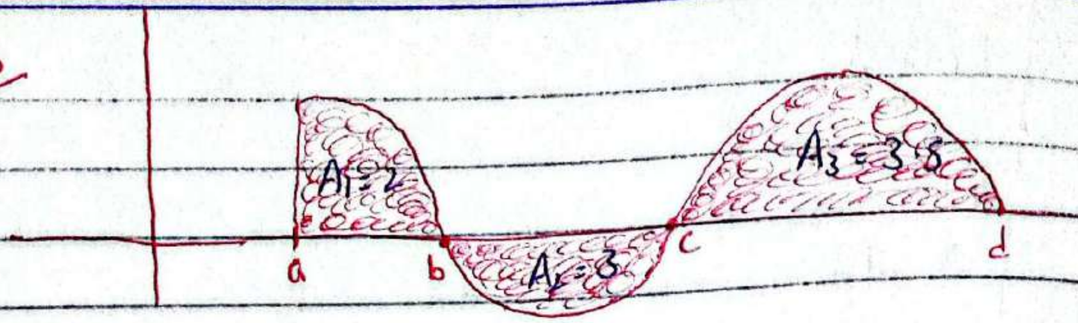
$$= \frac{x^2}{2}\Big|_{-1}^1 + 3x\Big|_1^2$$

$$= \left[\frac{(1)^2}{2} - \frac{(-1)^2}{2}\right] + \left[3(2) - 3(1)\right]$$

$$= \left[\cancel{\frac{1}{2}} - \cancel{\frac{1}{2}}\right] + \left[6 - 3\right]$$

$$= 0 + 3 = 3$$

Q#3



i) $\int_a^b F(x)\,dx = 2$

ii) $\int_b^c F(x)\,dx = 3$

iii) $\int_c^d F(x)\,dx = 3.5$

iv) $\int_a^c F(x)\,dx = \int_a^b F(x)\,dx + \int_b^c F(x)\,dx$

$= 2 + 3$

$= 5$

v) $\int_b^d F(x)\,dx = \int_b^c F(x)\,dx + \int_c^d F(x)\,dx$

$= 3 + 3.5$

$= 6.5$

vi) $\int_a^d F(x)\,dx = \int_a^b F(x)\,dx + \int_b^c F(x)\,dx + \int_c^d F(x)\,dx$

$= 2 + 3 + 3.5$

$= 8.5$ Ans.

Q#4 Find

$\int_1^5 [3F(x) - 2g(x)]\,dx$ If $\int_1^5 F(x)\,dx = 4$ and $\int_1^5 g(x)\,dx = 5$

Sol/ $= \int_1^5 3F(x)\,dx - \int_1^5 2g(x)\,dx$

$= 3\int_1^5 F(x)\,dx - 2\int_1^5 g(x)\,dx$

$= 3(4) - 2(5)$

$= 12 - 10$

$= 2$

Q#5 Find $\int_1^4 F(x)\,dx$ If

$\int_1^2 F(x)\,dx = 1$ and $\int_2^4 F(x)\,dx = 2$

Sol/ Find $\int_1^4 F(x)\,dx$

$$= \int_1^2 F(x)\,dx + \int_2^4 F(x)\,dx$$

$$= 1 + 2$$

$$= 3$$

Q#6 $\int_3^{-2} F(x)\,dx$ If

$\int_{-2}^{1} F(x)\,dx = 1$ and $\int_1^3 F(x)\,dx = -5$

Sol $= -\int_{-2}^{3} F(x)\,dx$

$$\boxed{\int_a^b F(x)\,dx = -\int_b^a F(x)\,dx}$$

$$= -\left[\int_{-2}^{1} F(x)\,dx + \int_1^3 F(x)\,dx\right]$$

$$= -\left[1 + (-5)\right]$$

$$= -\left[1-5\right] = -\left[-4\right] = 4 \quad \text{Ans.}$$

Q#7

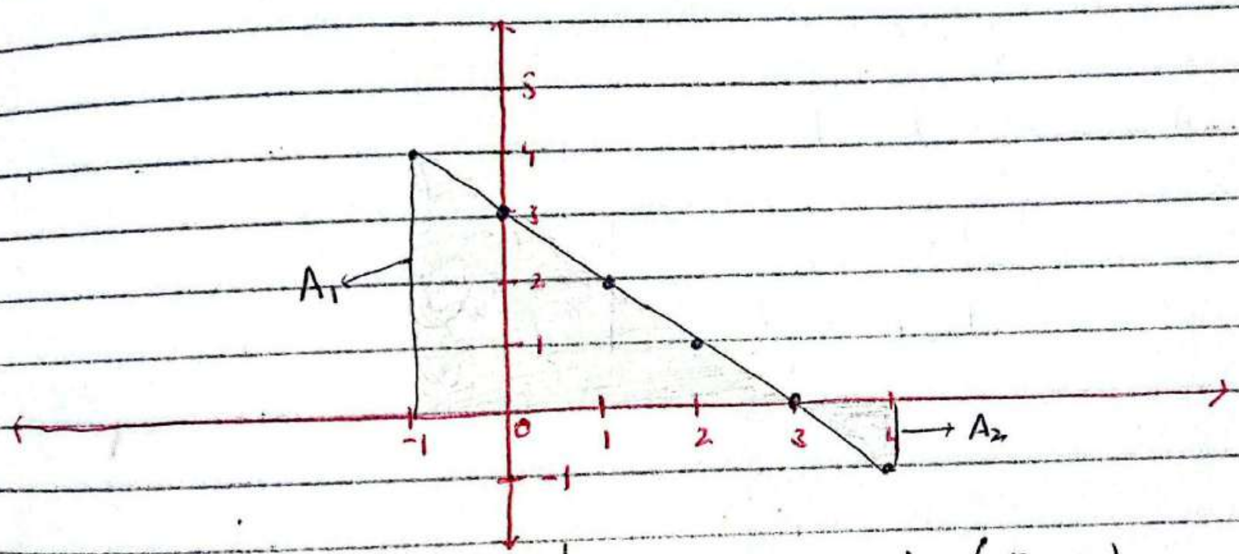
$$\int_{-1}^{4} (3-x)\, dx$$

Sol. $F(x) = 3 - x$ As $y = F(x)$

$y = 3 - x.$

$x = -1$ to 4

x	-1	0	1	2	3	4
y	4	3	2	1	0	-1



$A_1 = \frac{1}{2}(B \times H)$

$= \frac{1}{2}(4 \times 4)$

$= 8$

$A_2 = \frac{1}{2}(B \times H)$

$= \frac{1}{2}(1 \times +1)$

$= \frac{+1}{2}$ $A_2 = \frac{-1}{2}$ (Below X-axis

Total Area $= A_1 + A_2$

$= 8 - \frac{1}{2} = \frac{15}{2}$ unit^2 Ans

(ii) $\int_0^1 \left[2+\sqrt{1-x^2}\right] dx$

Sol: $\int_0^1 2\,dx + \int_0^1 \sqrt{1-x^2}\,dx$

$A_1 + A_2$

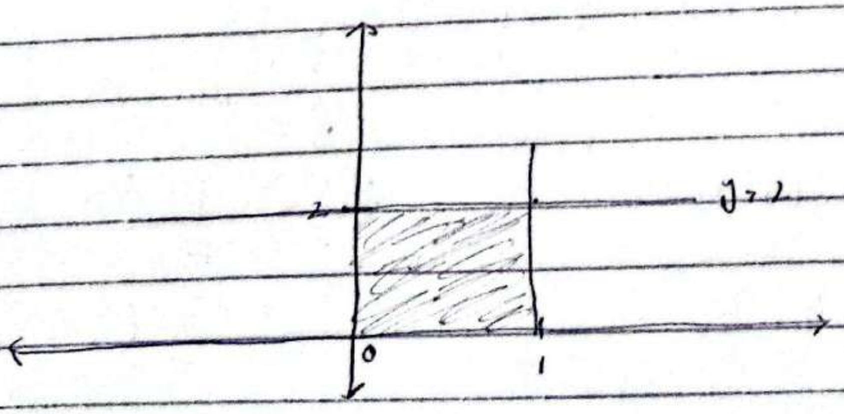
$A_1 = \int_0^1 2\,dx$

$F(x) = 2$ As $y = F(x)$

$y = 2$

$x = 0$ to 1

x	0	1
y	2	2



$A_1 = L \times W$

$= 1 \times 2$

$A_1 = 2$

$$A_2 = \int_0^1 \sqrt{1-x^2}\, dx$$

$y = \sqrt{1-x^2}$

Squaring b.sides

$y^2 = 1-x^2$

$x^2+y^2 = 1$

$(x-0)^2+(y-0)^2 = (1)^2$

or

$y^2 = 1-x^2$

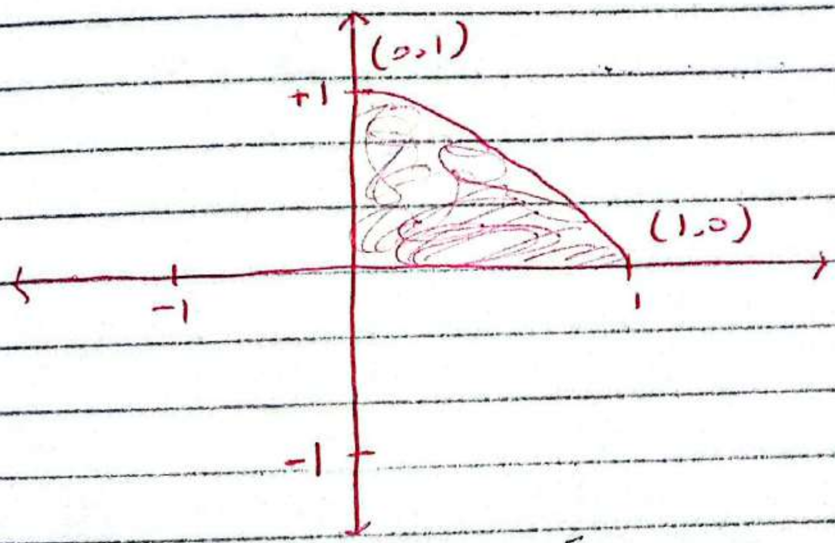
$y = \sqrt{1-x^2}$

$x = 0$ to 1

x	0	1
y	1	0

(0,1) (1,0)

Eq of circle with radius $r=1$ and centre (0,0)



Area of Circle = πr^2 [As Circle divids In to 4 Parts]

$$A = \frac{\pi r^2}{4} = \frac{\pi \cdot (1)}{4} = \frac{\pi}{4}$$

Total Area =

$A_1 + A_2$

$$2 + \frac{\pi}{4} = \frac{8+\pi}{4} \quad \text{Ans.}$$

(iii) $\int_{+2}^{+2} \sqrt{x^3-4}\, dx$

Sol

Property 1 : $\int_{a}^{a} f(x)\, dx = 0$

So $\int_{+2}^{+2} \sqrt{x^3-4}\, dx = 0$

Complete