Exc 2.4

- **Power Rule:** $\frac{d}{dx}$ → Derivative

$$\frac{d}{dx} x^n = n x^{n-1}$$

- **Product Rule:**

$$\frac{d}{dx}(U \cdot V) = U \frac{d}{dx} V + V \frac{d}{dx} U$$

- **Quotient Rule:**

$$\frac{d}{dx}\left(\frac{U}{V}\right) = \frac{V \frac{d}{dx} U - U \frac{d}{dx} V}{V^2}$$

Rules of Derivative:

i) Derivative of x is 1 when we take derivative w.r.to x

$$\frac{d}{dx}(x) = 1 \quad , \quad \frac{d}{dy}(y) = 1 \quad , \quad \frac{d}{dz}(z) = 1$$

ii) Derivative of constant is Zero when it is alone.

$$\frac{d}{dx}(5) = 0 \qquad \frac{d}{dx}(-c) = 0 \qquad \frac{d}{dx}(10) = 0$$

iii) Constant remain as it is when it is with some variable.

$$\frac{d}{dx}(5x) = 5\frac{d}{dx}(x) = 5(1) = 5.$$

iv) When whole power of x is involve first we apply power rule then we take derivative of inner value.

$$\frac{d}{dx}(x^2+1)^5 = 5(x^2+1)^{5-1}\frac{d}{dx}(x^2+1)$$

$$= 5(x^2+1)^4\left[\frac{d}{dx}x^2 + \frac{d}{dx}1\right]$$

$$= 5(x^2+1)^4\left[2x^{2-1} + 0\right]$$

$$= 5(x^2+1)^4 \cdot 2x \quad \text{Ans.}$$

v)

Variable	Constant
x	a
y	b
z	c
θ	1
t	2
	3
	π

Q#1 Find the derivative of the Function w.r.to x.

a) $y = x^9$

Sol Diff wr.to x

$$\frac{d}{dx} y = \frac{d}{dx} x^9$$

$$\frac{dy}{dx} = 9x^{9-1} \qquad \because \frac{d}{dx} x^n = n x^{n-1}$$

$$\frac{dy}{dx} = 9x^8 \quad \text{Ans.}$$

b) $f(x) = 4x^{1/3}$

Sol Diff w.r.to x.

$$\frac{d}{dx} f(x) = \frac{d}{dx} (4x^{1/3})$$

$$f'(x) = 4 \frac{d}{dx} x^{1/3}$$

$$= 4 \cdot \left[\frac{1}{3} x^{1/3-1} \right]$$

$$= \frac{4}{3} x^{-2/3} \quad \text{Ans.}$$

(c) $F(x) = 9$

Sol

Diff w.r.to x

$$\frac{d}{dx} F(x) = \frac{d}{dx} 9$$

$$F'(x) = 0 \quad \text{Ans.}$$

d) $F(x) = 6x^3 + 3x^2 - 10$

Sol

Diff w.r.to x.

$$\frac{d}{dx} F(x) = \frac{d}{dx} (6x^3 + 3x^2 - 10)$$

$$= \frac{d}{dx}(6x^3) + \frac{d}{dx}(3x^2) - \frac{d}{dx}(10)$$

$$= 6\frac{d}{dx}x^3 + 3\frac{d}{dx}x^2 - \frac{d}{dx}(10)$$

$$= 6.[3x^{3-1}] + 3[2x^{2-1}] = 0$$

$$F'(x) = 18x^2 + 6x \quad \text{Ans.}$$

Q#2 Find $f'(x)$

a) $F(x) = \sqrt{5}$

Sol

Diff w.r. to x.

$$\frac{d}{dx} F(x) = \frac{d}{dx} \sqrt{5}$$

$$F'(x) = 0 \quad \text{Ans.}$$

b) $F(x) = \sqrt{5}\, x$

Sol

Diff w.r. to x.

$$\frac{d}{dx} F(x) = \frac{d}{dx} (\sqrt{5}\, x)$$

$$F'(x) = \sqrt{5} \frac{d}{dx} x$$

$$= \sqrt{5} \cdot 1$$

$$F'(x) = \sqrt{5} \quad \text{Ans.}$$

(c) $F(x) = 5\sqrt{x}$

Sol

Diff w.r.to x.

$$\frac{d}{dx} F(x) = \frac{d}{dx} (5\sqrt{x})$$

$$f'(x) = 5 \frac{d}{dx} (x)^{1/2}$$

$$= 5 \cdot \left[\frac{1}{2} x^{1/2 - 1} \right]$$

$$= 5 \cdot \left[\frac{1}{2} x^{-1/2} \right]$$

$$f'(x) = \frac{5}{2\sqrt{x}} \quad \text{Ans}$$

(d) $f(x) = \sqrt{5x}$

Sol Diff w.r.to x

$$\frac{d}{dx} f(x) = \frac{d}{dx} (5x)^{1/2}$$

$$= \frac{1}{2} (5x)^{\frac{1}{2} - 1} \frac{d}{dx} 5x$$

$$= \frac{1}{2} (5x)^{-1/2} \cdot 5 \cdot \frac{d}{dx} x$$

$$= \frac{1}{2\sqrt{5x}} \cdot 5 \cdot 1$$

$$= \frac{5}{2\sqrt{5x}} \quad \text{Ans}$$

Q#3 Determine whether $F'(x)$

a) $F(x) = x^2(x^3+5)$

Sol

$F(x) = x^5 + 5x^2$.

Diff w.r.to x.

$$\frac{d}{dx}F(x) = \frac{d}{dx}(x^5 + 5x^2)$$

$$= \frac{d}{dx}x^5 + \frac{d}{dx}5x^2$$

$$= \frac{d}{dx}x^5 + 5\frac{d}{dx}x^2$$

$$= [5x^{5-1}] + 5[2x^{2-1}]$$

$$= 5x^4 + 10x \quad \text{Ans.}$$

b) $F(x) = (x+9)(x-9)$

Sol

$F(x) = x^2 - (9)^2$

$F(x) = x^2 - 81$

Diff w.r to x.

$$\frac{d}{dx}F(x) = \frac{d}{dx}(x^2 - 81)$$

$$F'(x) = \frac{d}{dx}x^2 - \frac{d}{dx}81$$

$$= 2x^{2-1} - 0$$

$$F'(x) = 2x$$

(c) $F(x) = (x^2 + x^3)^3$

Sol

Diff w.r. to x.

$$\frac{d}{dx} F(x) = \frac{d}{dx} (x^2+x^3)^3$$

$$= 3 \cdot (x^2+x^3)^{3-1} \cdot \frac{d}{dx} (x^2+x^3)$$

$$= 3 (x^2+x^3)^2 \cdot \frac{d}{dx} x^2 + \frac{d}{dx} x^3$$

$$= 3 (x^2+x^3)^2 \cdot [2x^{2-1}] + [3x^{3-1}]$$

$$= 3 (x^2+x^3)^2 \cdot (2x + 3x^2)$$

$$= 3 (x^2+x^3)^2 (2x + 3x^2) \quad \text{Ans.}$$

d) $F(x) = -3x^{-8} + 2\sqrt{x}$

Sol

Diff w.r. to x

$$\frac{d}{dx} F(x) = \frac{d}{dx} (-3x^{-8}) + \frac{d}{dx} (2\sqrt{x})$$

$$= -3 \frac{d}{dx} x^{-8} + 2 \frac{d}{dx} (x)^{1/2}$$

$$= -3 [-8x^{-8-1}] + 2 \left[\frac{1}{2} x^{\frac{1}{2}-1}\right]$$

$= 24x^{-9} + x^{-1/2}$

$F'(x) = \frac{24}{x^9} + \frac{1}{x^{1/2}}$

e) $F(x) = ax^3 + bx^2 + cx + d$

Sol Diff w.r.to x.

$\frac{d}{dx} F(x) = \frac{d}{dx}(ax^3 + bx^2 + cx + d)$

$F'(x) = \frac{d}{dx} ax^3 + \frac{d}{dx} bx^2 + \frac{d}{dx} cx + \frac{d}{dx} d$

$= a\frac{d}{dx} x^3 + b\frac{d}{dx} x^2 + c\frac{d}{dx} x + \frac{d}{dx} d$

$= a[3x^{3-1}] + b[2x^{2-1}] + c[1] + 0$

$= 3ax^2 + 2bx + c$ Ans.

(f) $F(x) = x^{24} + 2x^{1/2} + 3x^8 + 9x^4$

Sol Diff w.r.to x.

$\frac{d}{dx} F(x) = \frac{d}{dx}(x^{24} + 2x^{1/2} + 3x^8 + 9x^4)$

$F'(x) = \frac{d}{dx} x^{24} + \frac{d}{dx} 2x^{1/2} + \frac{d}{dx} 3x^8 + \frac{d}{dx} 9x^4$

$$F'(x) = [24x^{24-1}] + 2\left[\frac{1}{2}x^{1/2-1}\right] + 3[8x^{8-1}] + 9[4x^{4-1}]$$

$$F'(x) = 24x^{3} + x^{-1/2} + 24x^{7} + 36x^{3}$$

Q#4 Find $\frac{dy}{dx}$

a) $y = \frac{x + 2x^{3/2}}{\sqrt{x}}$

Sol

$$y = \frac{x}{\sqrt{x}} + \frac{2x^{3/2}}{\sqrt{x}}$$

$$y = \frac{x}{x^{1/2}} + \frac{2x^{3/2}}{x^{1/2}}$$

$$y = x^{1}.x^{-1/2} + 2x^{3/2}.x^{-1/2}$$

$$y = x^{1-\frac{1}{2}} + 2x^{\frac{3}{2}-\frac{1}{2}}$$

$$y = x^{1/2} + 2x$$

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}x^{1/2} + \frac{d}{dx}2x$$

$$= \frac{1}{2} x^{\frac{1}{2}-1} + 2 \frac{d}{dx} x$$

$$= \frac{1}{2} x^{-\frac{1}{2}} + 2 \cdot 1$$

$$= \frac{1}{2\sqrt{x}} + \frac{2}{1} \quad \text{Am}$$

b) $y = (x^3 - 5)(2x + 3)$

Sol

$$y = 2x^4 + 3x^3 - 10x - 15$$

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}(2x^4 + 3x^3 - 10x - 15)$$

$$= \frac{d}{dx} 2x^4 + \frac{d}{dx} 3x^3 - \frac{d}{dx} 10x - \frac{d}{dx} 15$$

$$= 2 \cdot [4x^3] + 3[3x^2] - 10[1] - 0$$

$$= 8x^3 + 9x^2 - 10 \quad \text{Am.}$$

(C) $y = (4x^2 - 3)(7x^2 + x)$

Sol $y = 28x^4 + 4x^3 - 21x^2 - 3x$

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}\left[28x^4 + 4x^3 - 21x^2 - 3x\right]$$

$$= \frac{d}{dx}28x^4 + \frac{d}{dx}4x^3 - \frac{d}{dx}21x^2 - \frac{d}{dx}3x$$

$$= 28\frac{d}{dx}x^4 + 4\frac{d}{dx}x^3 - 21\frac{d}{dx}x^2 - 3\frac{d}{dx}x$$

$$= 28\left[4x^3\right] + 4\left[3x^2\right] - 21\left[2x\right] - 3\left[1\right]$$

$$\frac{dy}{dx} = 112x^3 + 12x^2 - 42x - 3 \quad \text{Ans}$$

SLOPE OF TANGENT

$$m = \frac{dy}{dx}$$

Q#5 (a) $y = x^2 + 3x$

Sol

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}x^2 + \frac{d}{dx}3x$$

$$= 2x + 3\frac{d}{dx}x$$

$$\frac{dy}{dx} = 2x + 3$$

At $x = 1$

$$\frac{dy}{dx} = 2(1) + 3$$

$$\frac{dy}{dx} = 5 \quad \text{Ans.}$$

b) $y = x^4 - x^2$

Sol

Diff w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}(x^4 - x^2)$$

$$= \frac{d}{dx}x^4 - \frac{d}{dx}x^2$$

$$= 4x^{4-1} - 2x^{2-1}$$

$$\frac{dy}{dx} = 4x^3 - 2x$$

at $x = 1$

$$\left.\frac{dy}{dx}\right|_{x=1} = 4(1)^3 - 2$$

$$= 4 - 2$$

$$= 2$$

Complete