

Exe 3.5

PARTIAL FRACTION

Type I: When power of x is Linear
(Linear) (Linear).

$$\frac{1}{(x+3)(x+4)} = \frac{A}{x+3} + \frac{B}{x+4}$$

Type II: When whole power of x is involve
(Linear) (Linear Repeated)

$$\frac{1}{(x+3)(x+4)^2} = \frac{A}{x+3} + \frac{B}{x+4} + \frac{C}{(x+4)^2}$$

Type III: When self power of x is involve.
(Linear) (Quadratic)

$$\frac{1}{(x+3)(x^2+4)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+4}$$

Type IV: When whole power of x (self power)
is involved.
(Linear) (Quadratic Repeated)

$$\frac{1}{(x+3)(x^2+4)^2} = \frac{A}{x+3} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$$

Rule I: (Power Rule)

$$\int [f(n)]^n f'(n) dn = \frac{[f(n)]^{n+1}}{n+1} + C$$

Rule II:

$$\int \frac{f'(n) dn}{f(n)} = \ln |f(n)| + C$$

Q: #1

$$\int \frac{3n+7}{(n+2)(n+3)} dn$$

Sol

First we find Partial Fraction:

Consider

$$\frac{3n+7}{(n+2)(n+3)} = \frac{A}{n+2} + \frac{B}{n+3} \quad \text{--- (i)}$$

King b. side by $(n+2)(n+3)$

$$\frac{3n+7}{(n+2)(n+3)} \times (n+2)(n+3) = \frac{A}{\cancel{n+2}} \times (\cancel{n+2})(n+3) + \frac{B}{\cancel{n+3}} \times (n+2)(\cancel{n+3})$$

$$3n+7 = A(n+3) + B(n+2) \quad \text{--- (ii)}$$

Put $n+2 = 0$

$$n = -2$$

Put $n = -2$ in (ii)

$$3(-2) + 7 = A(-2+3) + B(0)$$

$$-6 + 7 = A(1)$$

$$1 = A \quad \boxed{A=1}$$

Put $n+3=0$

$$n = -3$$

Put in (ii)

$$3(-3) + 7 = A(0) + B(-3+2)$$

$$-9 + 7 = -B$$

$$-2 = -B$$

$$\boxed{B=2}$$

Put values of A and B in (i)

$$\frac{3n+7}{(n+2)(n+3)} = \frac{A}{n+2} + \frac{B}{n+3}$$

$$\frac{3n+7}{(n+2)(n+3)} = \frac{1}{n+2} + \frac{2}{n+3}$$

Integrating b.sides

$$\int \frac{3n+7}{(n+2)(n+3)} dx = \int \frac{1}{n+2} dx + \int \frac{2}{n+3} dx$$

$$= \int \frac{1}{n+2} dn + 2 \int \frac{1}{n+3} dn \quad \text{Rule II}$$

$$\frac{3n+7}{(n+2)(n+3)} dn = \ln|n+2| + 2 \ln|n+3| + C$$

Q#2. $\int \frac{4n+9}{n^2+n-12} dn$

Sol. First we find Partial Fraction.
Consider

$$\frac{4n+9}{n^2+n-12} = \frac{4n+9}{n^2+4n-3n-12} = \frac{4n+9}{n(n+4)-3(n+4)}$$

$$\Rightarrow \frac{4n+9}{(n+4)(n-3)} = \frac{A}{n+4} + \frac{B}{n-3} \quad \text{--- (i)}$$

xing b. side by $(n+4)(n-3)$

$$4n+9 = A(n-3) + B(n+4) \quad \text{--- (ii)}$$

Put $n+4=0$
 $n = -4$

$$4(-4)+9 = A(-4-3) + B(0)$$

$$-16+9 = -7A$$

$$\frac{-7}{-7} = A$$

$$\boxed{A=1}$$

Put $n - 3 = 0$

$n = 3$

$$4(3) + 9 = A(0) + B(3 + 4)$$

$$21 = 7B$$

$$\frac{21}{7} = B$$

$B = 3$

Put values of A and B in (1)

$$\frac{4n + 9}{(n + 4)(n - 3)} = \frac{1}{n + 4} + \frac{3}{n - 3}$$

Integrating b. sides

$$\int \frac{4n + 9}{(n + 4)(n - 3)} dn = \int \frac{1}{n + 4} dn + 3 \int \frac{1}{n - 3} dn$$

Rule II

$$\int \frac{4n + 9}{(n + 4)(n - 3)} dn = \ln |n + 4| + 3 \ln |n - 3| + C$$

Q #3 $\int \frac{21 - 8n}{n^2 + n - 6} dn$

Sol First we find Partial Fraction

Consider

$$= \frac{21-8n}{n^2+n-6} = \frac{21-8n}{n^2+3n-2n-6} = \frac{21-8n}{n(n+3)-2(n+3)}$$

$$= \frac{21-8n}{(n+3)(n-2)} = \frac{A}{n+3} + \frac{B}{n-2} \quad \text{--- (i)}$$

xing b. side by $(n+3)(n-2)$

$$\Rightarrow 21-8n = A(n-2) + B(n+3) \quad \text{--- (ii)}$$

Put $n+3=0$
 $n = -3$

$$21-8(-3) = A(-3-2) + B(0)$$

$$21+24 = A(-5)$$

$$\frac{45}{-5} = A$$

$$\boxed{A = -9}$$

Put $n-2=0$ $n = 2$

$$21-8(2) = A(0) + B(2+3)$$

$$21-16 = 5B$$

$$\frac{5}{5} = B$$

$$\boxed{B = 1}$$

Put values of A and B in (i)

$$\frac{21-8x}{(x+3)(x-2)} = \frac{-9}{x+3} + \frac{1}{x-2}$$

Integrating b. sides

$$\int \frac{21-8x}{(x+3)(x-2)} dx = -9 \int \frac{1}{x+3} dx + \int \frac{1}{x-2} dx$$

Rule II

$$= -9 \cdot \ln|x+3| + \ln|x-2| + C$$

Q #4 $\int \frac{3x+7}{(x+2)^2} dx$

Sol

First we find partial fraction
consider

$$\frac{3x+7}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} \quad \text{--- (i)}$$

xing b. side by $(x+2)^2$

$$3x+7 = A(x+2) + B \quad \text{--- (ii)}$$

$$3x+7 = Ax + 2A + B \quad \text{--- (iii)}$$

Put $x+2=0$

$$x = -2$$

$$3(-2) + 7 = A(0) + B$$

$$-6 + 7 = B$$

$$B = 1$$

Compare the coeff of x eq (iii)

$$3 = A \quad A = 3$$

Put the values of A and B in (i)

$$\frac{3x+7}{(x+2)^2} = \frac{3}{x+2} + \frac{1}{(x+2)^2}$$

Integrating b. side

$$\int \frac{3x+7}{(x+2)^2} dx = 3 \int \frac{1}{x+2} dx + \int \frac{1}{(x+2)^2} dx$$

$$= 3 \int \frac{1}{x+2} dx + \int (x+2)^{-2} dx$$

Rule II

Rule I

$$= 3 \ln|x+2| + \frac{(x+2)^{-2+1}}{-2+1} + C$$

$$= 3 \ln|x+2| - \frac{1}{x+2} + C$$

Q#5 $\int \frac{5n^2 - 5n + 2}{(n+1)(n-1)^2} dn$

Sol First we find Partial Fraction
Consider

$$\frac{5n^2 - 5n + 2}{(n+1)(n-1)^2} = \frac{A}{n+1} + \frac{B}{n-1} + \frac{C}{(n-1)^2} \quad \text{--- (1)}$$

Xing b. side by $(n+1)(n-1)^2$

$$5n^2 - 5n + 2 = A(n-1)^2 + B(n+1)(n-1) + C(n+1) \quad \text{--- (2)}$$

$$5n^2 - 5n + 2 = A(n^2 - 2n + 1) + B(n^2 - 1) + C(n+1)$$

$$5n^2 - 5n + 2 = An^2 - 2An + A + Bn^2 - B + Cn + C \quad \text{--- (3)}$$

• put $n+1=0$
 $n = -1$

$$5(-1)^2 - 5(-1) + 2 = A(-1-1)^2 + B(0) + C(2)$$

$$5 + 5 + 2 = A(4)$$

$$\frac{12}{4} = A$$

$$\boxed{A = 3}$$

• put $n-1=0$

$$n = 1$$

$$5(1)^2 - 5(1) + 2 = A(0) + B(0) + C(1+1)$$

$$8 - 5 + 2 = 2C$$

$$\frac{2}{2} = C \quad \boxed{C=1}$$

Compare coeff of n^2

$$5 = A + B$$

$$5 - 3 = B$$

$$\boxed{B=2}$$

Put the values of A, B and C in (i)

$$\frac{5n^2 - 5n + 2}{(n+1)(n-1)^2} = \frac{3}{n+1} + \frac{2}{n-1} + \frac{1}{(n-1)^2}$$

Integrating b. sides.

$$\int \frac{5n^2 - 5n + 2}{(n+1)(n-1)^2} dn = \int \frac{3}{n+1} dn + \int \frac{2}{n-1} dn + \int \frac{1}{(n-1)^2} dn$$

$$= 3 \int \frac{1}{n+1} dn + 2 \int \frac{1}{n-1} + \int (n-1)^{-2} dn$$

Rule II

Rule II

Rule I

$$= 3 \ln|n+1| + 2 \ln|n-1| + \frac{(n-1)^{-2+1}}{-2+1} + C$$

$$= 3 \ln|n+1| + 2 \ln|n-1| - \frac{1}{n-1} + C$$

Q #6 $\int \frac{9x^2 + 3x + 29}{(x+1)(x^2+4)} dx$

Sol. First we find Partial Fraction
Consider.

$$\frac{9x^2 + 3x + 29}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4} \quad \text{--- (1)}$$

Xing b. side by $(x+1)(x^2+4)$

$$9x^2 + 3x + 29 = A(x^2+4) + Bx + C(x+1) \quad \text{--- (2)}$$

$$9x^2 + 3x + 29 = Ax^2 + 4A + Bx^2 + Bx + Cx + C \quad \text{--- (3)}$$

Put $x+1 = 0$

$$x = -1$$

$$9(-1)^2 + 3(-1) + 29 = A(1+4) + 0$$

$$9 - 3 + 29 = 5A$$

$$\frac{35}{5} = A$$

$$\boxed{A = 7}$$

From (3) compare coeff of x^2

$$9 = A + B$$

$$9 - 7 = B$$

$$\boxed{B = 2}$$

Compare Coeff of x

$$3 = B + C$$

$$3 - 2 = C$$

$$\boxed{C = 1}$$

Put the values of A, B, C in (i)

$$\frac{9x^2 + 3x + 29}{(x+1)(x^2+4)} = \frac{7}{x+1} + \frac{2x+1}{x^2+4}$$

Integrating b. sides

$$\int \frac{9x^2 + 3x + 29}{(x+1)(x^2+4)} dx = \int \frac{7}{x+1} dx + \int \frac{2x+1}{x^2+4} dx$$

$$= 7 \int \frac{1}{x+1} dx + \int \frac{2x}{x^2+4} dx + \int \frac{1}{x^2+2^2} dx$$

$$= 7 \int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$= 7 \ln|x+1| + \ln|x^2+4| + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$$

Ans'

$$\textcircled{7} \int \frac{7x^2 + 7x + 4}{(2x+1)(x^2+x+1)} dx$$

Sol First we find Partial Fraction
Consider

$$\frac{7x^2 + 7x + 4}{(2x+1)(x^2+x+1)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+x+1} \quad \text{---(1)}$$

xing b. side by $(2x+1)(x^2+x+1)$

$$7x^2 + 7x + 4 = A(x^2+x+1) + Bx+C(2x+1) \quad \text{---(2)}$$

$$7x^2 + 7x + 4 = Ax^2 + Ax + A + 2Bx^2 + Bx + 2Cx + C$$

Put $2x+1 = 0$

$$x = -\frac{1}{2}$$

Put $x = -\frac{1}{2}$ in (2)

$$7\left(-\frac{1}{2}\right)^2 + 7\left(-\frac{1}{2}\right) + 4 = A\left(\left(-\frac{1}{2}\right)^2 - \frac{1}{2} + 1\right) + 0$$

$$\frac{7}{4} - \frac{7}{2} + \frac{4}{1} = A\left(\frac{1}{4} - \frac{1}{2} + 1\right)$$

$$\frac{7 - 14 + 16}{4} = A\left(\frac{1 - 2 + 4}{4}\right)$$

$$\frac{9}{4} = A\left(\frac{3}{4}\right)$$

$$\frac{9}{4} \times \frac{4}{3} = A$$

$$\boxed{A = 3}$$

Compare the coeff of n^2

$$7 = A + 2B$$

$$7 = 3 + 2B$$

$$7 - 3 = 2B$$

$$B = \frac{4}{2}$$

$$\boxed{B = 2}$$

Compare the coeff of n

$$7 = A + B + 2C$$

$$7 = 3 + 2 + 2C$$

$$7 - 5 = 2C$$

$$\boxed{C = 1}$$

Put the values of A, B, C in (i)

$$\frac{7n^2 + 7n + 4}{(2n+1)(n^2+n+1)} = \frac{3}{2n+1} + \frac{2n+1}{n^2+n+1}$$

Integrating b. sides:

$$\int \frac{7n^2 + 7n + 4}{(2n+1)(n^2+n+1)} dn = 3 \int \frac{1}{2n+1} dn + \int \frac{2n+1}{n^2+n+1} dn$$

$$= \frac{3}{2} \int \frac{2}{2n+1} dn + \int \frac{2n+1}{n^2+n+1} dn$$

$$= \frac{3}{2} \ln|2n+1| + \ln|n^2+n+1| + C \quad \text{Ans.}$$

$$\textcircled{8} \int \frac{x^3 + 4x^2 + 9x + 14}{x^2 + 4x + 3} dx \quad (\text{Improper form})$$

Sol First we find partial fraction.

Consider

$$\frac{x^3 + 4x^2 + 9x + 14}{x^2 + 4x + 3}$$

It is improper fraction so first we divide it.

$$\begin{array}{r} x \\ x^2 + 4x + 3 \overline{) x^3 + 4x^2 + 9x + 14} \\ \underline{+ x^3 + 4x^2 + 3x} \\ 6x + 14 \end{array}$$

So

$$\frac{x^3 + 4x^2 + 9x + 14}{x^2 + 4x + 3} = x + \frac{6x + 14}{x^2 + 4x + 3} \quad \text{--- (A)}$$

Now

$$\frac{6x + 14}{x^2 + 4x + 3} = \frac{6x + 14}{x^2 + 3x + x + 3} = \frac{6x + 14}{x(x+3) + 1(x+3)}$$

$$\frac{6x + 14}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3} \quad \text{--- (i)}$$

Bring b. side by $(n+1)(n+3)$

$$6n+14 = A(n+3) + B(n+1) \quad \text{--- (2)}$$

$$\text{Put } n+1=0 \Rightarrow n=-1$$

$$6(-1)+14 = A(-1+3)$$

$$-6+14 = 2A$$

$$\frac{8}{2} = A$$

$$\boxed{A=4}$$

$$\text{Put } n+3=0 \Rightarrow n=-3$$

$$6(-3)+14 = 0 + B(-3+1)$$

$$-18+14 = -2B$$

$$\frac{-4}{-2} = B$$

$$\boxed{B=2}$$

Put the values of A and B in (i)

$$\frac{6n+14}{(n+1)(n+3)} = \frac{4}{n+1} + \frac{2}{n+3}$$

Now Eq # (A) becomes:

$$\frac{n^3+4n^2+9n+14}{n^2+4n+3} = n + \frac{4}{n+1} + \frac{2}{n+3}$$

Integrating b. sides

$$\int \frac{n^3+4n^2+9n+14}{n^2+4n+3} dn = \int n dn + 4 \int \frac{1}{n+1} dn + 2 \int \frac{1}{n+3} dn$$

$$= \frac{n^2}{2} + 4 \ln|n+1| + 2 \ln|n+3| \quad \text{Ans}$$

$$(9) \int \frac{1}{n^2+9} dx$$

Sol First we find Partial Fraction.
Consider.

$$\frac{1}{n^2+9} = \frac{1}{(n)^2-(3)^2} = \frac{1}{(n+3)(n-3)}$$

$$\frac{1}{(n+3)(n-3)} = \frac{A}{n+3} + \frac{B}{n-3} \quad \text{--- (i)}$$

xing b. side by $(n+3)(n-3)$

$$1 = A(n-3) + B(n+3) \quad \text{--- (2)}$$

$$\text{Put } n+3=0 \Rightarrow n=-3$$

$$1 = A(-3-3) + B(0)$$

$$\boxed{A = \frac{-1}{6}}$$

$$\text{Put } n-3=0 \Rightarrow n=3$$

$$1 = A(3-3) + B(3+3)$$

$$1 = 6B$$

$$\boxed{B = \frac{1}{6}}$$

Put the values of A and B in (i)

$$\frac{1}{(n+3)(n-3)} = \frac{-1}{6(n+3)} + \frac{1}{6(n-3)}$$

Integrating:

$$\int \frac{1}{n^2+9} dn = \frac{-1}{6} \int \frac{1}{n+3} dn + \frac{1}{6} \int \frac{1}{n-3} dn$$

$$= \frac{-1}{6} \ln|n+3| + \frac{1}{6} \ln|n-3| + C \text{ Ans}$$

(10) $\int \frac{1}{n^3+2n^2+n} dn$

Sol $\int \frac{1}{n(n^2+2n+1)} dn$

$$\int \frac{1}{n(n+1)^2} dn$$

First we find Partial Fraction

consider:

$$\frac{1}{n(n+1)^2} = \frac{A}{n} + \frac{B}{n+1} + \frac{C}{(n+1)^2} \quad (1)$$

Multiplying both side by $n(n+1)^2$

$$1 = A(n+1)^2 + B \cdot n \cdot (n+1) + C \cdot n \quad (2)$$

$$1 = An^2 + 2An + A + Bn^2 + Bn + Cn \quad (3)$$

Put $x=0$

$$1 = A(0+1)^2 + 0 + 0$$

$$1 = A(1)$$

$$\boxed{A=1}$$

Put $x+1=0 \Rightarrow x=-1$

$$1 = 0 + 0 + C(-1)$$

$$\boxed{C=-1}$$

From (3) compare the coeff of x^2

$$0 = A + B$$

$$B = -A$$

$$B = -(1)$$

$$\boxed{B=-1}$$

Put the values of A, B, C in (i)

$$\frac{1}{x(x+1)^2} = \frac{1}{x} - \frac{1}{x+1} - \frac{1}{(x+1)^2}$$

Integrating

$$\int \frac{1}{x(x+1)^2} dx = \int \frac{1}{x} dx - \int \frac{1}{x+1} dx - \int \frac{1}{(x+1)^2} dx$$

$$= \int \frac{1}{x} dx - \int \frac{1}{x+1} dx - \int (x+1)^{-2} dx$$

$$= \ln |n| - \ln |n+1| - \frac{(n+1)^{-2+1}}{-2+1} + C$$

$$= \ln |n| - \ln |n+1| + \frac{1}{(n+1)} + C \quad \text{Ans}$$

(ii) $\int \frac{e^n}{(e^n+1)^2 (e^n-2)} dn$

Sol

Put $e^n = y$

$$\frac{d}{dn} e^n = \frac{dy}{dn}$$

$$e^n = \frac{dy}{dn}$$

$$e^n dn = dy$$

$$\int \frac{1}{(y+1)^2 (y-2)} dy$$

First we find Partial Fraction:

Consider

$$\frac{1}{(y-2)(y+1)^2} = \frac{A}{y-2} + \frac{B}{y+1} + \frac{C}{(y+1)^2} \quad (i)$$

Multiplying both sides by $(y-2)(y+1)^2$

$$1 = A(y+1)^2 + B(y-2)(y+1) + C(y-2) \quad \text{--- (2)}$$

$$1 = Ay^2 + 2Ay + A + B(y^2 - 2y + y - 2) + C(y-2)$$

$$1 = Ay^2 + 2Ay + A + By^2 - By - 2B + Cy - 2C \quad \text{--- (3)}$$

$$\text{Put } y-2 = 0$$

$$y = 2$$

$$1 = A(2+1)^2$$

$$\boxed{\frac{1}{9} = A}$$

$$\text{Put } y+1 = 0$$

$$y = -1$$

$$1 = 0 + 0 + C(-1-2)$$

$$\boxed{C = -\frac{1}{3}}$$

Compare the coeff of y^2

$$0 = A + B$$

$$0 = \frac{1}{9} + B$$

$$\boxed{B = -\frac{1}{9}}$$

Put the values of A, B, C in (i)

$$\frac{1}{(y-2)(y+1)^2} = \frac{1}{9(y-2)} - \frac{1}{9(y+1)} - \frac{1}{3(y+1)^2}$$

Integrating:

$$\int \frac{1}{(y-2)(y+1)^2} dy = \frac{1}{9} \int \frac{1}{y-2} dy - \frac{1}{9} \int \frac{1}{y+1} dy - \frac{1}{3} \int \frac{1}{(y+1)^2} dy$$

$$= \frac{1}{9} \ln|y-2| - \frac{1}{9} \ln|y+1| - \frac{1}{3} \int (y+1)^{-2} dy$$

$$= \frac{1}{9} \ln|y-2| - \frac{1}{9} \ln|y+1| - \frac{1}{3} \frac{(y+1)^{-2+1}}{-2+1} + C$$

$$= \frac{1}{9} \ln|y-2| - \frac{1}{9} \ln|y+1| + \frac{1}{3(y+1)} + C \text{ Ans.}$$

$$= \frac{1}{9} \ln|e^x-2| - \frac{1}{9} \ln|e^x+1| + \frac{1}{3(e^x+1)} + C \text{ Ans}$$

(2) $\int \frac{x}{(x+1)^2(x^2+1)} dx$

Sol First we find Partial Fraction

$$\frac{x}{(x+1)^2(x^2+1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+1} \text{ --- (i)}$$

Multiplying b. sides by $(x+1)^2(x^2+1)$

$$x = A(x+1)(x^2+1) + B(x^2+1) + Cx + D(x+1)^2 \quad \text{--- (4)}$$

$$x = A(x^3 + x + x^2 + 1) + B(x^2 + 1) + Cx + D(x^2 + 2x + 1)$$

$$x = Ax^3 + Ax + Ax^2 + A + Bx^2 + B + Cx^2 + 2Cx + C + Dx^2 + 2Dx + D$$

$$\text{Put } x+1=0$$

$$x = -1$$

$$-1 = B(1+1)$$

$$\boxed{B = \frac{-1}{2}}$$

Compare coeff of x^3

$$0 = A + C \quad \text{--- (4)}$$

Compare coeff of x^2

$$0 = A + B + D + 2C \quad \text{--- (5)}$$

Compare coeff of x

$$1 = A + C + 2D \quad \text{--- (6)}$$

Compare coeff of constant

$$0 = A + B + D \quad \text{--- (7)}$$

Put eq (4) in (6)

$$1 = 0 + 2D$$

$$\boxed{D = \frac{1}{2}}$$

Put the values of B and D in (7)

$$0 = A \frac{-1}{2} + \frac{1}{2}$$

$$\boxed{A = 0}$$

Put $A = 0$ in (4)

$$0 = 0 + C$$

$$\boxed{C = 0}$$

Put the values of A, B, C, D in (1)

$$\frac{x}{(x+1)^2(x^2+1)} = \frac{0}{x+1} - \frac{1}{2(x+1)^2} + \frac{1}{2(x^2+1)}$$

Integrating

$$\int \frac{x}{(x+1)^2(x^2+1)} dx = -\frac{1}{2} \int \frac{1}{(x+1)^2} dx + \frac{1}{2} \int \frac{1}{x^2+1} dx$$

$$= -\frac{1}{2} \int (x+1)^{-2} dx + \frac{1}{2} \int \frac{1}{(x)^2+(1)^2} dx$$

$$= -\frac{1}{2} \frac{(x+1)^{-2+1}}{-2+1} + \frac{1}{2} \cdot \frac{1}{1} \tan^{-1}\left(\frac{x}{1}\right) + C$$

$$= \frac{1}{2(x+1)} + \frac{1}{2} \tan^{-1}(x) + C \quad \text{Complete}$$