Exc 2.8

$\frac{dy}{dx}$ = First Derivative

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$ = 2nd Derivative.

$\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$ = 3rd Derivative

$\frac{d}{dx}\left(\frac{d^3y}{dx^3}\right)$ = 4th Derivative.

Find the second derivative of the function w.r. to variable involved.

i) $y = -x^3 + 6x + 9$

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}(-x^3 + 6x + 9)$$

$$= -\frac{d}{dx}x^3 + 6\frac{d}{dx}x + \frac{d}{dx}9$$

$$= -3x^2 + 6 \cdot 1 + 0$$

$$\frac{dy}{dx} = -3x^2 + 6$$

Again Diff w.r.to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(-3x^2 + 6)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-3x^2) + \frac{d}{dx}(6)$$

$$= -3\frac{d}{dx}x^2 + 0$$

$$= -3(2x)$$

$$\boxed{\frac{d^2y}{dx^2} = -6x}$$

Q#2 $F(x) = 30x^2 - x^3$

Sol

Diff w.r.to x.

$$\frac{d}{dx}F(x) = \frac{d}{dx}(30x^2 - x^3)$$

$$f'(x) = \frac{d}{dx}30x^2 - \frac{d}{dx}x^3$$

$$= 30\frac{d}{dx}x^2 - \frac{d}{dx}x^3$$

$$= 30[2x] - [3x^2]$$

$$f'(x) = 60x - 3x^2$$

Again Diff w.r.to x.

$$\frac{d}{dx}(f'(x)) = \frac{d}{dx}(60x - 3x^2)$$

$$f''(x) = \frac{d}{dx} 60x - \frac{d}{dx} 3x^2$$

$$= 60 \frac{d}{dx} x - 3 \frac{d}{dx} x^2$$

$$= 60 \cdot 1 - 3[2x]$$

$$f''(x) = 60 - 6x \quad \text{Ans.}$$

Q#3 $f(x) = (-5x+9)^2$

Sol

Diff w.r.to x

$$\frac{d}{dx} f(x) = \frac{d}{dx} (-5x+9)^2$$

$$f'(x) = 2(-5x+9)^{2-1} \frac{d}{dx} (-5x+9)$$

$$= 2(-5x+9) \cdot (-5)$$

$$= -10(-5x+9)$$

$$f'(x) = 50x - 90$$

Again Diff w.r.to x.

$$\frac{d}{dx} f'(x) = \frac{d}{dx} (50x - 90)$$

$$f''(x) = \frac{d}{dx} 50x - \frac{d}{dx} 90$$

$$= 50 \frac{d}{dx} x - 0$$

$f''(x) = 50.1$

$f''(x) = 50$ Ans.

Q #4 $y = 2x^6 + 5x^3 - 6x^2$

Sol

Diff w.r.to x.

$$\frac{dy}{dx} = \frac{d}{dx}(2x^6 + 5x^3 - 6x^2)$$

$$= \frac{d}{dx} 2x^6 + \frac{d}{dx} 5x^3 - \frac{d}{dx} 6x^2$$

$$= 2\frac{d}{dx} x^6 + 5\frac{d}{dx} x^3 - 6\frac{d}{dx} x^2$$

$$= 2[6x^5] + 5[3x^2] - 6[2x]$$

$$\frac{dy}{dx} = 12x^5 + 15x^2 - 12x$$

Again Diff w.r.to x.

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx} 12x^5 + 15x^2 - 12x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} 12x^5 + \frac{d}{dx} 15x^2 - \frac{d}{dx} 12x$$

$$= 12\frac{d}{dx} x^5 + 15\frac{d}{dx} x^2 - 12\frac{d}{dx} x$$

$$\frac{d^2y}{dx^2} = 12[5x^4] + 15[2x] - 12[1]$$

$$\frac{d^2y}{dx^2} = 60x^4 + 30x - 12$$

Q#5 $y = 20x^{-3}$

Sol Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx} 20x^{-3}$$

$$= 20 \frac{d}{dx} x^{-3}$$

$$= 20[-3x^{-3-1}]$$

$$\frac{dy}{dx} = -60x^{-4}$$

Again Diff w.r.to x.

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(-60x^{-4})$$

$$\frac{d^2y}{dx^2} = -60 \frac{d}{dx} x^{-4}$$

$$= -60[-4x^{-4-1}]$$

$$\frac{d^2y}{dx^2} = 240x^{-5} \quad \text{Ans}$$

Q #6 $y = \frac{2}{x^4}$

Sol

$$y = 2 \cdot x^{-4}$$

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx} 2 \cdot x^{-4}$$

$$= 2 \frac{d}{dx} x^{-4}$$

$$= 2 \left[-4 x^{-4-1} \right]$$

$$\frac{dy}{dx} = -8 x^{-5}$$

Again Diff w.r.to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-8 x^{-5}\right)$$

$$= -8 \frac{d}{dx} x^{-5}$$

$$= -8 \left[-5 x^{-5-1} \right]$$

$$\frac{d^2y}{dx^2} = 40 x^{-6}$$ Ans.

Q#7 $f(x) = x^2(3x-4)^3$

Sol

Diff w.r.to x.

$$\frac{d}{dx} f(x) = \frac{d}{dx} x^2(3x-4)^3$$

u.v

$$f'(x) = x^2 \frac{d}{dx}(3x-4)^3 + (3x-4)^3 \frac{d}{dx} x^2$$

$$f'(x) = x^2 \left[3(3x-4)^{3-1} \frac{d}{dx}(3x-4)\right] + (3x-4)^3 [2x]$$

$$f'(x) = 3x^2(3x-4)^2 \cdot 3 + 2x(3x-4)^3$$

$$f'(x) = 9x^2(3x-4)^2 + 2x(3x-4)^3$$

Again Diff w.r to x

$$\frac{d}{dx} f'(x) = 9\frac{d}{dx} x^2(3x-4)^2 + 2\frac{d}{dx} x \cdot (3x-4)^3$$

$$f''(x) = 9\left[x^2 \frac{d}{dx}(3x-4)^2 + (3x-4)^2 \frac{d}{dx} x^2\right] + 2\left[x \frac{d}{dx}(3x-4)^3 + (3x-4)^3 \frac{d}{dx} x\right]$$

$$9\left[x^2 \cdot 2(3x-4) \cdot 3 + (3x-4)^2 \cdot 2x\right] + 2\left[x \cdot 3(3x-4)^2 \cdot 3 + (3x-4)^3\right]$$

$$9\left[6x^2(3x-4) + 2x(3x-4)^2\right] + 2\left[9x(3x-4)^2 + (3x-4)^3\right]$$

$$54x^2(3x-4) + 18x(3x-4)^2 + 18x(3x-4)^2 + 2(3x-4)^3$$

$$f''(x) = 36x(3x-4)^2 + 54x^2(3x-4) + 2(3x-4)^3$$

Q #8 $F(x) = (x^2+5x-1)^4$

Diff w.r.to x.

$$\frac{d}{dx} F(x) = \frac{d}{dx} (x^2+5x-1)^4$$

$$= 4(x^2+5x-1)^{4-1} \frac{d}{dx}(x^2+5x-1)$$

$$= 4(x^2+5x-1)^3 \cdot [2x+5-0]$$

$$= 4(x^2+5x-1)^3 (2x+5)$$

$$F'(x) = (8x+20)(x^2+5x-1)^3$$

Again Diff w.r.to x.

$$\frac{d}{dx}(F'(x) = \frac{d}{dx} (8x+20)(x^2+5x-1)^3$$

$$u \cdot v$$

$$F''(x) = (8x+20)\frac{d}{dx}(x^2+5x-1)^3 + (x^2+5x-1)^3 \frac{d}{dx}(8x+20)$$

$$= (8x+20)\left[3(x^2+5x-1)^2 \cdot (2x+5)\right] + (x^2+5x-1)^3 [8]$$

$$= (8x+20)\left[(6x+15)(x^2+5x-1)^2\right] + 8(x^2+5x-1)^3$$

Q#9 $f(x) = \cos \log x$.

Sol

Diff w.r.to x.

$\frac{d}{dx} f(x) = \frac{d}{dx} \cos \log x$.

$f'(x) = -\sin \log x \frac{d}{dx} \log x$.

$= -\sin \log x \cdot \log$

$f'(x) = -\log \sin \log x$.

Again Diff w.r.to x.

$\frac{d}{dx} f'(x) = \frac{d}{dx}(-\log \sin \log x)$

$f''(x) = -\log \frac{d}{dx} \sin \log x$

$= -\log \cos \log x \cdot \frac{d}{dx} \log x$

$= -\log \cdot \cos \log x \cdot \log$

$f''(x) = -\log \log \cos \log x$

Q#10 $f(x) = \tan \frac{x}{2}$

Sol

Diff w.r.to x

$$\frac{d}{dx} f(x) = \frac{d}{dx} \tan\frac{x}{2}$$

$$f'(x) = \sec^2\frac{x}{2} \frac{d}{dx}\left(\frac{x}{2}\right)$$

$$f'(x) = \sec^2\frac{x}{2} \cdot \frac{1}{2}$$

$$f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$$

Again Diff w.r.t x

$$\frac{d}{dx} f'(x) = \frac{1}{2}\frac{d}{dx}\sec^2\frac{x}{2}$$

$$= \frac{1}{2}\left[2 \sec^{2-1}\frac{x}{2} \cdot \frac{d}{dx}\sec\frac{x}{2}\right]$$

$$= \frac{1}{2}\left[2\sec\frac{x}{2} \cdot \sec\frac{x}{2}\tan\frac{x}{2} \cdot \frac{d}{dx}\frac{x}{2}\right]$$

$$= \frac{1}{2}\left[2\sec\frac{x}{2} \cdot \sec\frac{x}{2} \cdot \tan\frac{x}{2} \cdot \frac{1}{2}\right]$$

$$f''(x) = \frac{1}{2}\sec^2\frac{x}{2}\tan\frac{x}{2}$$

Q#11 $F(\theta) = \sin^2 5\theta$

Sol

Diff w.r.to θ.

$$\frac{d}{d\theta} F(\theta) = \frac{d}{d\theta} \sin^2 5\theta$$

$$F'(\theta) = 2 \sin 5\theta \cdot \frac{d}{d\theta} \sin 5\theta$$

$$F'(\theta) = 2 \sin 5\theta \cdot \cos 5\theta \cdot \frac{d}{d\theta} 5\theta$$

$$F'(\theta) = 2 \sin 5\theta \cos 5\theta \cdot 5$$

$$F'(\theta) = 10 \sin 5\theta \cos 5\theta$$

Again Diff w.r.to θ.

$$\frac{d}{d\theta} F'(\theta) = \frac{d}{d\theta} 10 \sin 5\theta \cos 5\theta$$

$$= 10 \frac{d}{d\theta} \sin 5\theta \cos 5\theta$$

$$= 10 \cdot \left[\sin 5\theta \frac{d}{d\theta} \cos 5\theta + \cos 5\theta \frac{d}{d\theta} \sin 5\theta \right]$$

$$= 10 \left[\sin 5\theta \cdot (-\sin 5\theta \cdot 5) + \cos 5\theta \cdot (\cos 5\theta \cdot 5) \right]$$

$$= 10 \left[-5 \sin^2 5\theta + 5 \cos^2 5\theta \right]$$

$$= -50 \left[\sin^2 5\theta - \cos^2 5\theta \right]$$ Ans

(12) $F(\theta) = \frac{1}{3+2\cos\theta}$.

Sol^n

$$F(\theta) = (3+2\cos\theta)^{-1}$$

Diff w.r.to θ.

$$\frac{d}{d\theta}F(\theta) = \frac{d}{d\theta}(3+2\cos\theta)^{-1}$$

$$= -1(3+2\cos\theta)^{-1-1}\frac{d}{d\theta}(3+2\cos\theta)$$

$$= -(3+2\cos\theta)^{-2}\cdot[0+2\cdot-\sin\theta]$$

$$F'(\theta) = (2\sin\theta)(3+2\cos\theta)^{-2}$$

Again Diff w.r.to θ.

$$\frac{d}{d\theta}F'(\theta) = \frac{d}{d\theta}(2\sin\theta)(3+2\cos\theta)^{-2}$$

$$= 2\sin\theta\frac{d}{d\theta}(3+2\cos\theta)^{-2} + (3+2\cos\theta)^{-2}\frac{d}{d\theta}2\sin\theta$$

$$= 2\sin\theta[-2(3+2\cos\theta)^{-2-1}\cdot(-2\sin\theta)] + (3+2\cos\theta)^{-2}\cdot 2\cos\theta$$

$$= 8\sin^2\theta(3+2\cos\theta)^{-3} + 2\cos\theta(3+2\cos\theta)^{-2}$$

$$= 2(3+2\cos\theta)^{-2}[4\sin^2\theta(3+2\cos\theta)^{-1} + \cos\theta]$$

Ans.

(13) $F(x) = e^{2x}(x^2+1)$

Sol

Diff w.r to x.

$$\frac{d}{dx}F(x) = \frac{d}{dx}e^{2x}(x^2+1)$$

$$F'(x) = e^{2x}\frac{d}{dx}(x^2+1) + (x^2+1)\frac{d}{dx}e^{2x}$$

$$= e^{2x}(2x) + (x^2+1)e^{2x}\cdot 2$$

$$F'(x) = 2xe^{2x} + 2e^{2x}(x^2+1)$$

Again Diff w.r to x.

$$\frac{d}{dx}F'(x) = \frac{d}{dx}2xe^{2x} + \frac{d}{dx}2e^{2x}(x^2+1)$$

$$= 2\left[x\frac{d}{dx}e^{2x} + e^{2x}\frac{d}{dx}x\right] + 2\left[e^{2x}\frac{d}{dx}(x^2+1) + (x^2+1)\frac{d}{dx}e^{2x}\right]$$

$$= 2\left[x\cdot e^{2x}\cdot 2 + e^{2x}\right] + 2\left[e^{2x}\cdot 2x + (x^2+1)\cdot e^{2x}\cdot 2\right]$$

$$= 4xe^{2x} + 2e^{2x} + 4xe^{2x} + 4e^{2x}(x^2+1)$$

$$= 8xe^{2x} + 2e^{2x} + 4x^2e^{2x} + 4e^{2x}$$

$$= 8xe^{2x} + 2e^{2x} + 4x^2e^{2x} + 4e^{2x}$$

$$F''(x) = 4x^2e^{2x} + 8xe^{2x} + 6e^{2x} \quad : \text{Ans}$$

(14) $F(x) = (x^2+1)\ln(x^2+1)$.

Sol:

Diff w.r to x.

$$\frac{d}{dx}F(x) = \frac{d}{dx}(x^2+1)\ln(x^2+1)$$

$$= (x^2+1)\cdot\frac{d}{dx}\ln(x^2+1) + \ln(x^2+1)\frac{d}{dx}(x^2+1)$$

$$= (x^2+1)\cdot\frac{1}{x^2+1}\cdot 2x + \ln(x^2+1)\cdot 2x$$

$$F'(x) = 2x + 2x\ln(x^2+1)$$

$$F'(x) = 2x\left[1 + \ln(x^2+1)\right]$$

$u \cdot v$

Again Diff w.r to x.

$$\frac{d}{dx}F'(x) = \frac{d}{dx}2x\,(1+\ln(x^2+1))$$

$u \cdot v$

$$= 2x\frac{d}{dx}(1+\ln(x^2+1)) + (1+\ln(x^2+1))\frac{d}{dx}2x$$

$$= 2x\left[0 + \frac{1}{x^2+1}\cdot 2x\right] + (1+\ln(x^2+1))\cdot 2$$

$$= \frac{4x^2}{x^2+1} + 2(1+\ln(x^2+1)) \quad \text{Ans.}$$

Find the indicated derivative.

(15) $y = 4x^7 + x^6 - x^4$; $\frac{d^4y}{dx^4}$

Sol $y = 4x^7 + x^6 - x^4$

$\frac{dy}{dx} = 28x^6 + 6x^5 - 4x^3$

$\frac{d^2y}{dx^2} = 168x^5 + 30x^4 - 12x^2$

$\frac{d^3y}{dx^3} = 840x^4 + 120x^2 - 24x$

$\frac{d^4y}{dx^4} = 3360x^3 + 360x^2 - 24$ Ans.

(16) $y = \frac{2}{x}$; $\frac{d^5y}{dx^5}$

$y = 2 \cdot x^{-1}$

$\frac{dy}{dx} = -2x^{-2}$

$\frac{d^2y}{dx^2} = 4x^{-3}$

$\frac{d^3y}{dx^3} = -12x^{-4}$

$\frac{d^4y}{dx^4} = 48x^{-5}$

$\frac{d^5y}{dx^5} = -240x^{-6}$ Ans

(17) $f(x) = \cos \pi x$ $f'''(x)$

Sol $f'(x) = -\sin \pi x \cdot \pi$

$f'(x) = -\pi \sin \pi x$

$f''(x) = -\pi \cos \pi x \cdot \pi$

$f''(x) = -\pi^2 \cos \pi x$

$f'''(x) = -\pi^2 \cdot -\sin \pi x \cdot \pi$

$f'''(x) = \pi^3 \sin \pi x$ Ans.

(18) $f(x) = \frac{1}{\sec(2x+1)}$ $f^4(x)$

Sol $f(x) = \cos(2x+1)$

$f'(x) = -\sin(2x+1) \cdot 2$

$f'(x) = -2 \sin(2x+1)$

$f''(x) = -2 \cos(2x+1) \cdot 2$

$f''(x) = -4 \cos(2x+1)$

$f'''(x) = -4 \cdot -\sin(2x+1) \cdot 2$

$F'''(x) = 8\sin(2x+1)$

$F^{IV}(x) = 8 \cdot \cos(2x+1) \cdot 2$

$F^{IV}(x) = 16\cos(2x+1)$ Ans.

(9) $F(x) = x^3 + 2x$

a) Find $F'(x)$ and $F''(x)$

$F(x) = x^3 + 2x$.

$F'(x) = 3x^2 + 2$

$F''(x) = 6x$

b)

$$F''(x) = \lim_{\Delta x \to 0} \frac{F'(x+\Delta x) - F'(x)}{\Delta x}$$

$F'(x) = 3x^2 + 2$

$F'(x+\Delta x) = 3(x+\Delta x)^2 + 2$

$$F''(x) = \lim_{\Delta x \to 0} \frac{3(x+\Delta x)^2 + 2 - 3x^2 - 2}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{3x^2 + 3\Delta x^2 + 6x\Delta x + 2 - 3x^2 - 2}{\Delta x}$$

$$\lim_{\Delta n \to 0} \frac{3\Delta n^2 + 6n\Delta n}{\Delta n}$$

$$\lim_{\Delta n \to 0} \frac{\Delta n(3\Delta n + 6n)}{\Delta n}$$

$$= 3(0) + 6n$$

$$F''(n) = 6n$$

(20) (a) $\frac{d^2}{dn^2}(Fg) = F''g + 2F'g' + Fg''$

Proof

$$\frac{d}{dn}(F \cdot g) = F\frac{d}{dn}g + g\frac{d}{dn}F$$

$$= Fg' + gF'$$

$$\frac{d^2}{dn^2}(F \cdot g) = \frac{d}{dn}Fg' + \frac{d}{dn}gF'$$

$$= [Fg'' + g'F'] + [gF'' + F'g']$$

$$= Fg'' + g'F' + gF'' + F'g'$$

$$= F''g + 2F'g' + Fg''$$ Proved

(b) $\frac{d^3}{dx^3} fg = f'''g + 3f''g' + 3f'g'' + fg''$

Sol

$$\frac{d^2}{dx^2} f \cdot g = f''g + 2f'g' + fg''$$

Again Diff

$$\frac{d^3}{dx^3} f \cdot g = \frac{d}{dx} f''g + 2\frac{d}{dx} f'g' + \frac{d}{dx} fg''$$

$$= \left[f'' \cdot g' + g f'''\right] + 2\left[f'g'' + g'f''\right] + \left[fg''' + g''f'\right]$$

$$= f''g' + gf''' + 2f'g'' + 2f''g' + fg''' + f'g''$$

$$= 3f''g' + 3f'g'' + fg'' + f'''g \quad \text{proved.}$$

Complete.