Exc 3.2

Basic Formulas

Derivative	Anti-Derivative
$\frac{d}{dx}\sin x = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}\cos x = -\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}\sec x = \sec x\tan x$	$\int \sec x\tan x\,dx = \sec x + C$
$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x\cot x$	$\int \operatorname{cosec} x\cot x\,dx = -\operatorname{cosec} x + C$
$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x\,dx = -\cot x + C$
	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x\,dx = e^x + C$
	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C = \ln\lvert\sec x\rvert + C$
	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$

Identities:

$\sin^2\theta + \cos^2\theta = 1$ $\quad\quad$ $1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta.$

Formulas:

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

General Form (Some Basic)

- $\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$
- $\int \sin(ax+b)\, dx = -\frac{\cos(ax+b)}{a} + C$
- $\int \cos(ax+b)\, dx = \frac{\sin(ax+b)}{a} + C$
- $\int \tan(ax+b)\, dx = \frac{1}{a} \ln|\sec(ax+b)| + C$

  $= -\frac{1}{a} \ln|\cos(ax+b)| + C$
- $\int \cot(ax+b)\, dx = \frac{\ln|\sin(ax+b)|}{a} + C$
- $\int \sec(ax+b)\, dx = \frac{\ln|\sec(ax+b) + \tan(ax+b)|}{a} + C$

- $\int e^{ax+b}\, dx = \frac{e^{ax+b}}{a} + C$

Examples

Function : x^5

Integral

$$\int x^5 \, dx = \frac{x^{5+1}}{5+1} + C = \frac{x^6}{6} + C$$

Derivative

$$= \frac{d}{dx} \frac{x^6}{6}$$

$$= \frac{1}{6} \cdot \frac{d}{dx} x^6$$

$$= \frac{1}{6} \cdot 6 x^5$$

$$= x^5$$

Function : $\frac{1}{(2x+3)^4}$

Integral

$$\int \frac{1}{(2x+3)^4} \, dx = \int (2x+3)^{-4} \, dx$$

$$= \frac{(2x+3)^{-4+1}}{2(-4+1)} + C = \frac{(2x+3)^{-3}}{2 \cdot (-3)} + C$$

$$= -\frac{1}{6(2x+3)^3} + C$$

Derivative:

$$= \frac{d}{dx}\left(\frac{-1}{6(2x+3)^3}\right)$$

$$= \frac{-1}{6}\frac{d}{dx}(2x+3)^{-3}$$

$$= \frac{-1}{6}\cdot\left[-3(2x+3)^{-3-1}\frac{d}{dx}(2x+3)\right]$$

$$= \frac{3\cdot 2}{6(2x+3)^4} = \frac{1}{(2x+3)^4}$$

Function: $\cos 2x$.

Integral:

$$\int \cos 2x = \frac{\sin 2x}{2} + C$$

Derivative:

$$= \frac{d}{dx}\frac{\sin 2x}{2}$$

$$= \frac{1}{2}\frac{d}{dx}\cdot\sin 2x.$$

$$= \frac{1}{2}\cdot\cos 2x\cdot 2$$

$$= \cos 2x$$

Function: $\sin 3x$

Integral:

$$\int \sin 3x \, dx = -\frac{\cos 3x}{3} + C$$

$$= -\frac{1}{3} \cos 3x + C$$

Derivative:

$$= \frac{d}{dx} -\frac{1}{3} \cos 3x$$

$$= -\frac{1}{3} \frac{d}{dx} \cos 3x$$

$$= -\frac{1}{\cancel{3}} \cdot -\sin 3x \cdot \cancel{3}$$

$$= \sin 3x$$

Function: $\operatorname{cosec}^2 x$

Integral:

$$\int \operatorname{cosec}^2 x \, dx = -\cot x + C$$

Derivative:

$$= \frac{d}{dx}(-\cot x)$$

$$= -\frac{d}{dx} \cot x$$

$$= - \cdot -\operatorname{cosec}^2 x$$

$$= \operatorname{cosec}^2 x$$

Function: $\sec 5x \tan 5x$

Integral:

$$\int \sec 5x \tan 5x = \frac{\sec 5x}{5} + C$$

Derivative:

$$= \frac{d}{dx} \frac{\sec 5x}{5}$$

$$= \frac{1}{5} \frac{d}{dx} \sec 5x.$$

$$= \frac{1}{5} \cdot \sec 5x \tan 5x \cdot 5$$

$$= \sec 5x \tan 5x.$$

Function: e^{ax+b}

Integral:

$$\int (e^{ax+b})\,dx = \frac{e^{ax+b}}{a} + C$$

Derivative:

$$= \frac{d}{dx} \frac{e^{ax+b}}{a}$$

$$= \frac{1}{a} \frac{d}{dx} e^{ax+b}$$

$$= \frac{1}{a} \cdot e^{ax+b} \cdot a$$

$$= e^{ax+b}$$

Function : 3^{ax}

Integral:

$$\int 3^{ax}\,dx = \frac{3^{ax}}{a \cdot \ln 3} + C$$

Derivative:

$$= \frac{d}{dx}\,\frac{3^{ax}}{a \cdot \ln 3}$$

$$= \frac{1}{a\ln 3}\,\frac{d}{dx}\,3^{ax}$$

$$= \frac{1}{\cancel{a}\,\cancel{\ln 3}} \cdot 3^{ax} \cdot \cancel{\ln 3} \cdot \cancel{a}$$

$$= 3^{ax}$$

Function : $\frac{1}{ax+b}$

Integral:

$$\int \frac{1}{(ax+b)}\,dx = \frac{1}{a}\ln(ax+b) + C$$

Derivative:

$$= \frac{d}{dx}\,\frac{1}{a}\ln(ax+b)$$

$$= \frac{1}{a}\,\frac{d}{dx}\ln(ax+b)$$

$$= \frac{1}{\cancel{a}} \cdot \frac{1}{ax+b} \cdot \cancel{a} = \frac{1}{ax+b}$$

IMPORTANT RULES

Rule - I

$$\int [F(x)]^n \cdot F'(x)\, dx = \frac{[F(x)]^{n+1}}{n+1} + C$$

Rule - II

$$\int \frac{F'(x)}{F(x)}\, dx = \ln |F(x)| + C$$

Exc 3.2

Evaluate the integrals and recheck your answer by differentiating

1) $\int (\sin \pi x - 3 \sin 3x)\, dx$

Sol

$= \int \sin \pi x \, dx - \int 3 \sin 3x \, dx$

$= \int \sin \pi x \, dx - 3 \int \sin 3x \, dx$

$= -\frac{\cos \pi x}{\pi} - 3 \cdot \frac{-\cos 3x}{3} + C$

$= -\frac{\cos \pi x}{\pi} + \cos 3x + C$ —(i) Ans.

Rechecking

Differentiate.

$\frac{d}{dx}\left(-\frac{\cos \pi x}{\pi}\right) + \frac{d}{dx}(\cos 3x) + \frac{d}{dx}(C)$

$\frac{-1}{\pi} \frac{d}{dx} \cos \pi x + \frac{d}{dx} \cos 3x + 0$

$\frac{-1}{\pi} \cdot -\sin \pi x \cdot \pi + -\sin 3x \cdot 3$

$\sin \pi x - 3 \sin 3x$

(2) $\int -\sec^2\left(\frac{3y}{2}\right) dy$

Sol

$= -\int \sec^2\left(\frac{3y}{2}\right) dy$

$= -\frac{\tan(3y/2)}{3/2} + C$

$= -\frac{2}{3}\tan(3y/2) + C$

$\int \sec^2 x \, dx = \tan x + C$

$\int \sec^2(3x)\,dx = \frac{\tan 3x}{3} + C$

Rechecking

Differentiate

$= \frac{d}{dx}\left(-\frac{2}{3}\tan(3y/2)\right) + \frac{d}{dx}(C)$

$= -\frac{2}{3}\tan\left(\frac{3y}{2}\right) + 0$

$= -\frac{2}{3} \cdot \sec^2\left(\frac{3y}{2}\right) \cdot \frac{3}{2}$

$= -\sec^2\left(\frac{3y}{2}\right)$ Ans.

(3) $\int (1 - 8\,\mathrm{cosec}^2 2x)\,dx$

$\int \mathrm{cosec}^2 x\,dx = -\cot x +$

$\int \mathrm{cosec}^2 2x\,dx = \frac{-\cot 2x}{2} +$

Sol: $= \int 1\,dx - 8\int \mathrm{cosec}^2 2x\,dx$

$= x - 8\cdot\left(-\frac{\cot 2x}{2}\right) + C$

$= x + 4\cot 2x + C$

Rechecking : Differentiate

$x + 4\cot 2x + C$

$= \frac{d}{dx}x + 4\frac{d}{dx}\cot 2x + \frac{d}{dx}C$

$= 1 + 4\cdot -\mathrm{cosec}^2 2x\cdot 2 + 0$

$= 1 - 8\,\mathrm{cosec}^2 2x$

(4) $\int \frac{1}{2}(\mathrm{cosec}^2 x - \mathrm{cosec}\,x\cot x)\,dx$

Sol: $= \frac{1}{2}\left[\int \mathrm{cosec}^2 x\,dx - \int \mathrm{cosec}\,x\cot x\,dx\right]$

$= \frac{1}{2}\left[\int \mathrm{cosec}^2 x\,dx + \int -\mathrm{cosec}\,x\cot x\,dx\right]$

$$= \frac{1}{2}\left[-\cot x - (-\operatorname{cosec} x)\right] + c$$

$$= \frac{1}{2}\left[-\cot x + \operatorname{cosec} x\right] + c$$

Rechecking : Differentiate

$$= \frac{1}{2}\left[-\cot x + \operatorname{cosec} x\right] + c$$

Diff w.r. to x:

$$= \frac{1}{2}\left[-\frac{d}{dx}\cot x + \frac{d}{dx}\operatorname{cosec} x\right] + \frac{d}{dx} c$$

$$= \frac{1}{2}\left[-(-\operatorname{cosec}^2 x) + (-\operatorname{cosec} x \cot x)\right] + 0$$

$$= \frac{1}{2}\left[\operatorname{cosec}^2 x - \operatorname{cosec} x \cot x\right]$$

Ans.

(6) $\int \frac{\cos^2 z}{7} dz$

Sol

$$\frac{1}{7}\int \cos^2 z \, dz$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$= \frac{1}{7}\int \frac{1+\cos 2z}{2} dz$$

$$= \frac{1}{14}\left[\int 1\,dZ + \int \cos 2Z\, dZ\right]$$

$$= \frac{1}{14}\cdot\left[Z + \frac{\sin 2Z}{2}\right] + C$$

Rechecking:

$$\frac{1}{14}\left[Z + \frac{\sin 2Z}{2}\right] + C$$

$$= \frac{1}{14}\left[\frac{d}{dZ}Z + \frac{1}{2}\frac{d}{dZ}\sin 2Z\right] + \frac{d}{dZ}C$$

$$= \frac{1}{14}\left[1 + \frac{1}{\cancel{2}}\cdot \cos 2Z\cdot \cancel{2}\right] + 0$$

$$= \frac{1}{14}\left[1 + \cos 2Z\right]$$

$$= \frac{1}{7}\left[\frac{1+\cos 2Z}{2}\right]$$

$$= \frac{1}{7}\cdot \cos^2 Z$$

$$= \frac{\cos^2 Z}{7} \quad \text{Ans:}$$

(6) $\int (1+\tan^2\theta)\, d\theta$

Sol $\int \sec^2\theta \, d\theta \quad \because 1+\tan^2\theta = \sec^2\theta$

$\tan\theta + C.$

Rechecking

$\tan\theta + C$

Diff w.r.t. θ.

$= \frac{d}{d\theta}\tan\theta + \frac{d}{d\theta} C$

$= \sec^2\theta + 0$

$= \sec^2\theta$

$= 1+\tan^2\theta$ Proved.

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(7) $\int \left(\frac{1+\cos 4t}{2}\right) dt$

Sol $= \int \left(\frac{1}{2} + \frac{\cos 4t}{2}\right) dt$

$= \int \frac{1}{2}\, dt + \int \frac{\cos 4t}{2}\, dt$

$= \frac{1}{2}\int 1\, dt + \frac{1}{2}\int \cos 4t\, dt$

$= \frac{1}{2} \cdot t + \frac{1}{2} \cdot \frac{\sin 4t}{4} + C$

$$= \frac{1}{2}t + \frac{1}{8}\sin 4t + C$$

Rechecking

$$= \frac{d}{dt}\frac{1}{2}t + \frac{d}{dt}\frac{1}{8}\sin 4t + \frac{d}{dt}C$$

$$= \frac{1}{2}\frac{d}{dt}t + \frac{1}{8}\frac{d}{dt}\sin 4t + \frac{d}{dt}C$$

$$= \frac{1}{2}\cdot 1 + \frac{1}{8}\cdot \cos 4t \cdot 4 + 0$$

$$= \frac{1}{2} + \frac{1}{2}\cos 4t$$

$$= \frac{1+\cos 4t}{2}$$

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(8) $\int \sec^2(5x-1)\,dx$

Sol

$$= \frac{\tan(5x-1)}{5} + c$$

$\int \sec^2\theta\, d\theta = \tan\theta + c$

Rechecking

$$\frac{d}{dx}\frac{\tan(5x-1)}{5} + \frac{d}{dx}c$$

$= \frac{1}{5} \frac{d}{dx} \tan(5x-1) + 0$

$= \frac{1}{\cancel{5}} \cdot \sec^2(5x-1) \cdot 5$

$= \sec^2(5x-1)$ Ans.

(9) $\int (\tan 5x + \cos 7x)\, dx$

Sol $= \int \tan 5x \, dx + \int \cos 7x \, dx$

$= \frac{\ln|\sec 5x|}{5} + \frac{\sin 7x}{7} + C$

Rechecking

$\frac{\ln|\sec 5x|}{5} + \frac{\sin 7x}{7} + C$

Diff w.r. to x.

$= \frac{1}{5} \frac{d}{dx} \ln|\sec 5x| + \frac{1}{7} \frac{d}{dx} \sin 7x + \frac{d}{dx} C$

$= \frac{1}{\cancel{5}} \cdot \frac{1}{\cancel{\sec 5x}} \cdot \cancel{\sec 5x} \tan 5x \cdot \cancel{5} + \frac{1}{\cancel{7}} \cdot \cos 7x \cdot \cancel{7}$

$= \tan 5x + \cos 7x$

(10) $\int (\cot 9y - 3)\, dy$

Sol

$= \int \cot 9y \, dy - 3\int 1 \, dy$

$= \frac{\ln|\sin 9y|}{9} - 3y + C$

Rechecking

$\frac{\ln|\sin 9y|}{9} - 3y + C$

Diff w.r.to y

$= \frac{1}{9} \frac{d}{dy} \ln|\sin 9y| - \frac{d}{dy} 3y + \frac{d}{dy} C$

$= \frac{1}{9} \cdot \frac{1}{\sin 9y} \cdot \cos 9y \cdot 9 - 3$

$= \frac{\cos 9y}{\sin 9y} - 3$

$= \cot 9y - 3$

Q#11 $\int (\tan^2 2\theta + \cot^2 2\theta) d\theta$

Sol

$= \int \tan^2 2\theta \, d\theta + \int \cot^2 2\theta \, d\theta$

$= \int (\sec^2 2\theta - 1) \, d\theta + \int (\operatorname{cosec}^2 2\theta - 1) \, d\theta$

$\sec^2\theta = 1 + \tan^2\theta, \quad \operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\int \sec^2 2\theta \, d\theta - \int 1 \, d\theta + \int \operatorname{cosec}^2 2\theta \, d\theta - \int 1 \, d\theta$

$\frac{\tan 2\theta}{2} - \theta + \left(\frac{-\cot 2\theta}{2}\right) - \theta + c$

$\frac{\tan 2\theta}{2} - \frac{\cot 2\theta}{2} - 2\theta + c$ Ans.

Q #12 $\int \sin^2\left(\frac{11}{2} y\right) dy$

Sol

$\sin^2 y = \frac{1 - \cos 2y}{2}$

$\int \frac{1 - \cos 2\left(\frac{11}{2} y\right)}{2} dy$

$\int \frac{1 - \cos 11y}{2} dy$

$\int \frac{1}{2} dy - \int \frac{\cos 11y}{2} dy$

$$= \frac{1}{2}\theta - \frac{1}{2}\frac{\sin 11\theta}{11} + C$$

$$= \frac{1}{2}\left[\theta - \frac{\sin 11\theta}{11}\right] + C$$

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(13) $\int (\csc 11x \tan 11x)\, dx$

$$\int \frac{1}{\sin 11x} \cdot \frac{\sin 11x}{\cos 11x}\, dx$$

$$\int \frac{1}{\cos 11x}\, dx$$

$$\int \sec 11x\, dx$$

$$\int \sec x\, dx = \ln|\sec x + \tan x| + C$$

$$= \frac{\ln|\sec 11x + \tan 11x|}{11} + C$$

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(14) $\int \cos\theta\,(\tan\theta + \sec\theta)\, d\theta$

$$\int \cos\theta \cdot \tan\theta + \cos\theta \cdot \sec\theta\, d\theta$$

$$\int \left(\cos\theta \cdot \frac{\sin\theta}{\cos\theta} + \cos\theta \cdot \frac{1}{\cos\theta}\right) d\theta$$

$$\int (\sin\theta + 1)\, d\theta$$

$$= \int \sin\theta \, d\theta + \int 1 \, d\theta$$

$$= -\cos\theta + \theta + C \quad \text{Ans.}$$

(15) $\int \csc^2\left(\frac{x-1}{3}\right) dx$

Sol

$$= \frac{-\cot\left(\frac{x-1}{3}\right)}{1/3} + C$$

$$= -3\cot\left(\frac{x-1}{3}\right) + C \quad \text{Ans.}$$

(16) $\int (\cos x)^{1/5} \sin x \, dx$

Sol

Rule I

$$\int [F(x)]^n F'(x) \, dx = \frac{[F(x)]^{n+1}}{n+1} + C$$

$$\int (\cos x)^{1/5} \cdot \sin x \, dx$$

Xing and ÷ing by −1

$$= \frac{1}{-1} \int (\cos x)^{1/5} \cdot (-\sin x) \, dx$$

$$= \frac{1}{-1} \cdot \frac{(\cos x)^{\frac{1}{5}+1}}{\frac{1}{5}+1} + C$$

$$= \frac{1}{-1} \frac{(\cos x)^{4/5}}{4/5} + C$$

$$= \frac{-5}{4} (\cos x)^{4/5} + C$$

(17) $\int e^{y} \sin e^{y} \, dy$

Sol

Put $e^{y} = z$

$e^{y} = \frac{dz}{dy}$

$e^{y} dy = dz$

$\int \sin z \, dz$

$-\cos z + C$

$-\cos e^{y} + C$ Ans

(18) $\int 9 \tan(x+7) \, dx$

Sol

$= 9 \int \tan(x+7) \, dx$

$= 9 \cdot \frac{-\ln|\cos(x+7)|}{1} + C$

$-9 \ln |\cos(x+7)| + C$ Ans.

Complete