Exc 2.6

Derivative of Trignometric Functions:

- $\frac{d}{dx}\sin x = \cos x$
- $\frac{d}{dx}\cos x = -\sin x$
- $\frac{d}{dx}\tan x = \sec^2 x$
- $\frac{d}{dx}\sec x = \sec x \tan x$
- $\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$
- $\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

Note: Whenever we take derivative of trigonometric function then we always take derivative of its angle.

$$\frac{d}{dx}\sin 4x = \cos 4x \frac{d}{dx} 4x$$

$$= \cos 4x \cdot 4 \frac{d}{dx} x$$

$$= 4\cos 4x$$

Find the Derivative of the given Function w.r.to x.

i) $y = x^2 - \cos x$

Sol

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 - \cos x)$$

$$\frac{dy}{dx} = \frac{d}{dx}x^2 - \frac{d}{dx}\cos x$$

$$\frac{dy}{dx} = \left[2x^{2-1}\right] - \left[-\sin x\right]$$

$$\frac{dy}{dx} = 2x + \sin x$$ Ans.

(2) $y = 4x^3 + x + \sin x$.

Sol

Diff w.r.to x.

$$\frac{dy}{dx} = \frac{d}{dx}(4x^3 + x + \sin x)$$

$$= \frac{d}{dx}4x^3 + \frac{d}{dx}x + \frac{d}{dx}\sin x.$$

$$= 4\frac{d}{dx}x^3 + \frac{d}{dx}x + \frac{d}{dx}\sin x$$

$$= \left[4 \cdot 3 \cdot x^{3-1} + 1 + \cos x\right]$$

$= 12x^2 + 1 + \cos x$ Ans

(3) $y = 3\cos x - 5\cot x$

Sol Diff w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}(3\cos x - 5\cot x)$$

$$= \frac{d}{dx}3\cos x - \frac{d}{dx}5\cot x$$

$$= 3\frac{d}{dx}\cos x - 5\frac{d}{dx}\cot x$$

$$= 3[-\sin x] - 5[-\csc^2 x]$$

$$\frac{dy}{dx} = -3\sin x + 5\csc^2 x$$

(4) $y = \sin x \cos x$

Sol Diff w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x \cos x)$$

$$= \sin x \frac{d}{dx}\cos x + \cos x \frac{d}{dx}\sin x$$

$$\frac{dy}{dx} = \sin x[-\sin x] + \cos x[\cos x]$$

$$\frac{dy}{dx} = -\sin^2 x + \cos^2 x \quad \text{OR} \quad \cos 2x$$

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(5) $y = (x^2 + \sin x)\sec x.$

Sol Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + \sin x)\sec x.$$

Using Product rule.

$$\frac{dy}{dx} = (x^2 + \sin x)\frac{d}{dx}\sec x + \sec x \frac{d}{dx}(x^2 + \sin x)$$

$$= (x^2 + \sin x).(\sec x \tan x) + \sec x(2x + \cos x).$$

$$= x^2 \sec x \tan x + \sin x \sec x \tan x + 2x \sec x + \cos x \sec x$$

$$= x^2 \sec x \tan x + \sin x . \frac{1}{\cos x} . \tan x + 2x \sec x + \cos x . \frac{1}{\cos x}$$

$$= x^2 \sec x \tan x + \tan x \tan x + 2x \sec x + 1$$

$$\frac{dy}{dx} = x^2 \tan x \sec x + \tan^2 x + 2x \sec x + 1 \quad \text{Ans.}$$

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(6) $y = \dfrac{5 - \cos x}{5 + \sin x}$

Sol

Diff w.r.to x.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{5-\cos x}{5+\sin x}\right)$$

Apply Quotient Rule.

$$= \frac{(5+\sin x)\frac{d}{dx}(5-\cos x) - (5-\cos x)\frac{d}{dx}(5+\sin x)}{(5+\sin x)^2}$$

$$= \frac{5+\sin x\,[0-(-\sin x)] - (5-\cos x)\,[0+\cos x]}{(5+\sin x)^2}$$

$$= \frac{5+\sin x\,[\sin x] - (5-\cos x)\,[\cos x]}{(5+\sin x)^2}$$

$$= \frac{5\sin x + \sin^2 x - 5\cos x + \cos^2 x}{(5+\sin x)^2}$$

$$= \frac{5\sin x - 5\cos x + \sin^2 x + \cos^2 x}{(5+\sin x)^2}$$

$$\frac{dy}{dx} = \frac{5(\sin x - \cos x) + 1}{(5+\sin x)^2} \quad \text{Ans.}$$

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(7) $y = \dfrac{\sec x}{1+\tan x}$

Sol

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sec x}{1+\tan x}\right)$$

$$= \frac{1+\tan x \frac{d}{dx}\sec x - \sec x \frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$= \frac{(1+\tan x)[\sec x \tan x] - \sec x\,[0 + \sec^2 x]}{(1+\tan x)^2}$$

$$= \frac{\sec x \tan x + \sec x \tan^2 x - \sec^3 x}{(1+\tan x)^2}$$

$$\frac{dy}{dx} = \frac{\sec x\,[\tan x + \tan^2 x - \sec^2 x]}{(1+\tan x)^2}$$ Ans.

Q#8 $\quad y = \dfrac{\sin x}{x^2 + \sin x}$.

Sol

Diff w.r.to x.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin x}{x^2+\sin x}\right)$$

$$= \frac{(x^2+\sin x)\frac{d}{dx}\sin x - \sin x \frac{d}{dx}(x^2+\sin x)}{(x^2+\sin x)^2}$$

$$= \frac{(x^2+\sin x)[\cos x] - \sin x\,[2x + \cos x]}{(x^2+\sin x)^2}$$

$$= \frac{x^2\cos x + \cancel{\sin x\cos x} - 2x\sin x - \cancel{\sin x\cos x}}{(x^2+\sin x)^2}$$

$$\frac{dy}{dx} = \frac{x^2\cos x - 2x\sin x}{(x^2+\sin x)^2}$$

$$\frac{dy}{dx} = \frac{x(x\cos x - 2\sin x)}{(x^2+\sin x)^2} \quad \text{Ans.}$$

Q#9 $\quad y = \dfrac{\cot x}{x+1}$

Sol

Diff w.r.to x.

$$\frac{dy}{dx} = \frac{(x+1)\frac{d}{dx}\cot x - \cot x\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{(x+1)\cdot[-\operatorname{cosec}^2 x] - \cot x\,[1+0]}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{-x\operatorname{cosec}^2 x - \operatorname{cosec}^2 x - \cot x}{(x+1)^2} \quad \text{Ans.}$$

Q#10 $\quad y = (1+\cos x)(x-\sin x)$

Sol

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}(1+\cos x)(x-\sin x)$$

$$= (1+\cos x)\frac{d}{dx}(x-\sin x) + (x-\sin x)\frac{d}{dx}(1+\cos x)$$

$$= (1+\cos x)(1-\cos x) + (x-\sin x)(0-\sin x)$$

$$= 1 - \cos^2 x - x\sin x + \sin^2 x$$

Ans.

Derivative of Inverse Trignometric Functions.

i) $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$

iii) $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

iv) $\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$

v) $\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$

vi) $\frac{d}{dx}\operatorname{cosec}^{-1}x = \frac{-1}{x\sqrt{x^2-1}}$

Note: When ever we take derivative of any trignometric function then we always take derivative of its angle.

$$\frac{d}{dx}\sin^{-1}(2x) = \frac{1}{\sqrt{1-(2x)^2}}\frac{d}{dx}(2x)$$

$$= \frac{2}{\sqrt{1-(2x)^2}}$$

Q#11

$$y = \sin^{-1}(5x-1)$$

Sol

Diff w.r.to x.

$$\frac{dy}{dx} = \frac{d}{dx}\sin^{-1}(5x-1)$$

Using $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(5x-1)^2}}\cdot\frac{d}{dx}(5x-1)$$

$$= \frac{1}{\sqrt{1-(5x-1)^2}}\left[\frac{d}{dx}5x - \frac{d}{dx}1\right]$$

$$= \frac{1}{\sqrt{1-\left((5x)^2+(1)^2-2(5x)(1)\right)}}\cdot\left[5\frac{d}{dx}x - 0\right]$$

$$= \frac{1}{\sqrt{1-(25x^2+1-10x)}}\left[5\cdot(1)-0\right]$$

$$= \frac{5}{\sqrt{\cancel{1}-25x^2-\cancel{1}+10x}} = \frac{5}{\sqrt{-25x^2+10x}}$$

$$= \frac{5}{\sqrt{10x-25x^2}} \quad \text{Ans.}$$

Q#12

$$y = 4\cot^{-1}\left(\frac{x}{2}\right)$$

Sol Diff w.r.to x.

$$\frac{dy}{dx} = \frac{d}{dx}4\cot^{-1}\left(\frac{x}{2}\right)$$

$$= 4\frac{d}{dx}\cot^{-1}\left(\frac{x}{2}\right)$$

Using $\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$

$$= 4\cdot\frac{-1}{1+\left(\frac{x}{2}\right)^2}\cdot\frac{d}{dx}\frac{x}{2}$$

$$= \frac{-4}{1+\frac{x^2}{4}}\cdot\frac{1}{2}\frac{d}{dx}x$$

$$= \frac{-4}{\frac{4+x^2}{4}} \cdot \frac{1}{2} \cdot 1$$

$$= \frac{-4 \cdot 4^2}{4+x^2} \cdot \frac{1}{\cancel{2}\,1}$$

$$\frac{dy}{dx} = \frac{-8}{4+x^2} \quad \text{Ans.}$$

Q#13 $\quad y = \dfrac{\sin^{-1}x}{\sin x}$

Sol Diff w.r.to x.

$$\frac{dy}{dx} = \frac{d}{dx}\frac{\sin^{-1}x}{\sin x}$$

$$= \frac{\sin x \frac{d}{dx}\sin^{-1}x - \sin^{-1}x \frac{d}{dx}\sin x}{(\sin x)^2}$$

$$= \frac{\sin x \cdot \left[\frac{1}{\sqrt{1-x^2}}\right] - \sin^{-1}x \cdot \cos x}{\sin^2 x}$$

$$= \frac{\frac{\sin x}{\sqrt{1-x^2}} - \frac{\cos x \sin^{-1}x}{1}}{\sin^2 x}$$

$$= \frac{1}{\sin^2 x}\left[\frac{\sin x - \sqrt{1-x^2}\cos x \sin^{-1}x}{\sqrt{1-x^2}}\right] \quad \text{Ans.}$$

Q#14 $y = \dfrac{\sec^{-1}x}{x}$

Sol Diff w.r.to x,

$$\frac{dy}{dx} = \frac{d}{dx}\frac{\sec^{-1}x}{x}$$

$$= \frac{x\frac{d}{dx}\sec^{-1}x - \sec^{-1}x\frac{d}{dx}x}{x^2}$$

$$= \frac{1}{x^2}\left[\cancel{x}\cdot\frac{1}{\cancel{x}\sqrt{x^2-1}} - \sec^{-1}x\cdot 1\right]$$

$$= \frac{1}{x^2}\left[\frac{1}{\sqrt{x^2-1}} - \sec^{-1}x\right]$$

$$= \frac{1}{x^2}\left[\frac{1-\sqrt{x^2-1}\sec^{-1}x}{\sqrt{x^2-1}}\right] \quad \text{Ans.}$$

Q#15 $y = x\sin^{-1}x + x\cos^{-1}x$

Sol

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}(x\cdot\sin^{-1}x) + \frac{d}{dx}(x\cos^{-1}x)$$

$$\left[x\cdot\frac{d}{dx}\sin^{-1}x + \sin^{-1}x\frac{d}{dx}x\right] + \left[x\frac{d}{dx}\cos^{-1}x + \cos^{-1}x\frac{d}{dx}x\right]$$

$$\left[x\cdot\cancel{\frac{1}{\sqrt{1-x^2}}} + \sin^{-1}x\cdot 1\right] + \left[x\cdot\cancel{\frac{-1}{\sqrt{1-x^2}}} + \cos^{-1}x\cdot 1\right]$$

$$= \sin^{-1}x + \cos^{-1}x \quad \text{Ans.}$$

$$\left[\frac{x+\sin^{-1}x\sqrt{1-x^2}}{\sqrt{1-x^2}}\right]+\left[\frac{-x+\cos^{-1}x\sqrt{1-x^2}}{\sqrt{1-x^2}}\right]$$

$$\frac{\cancel{x}+\sin^{-1}x\sqrt{1-x^2}\cancel{-x}+\cos^{-1}x\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

$$=\frac{\sin^{-1}x\sqrt{1-x^2}+\cos^{-1}x\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

$$=\cancel{\sqrt{1-x^2}}\left[\frac{\sin^{-1}x+\cos^{-1}x}{\cancel{\sqrt{1-x^2}}}\right]$$

$$=\sin^{-1}x+\cos^{-1}x.$$

Q#16

$$y=\frac{1}{\tan^{-1}x^2}$$

Sol

Diff w.r.to x.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{\tan^{-1}x^2}\right)$$

$$=\frac{\tan^{-1}x^2\frac{d}{dx}(1)+1\frac{d}{dx}\tan^{-1}x^2}{(\tan^{-1}x^2)^2}$$

$$=\frac{1}{(\tan^{-1}x^2)^2}\left[\tan^{-1}x^2\cdot 0+1\cdot\frac{1}{1+(x^2)^2}\cdot\frac{d}{dx}x^2\right]$$

$$= \frac{1}{(\tan^{-1} x^2)^2} \left[0 - \frac{2x}{1+x^4} \right]$$

$$= \frac{1}{(\tan^{-1} x^2)^2} \cdot \frac{-2x}{1+x^4} \qquad \text{Ans.}$$

Complete.