Exc 2.5

Q#1 Find $\frac{dy}{dx}$ If

(i) $y = \frac{1}{x}$

Sol $y = x^{-1}$

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}(x^{-1})$$

$$= -1\, x^{-1-1}$$

$$= -x^{-2}$$

$$\frac{dy}{dx} = \frac{-1}{x^2}$$ Ans.

(ii) $y = (x^2-7)(x^2+4x+2)$

Sol

Diff w.r.to x.

$$\frac{dy}{dx} = \frac{d}{dx}(x^2-7)(x^2+4x+2)$$

Applg Product Rule:

$$\frac{dy}{dx} = (x^2-7)\frac{d}{dx}(x^2+4x+2) + (x^2+4x+2)\frac{d}{dx}(x^2-7)$$

$$= (x^2-7)\left[\frac{d}{dx}x^2 + 4\frac{d}{dx}x + \frac{d}{dx}2\right] + (x^2+4x+2)\left[\frac{d}{dx}x^2 - \frac{d}{dx}7\right]$$

$$= (x^2-7)[2x+4\cdot 1+0] + (x^2+4x+2)[2x-0]$$

$= (x^2-7)(2x+4) + (x^2+4x+2)\cdot 2x$

$= 2x^3+4x^2-14x-28+2x^3+8x^2+4x$

$= 4x^3+12x^2-10x-28$

(iii) $y = (7x+1)(x^4-x^2-9x)$

Sol Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}(7x+1)(x^4-x^2-9x)$$

$$= (7x+1)\frac{d}{dx}(x^4-x^2-9x) + (x^4-x^2-9x)\frac{d}{dx}(7x+1)$$

$$= (7x+1)\left[\frac{d}{dx}x^4 - \frac{d}{dx}x^2 - 9\frac{d}{dx}x\right] + (x^4-x^2-9x)\left[7\frac{d}{dx}x + \frac{d}{dx}1\right]$$

$= (7x+1)[4x^3-2x-9] + (x^4-x^2-9x)[7+0]$

$= 28x^4-14x^2-63x+4x^3-2x-9+7x^4-7x^2-63x$

$= 35x^4-21x^2+4x^3-128x-9$ Ans.

(iv) $y = \frac{3x+4}{x^2+1}$

Sol Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3x+4}{x^2+1}\right)$$

$$\frac{dy}{dx} = \frac{(x^2+1)\frac{d}{dx}(3x+4) - (3x+4)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{(x^2+1)[3+0] - (3x+4)[2x]}{(x^2+1)^2}$$

$$= \frac{3x^2+3-6x^2-8x}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{-3x^2-8x+3}{(x^2+1)^2} \quad \text{Ans:}$$

(V) $y = \frac{x-2}{x^4+x+1}$

Sol

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x-2}{x^4+x+1}\right)$$

$$\frac{dy}{dx} = \frac{(x^4+x+1)\frac{d}{dx}(x-2) - (x-2)\frac{d}{dx}(x^4+x+1)}{(x^4+x+1)^2}$$

$$= \frac{(x^4+x+1)[1-0] - (x-2)[4x^3+1]}{(x^4+x+1)^2}$$

$$= \frac{[x^4+x+1] - [4x^4+x-8x^3-2]}{(x^4+x+1)^2}$$

$$= \frac{x^4 + x + 1 - 4x^4 - x + 8x^3 + 2}{(x^4 + x + 1)^2}$$

$$\frac{dy}{dx} = \frac{-3x^4 + 8x^3 + 3}{(x^4 + x + 1)^2}$$ Ans.

vii) $$y = \frac{3x^2 + 5}{3x - 1}$$

Sol

Diff w.r. to x.

$$\frac{dy}{dx} = \frac{(3x-1)\frac{d}{dx}(3x^2+5) - (3x^2+5)\frac{d}{dx}(3x-1)}{(3x-1)^2}$$

$$= \frac{(3x-1)[6x+0] - (3x^2+5)[3-0]}{(3x-1)^2}$$

$$= \frac{18x^2 - 6x - 9x^2 - 15}{(3x-1)^2}$$

$$\frac{dy}{dx} = \frac{9x^2 - 6x - 15}{(3x-1)^2}$$

(vii) $$y = \left(\frac{1}{x} + \frac{1}{x^2}\right)(3x^2 + 27)$$

Sol

$$y = \left(\frac{x+1}{x^2}\right)(3x^2 + 27)$$

$$y = \frac{3x^3 + 27x + 3x^2 + 27}{x^2}$$

$$y = \frac{3x^3 + 3x^2 + 27x + 27}{x^2}$$

Diff w.r.to x

$$\frac{dy}{dx} = \frac{x^2 \frac{d}{dx}(3x^3 + 3x^2 + 27x + 27) - (3x^3 + 3x^2 + 27x + 27)\frac{d}{dx}x^2}{(x^2)^2}$$

$$\frac{x^2[9x^2 + 6x + 27] - (3x^3 + 3x^2 + 27x + 27)[2x]}{x^4}$$

$$\frac{9x^4 + 6x^3 + 27x^2 - 6x^4 - 6x^3 - 54x^2 - 54x}{x^4}$$

$$\frac{3x^4 - 27x^2 - 54x}{x^4}$$

$$\frac{dy}{dx} = \frac{x(3x^3 - 27x - 54)}{x^{4-3}}$$

$$\frac{dy}{dx} = \frac{3x^3 - 27x - 54}{x^3}$$ Ans.

viii) $$y = \frac{2 - 3x}{7 - x}$$

Sol

Diff w.r.to x

$$\frac{dy}{dx} = \frac{(7-x)\frac{d}{dx}(2-3x) - (2-3x)\frac{d}{dx}(7-x)}{(7-x)^2}$$

$$\frac{dy}{dx} = \frac{(7-x)[0-3] - (2-3x)[0-1]}{(7-x)^2}$$

$$\frac{dy}{dx} = \frac{-21 + 3x + 2 - 3x}{(7-x)^2}$$

$$\frac{dy}{dx} = \frac{-19}{(7-x)^2}$$

(IX) $y = \dfrac{x^2 - 10x + 2}{x^3 - x}$

Sol

Diff w.r.to x

$$\frac{dy}{dx} = \frac{(x^3-x)\frac{d}{dx}(x^2-10x+2) - (x^2-10x+2)\frac{d}{dx}(x^3-x)}{(x^3-x)^2}$$

$$= \frac{(x^3-x)[2x-10] - (x^2-10x+2)[3x^2-1]}{(x^3-x)^2}$$

$$= \frac{2x^4 - 2x^2 - 10x^3 + 10x - [3x^4 - 30x^3 + 6x^2 - x^2 + 10x - 2]}{(x^3-x)^2}$$

$$= \frac{2x^4 - 2x^2 - 10x^3 + 10x - 3x^4 + 30x^3 - 6x^2 + x^2 - 10x + 2}{(x^3-x)^2}$$

$$\frac{dy}{dx} = \frac{-x^4 + 20x^3 - 7x^2 + 2}{(x^3 - x)^2}$$ Ans.

(X)

$$y = \frac{x^4 + 2x^3 - 1}{x^2}$$

Sol

Diff w.r.to x

$$\frac{dy}{dx} = \frac{x^2 \frac{d}{dx}(x^4 + 2x^3 - 1) - (x^4 + 2x^3 - 1)\frac{d}{dx}x^2}{(x^2)^2}$$

$$\frac{dy}{dx} = \frac{x^2\left[\frac{d}{dx}x^4 + 2\frac{d}{dx}x^3 - \frac{d}{dx}1\right] - (x^4 + 2x^3 - 1)\left[\frac{d}{dx}x^2\right]}{x^4}$$

$$= \frac{x^2\left[4x^3 + 6x^2 - 0\right] - (x^4 + 2x^3 - 1)\left[2x\right]}{x^4}$$

$$= \frac{4x^5 + 6x^4 - 2x^5 - 4x^4 + 2x}{x^4}$$

$$= \frac{4x^5 - 2x^5 + 6x^4 - 4x^4 + 2x}{x^4}$$

$$\frac{dy}{dx} = \frac{2x^5 + 2x^4 + 2x}{x^4} = \frac{x(2x^4 + 2x^3 + 2)}{x^4 x^3}$$

$$\frac{dy}{dx} = \frac{2x^4 + 2x^3 + 2}{x^3}$$ Ans.

(xi) $y = \frac{10}{(x^3-10)^9}$

Sol

$y = 10 \cdot (x^3-10)^{-9}$

Diff w.r.to x

$\frac{dy}{dx} = \frac{d}{dx} 10 (x^3-10)^{-9}$

$= 10 \frac{d}{dx} (x^3-10)^{-9}$

$= 10 \left[-9 (x^3-10)^{-9-1} \frac{d}{dx} (x^3-10) \right]$

$= 10 \left[-9 (x^3-10)^{-10} \cdot (3x^2-0) \right]$

$= -270 x^2 (x^3-10)^{-10}$ Ans.

(xii) $y = \frac{(x^2+1)^2}{3x-2}$

Sol

$y = \frac{x^4+1+2x^2}{3x-2}$

Diff w.r.to x

$\frac{dy}{dx} = \frac{(3x-2) \frac{d}{dx}(x^4+1+2x^2) - (x^4+1+2x^2) \frac{d}{dx}(3x-2)}{(3x-2)^2}$

$$\frac{dy}{dx} = \frac{(3x-2)\left[\frac{d}{dx}x^4 + \frac{d}{dx}1 + 2\frac{d}{dx}x^2\right] - (x^4+1+2x^2)\frac{d}{dx}3x - \frac{d}{dx}2}{(3x-2)^2}$$

$$\frac{dy}{dx} = \frac{(3x-2)\left[4x^3+0+4x\right] - (x^4+1+2x^2)\left[3-0\right]}{(3x-2)^2}$$

$$\frac{dy}{dx} = \frac{(3x-2)(4x^3+4x) - 3(x^4+2x^2+1)}{(3x-2)^2}$$

$$\frac{dy}{dx} = \frac{12x^4+12x^2-8x^3-8x-3x^4-6x^2-3}{(3x-2)^2}$$

$$\frac{dy}{dx} = \frac{9x^4-8x^3+6x^2-8x-3}{(3x-2)^2} \quad \text{Ans.}$$

(XIII) $y = \dfrac{(x+1)^2}{(x-1)^2}$

Sol: Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}\frac{(x+1)^2}{(x-1)^2}$$

$$= \frac{(x-1)^2\frac{d}{dx}(x+1)^2 - (x+1)^2\frac{d}{dx}(x-1)^2}{(x-1)^4}$$

$$= \frac{x^2-2x+1\left[2(x+1)^{2-1}\frac{d}{dx}(x+1)\right] - (x+1)^2\left[2(x-1)^{2-1}\frac{d}{dx}(x-1)\right]}{}$$

$$\frac{(x^2-2x+1)[2(x+1)] - (x^2+2x+1)[2(x-1)]}{(x-1)^4}$$

$$\frac{(x^2-2x+1)(2x+2) - (x^2+2x+1)(2x-2)}{(x-1)^4}$$

$$\frac{2x^3-4x^2+2x+2x^2-4x+2 - [2x^4+4x^2+2x-2x^2-4x-2]}{(x-1)^4}$$

$$\frac{2x^3-4x^2+2x+2x^2-4x+2-2x^4-4x^2-2x+2x^2+4x+2}{(x-1)^4}$$

$$\frac{-2x^4+2x^3-4x^2+4}{(x-1)^4}$$ Ans.

Q #14 Find slope of tangent.

$$y = \frac{4x-1}{x} \quad ; \quad x = -1$$

Sol

$$y = \frac{4x-1}{x}$$

Diff w.r.to x

$$\frac{dy}{dx} = \frac{x\frac{d}{dx}(4x-1) - (4x-1)\frac{d}{dx}x}{x^2}$$

$$\frac{dy}{dx} = \frac{x[4-0] - (4x-1)\cdot[1]}{x^2}$$

$$\frac{dy}{dx} = \frac{4x - 4x + 1}{x^2}$$

$$\frac{dy}{dx} = \frac{1}{x^2}$$

At $x = -1$

$$\frac{dy}{dx} = \frac{1}{(-1)^2} = \frac{1}{1}$$

$$\boxed{\frac{dy}{dx} = 1}$$

(15) $y = \frac{54}{x^2+1}$, $x = 2$

Sol

$$y = \frac{54}{x^2+1}$$

Diff w.r.to x.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{54}{x^2+1}\right)$$

$$= \frac{(x^2+1)\frac{d}{dx}(54) - (54)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)[0] - (54)[2x]}{(x^2+1)^2}$$

$$\frac{dy}{dx} = -\frac{108x}{(x^2+1)^2}$$

At $x = 2$

$$\frac{dy}{dx} = -\frac{108(2)}{(2^2+1)^2}$$

$$\frac{dy}{dx} = -\frac{216}{25} \quad \text{Ans.}$$

(16) $y = \frac{2x+5}{x+2}$, $x = 1$

Sol

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x+5}{x+2}\right)$$

$$\frac{dy}{dx} = \frac{(x+2)\frac{d}{dx}(2x+5) - (2x+5)\frac{d}{dx}(x+2)}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{(x+2)[2\cdot 1 + 0] - (2x+5)[1+0]}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{2x + 4 - 2x - 5}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{-1}{(x+2)^2}$$

At $x = 1$

$$\frac{dy}{dx} = \frac{-1}{(1+2)^2}$$

$$= \frac{-1}{9} \quad \text{Ans.}$$

(17) $y = (2\sqrt{x}+1)(x^3-6)$, $x=0$

Sol: Diff w.r.t x

$$\frac{dy}{dx} = (2\sqrt{x}+1)\frac{d}{dx}(x^3-6) + (x^3-6)\frac{d}{dx}(2\sqrt{x}+1)$$

$$= (2\sqrt{x}+1)\left[3x^2-0\right] + (x^3-6)\left[2\cdot\frac{1}{2}(x)^{\frac{1}{2}-1} + 0\right]$$

$$= (2\sqrt{x}+1)(3x^2) + (x^3-6)(x^{-1/2})$$

$$= 6x^{\frac{1}{2}}\cdot x^2 + 3x^2 + x^{3-\frac{1}{2}} - 6x^{-1/2}$$

$$= 6x^{2+\frac{1}{2}} + 3x^2 + x^{5/2} - 6x^{-1/2}$$

$$= 6x^{5/2} + 3x^2 + x^{5/2} - \frac{6}{x^{1/2}}$$

At $x=0$.

$$= 6(0) + 3(0) + 0 - \frac{6}{0}$$

$$= 0+0+0-\infty$$

$$= \infty$$

Undefined.

Complete.