MathCity.org
Merging man and maths

1

# Exc 2.10

## Q#1

Given $m = \dfrac{m_o}{\sqrt{1-\dfrac{v^2}{c^2}}}$ and $V \to \bar{c}$ (L.H.L)

$$\lim_{V\to\bar{c}} m = \lim_{V\to\bar{c}} \frac{m_o}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

Apply Limit

$$= \frac{m_o}{\sqrt{1-\dfrac{c^2}{c^2}}} = \frac{m_o}{\sqrt{1-1}}$$

$$= \frac{m_o}{0} = \infty$$ (undefined.

## Q#2

$$F(x) = \begin{cases} Kx+1 & x \leq 3 \\ 2-Kx & x > 3 \end{cases}$$

is continuous at 3. What is K?

Sol/

$F(x) = Kx+1$ $\quad$ $F(x) = kx+1$ $\quad$ $F(x) = 2-Kx$

3

L H L

CS CamScanner

$$\lim_{x \to 3^-} F(x) = \lim_{x \to 3^-} Kx+1$$
$$= K(3)+1$$
$$= 3K+1$$

R.H.L

Available at MathCity.org

$$\lim_{x \to 3^+} F(x) = \lim_{x \to 3^+} 2-Kx$$
$$= 2-K(3)$$
$$= 2-3K$$

Since given Function is Continuous So

L.H.L = R.H.L

$$3K+1 = 2-3K$$

$$3K+3K = 2-1$$
$$6K = 1$$
$$\boxed{K = \frac{1}{6}}$$

Q#3

$$V = \left(\frac{4}{3}\pi\right) r^3 \quad \text{---} (1)$$

Sol

$$\text{Surface area} = \frac{dv}{dr} = ?$$

Diff w.r. to r

$$\frac{dv}{dr} = \frac{d}{dr}\left(\frac{4\pi}{3}\right) r^3$$

$$= \frac{4\pi}{3} \frac{d}{dr} r^3$$

$$= \frac{4\pi}{3}\left(3r^{3-1}\right)$$

$$= \frac{4\pi}{\cancel{3}} \cdot \cancel{3} r^2$$

$$\frac{dv}{dr} = 4\pi r^2$$

**Available at MathCity.org**

Surface area $= 4\pi r^2$

Q#4

$$S(t) = \frac{1}{2} g t^2 + V_o t + S_o$$

Sol

Instantaneous rate of change

$$\frac{ds}{dt} = ? \qquad S_o, V_o, g \text{ constant.}$$

Diff w.r to $t$

$$\frac{ds}{dt} = \frac{d}{dt} \frac{1}{2} g t^2 + V_o t + S_o$$

CS CamScanner

MathCity.org
Merging man and maths



$$= \frac{1}{2} g \frac{d}{dt} t^2 + V_0 \frac{d}{dt} t + \frac{d}{dt} S_0$$

$$= \frac{1}{\cancel{2}} g \cdot \cancel{2} t + V_0 \cdot 1 + 0$$

$$\frac{ds}{dt} = gt + V_0$$

at $t = 4$

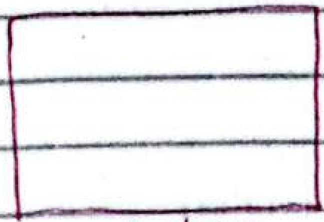
$$\left.\frac{ds}{dt}\right|_{t=4} = g(4) + V_0$$

$$= 4g + V_0$$

5

Length $= x = 10$ cm

Small Change $= \Delta x = dx = \pm 0.3$ cm



$x = 10$cm

Area of Square $= A = L \times W$

$$A = x \times x$$

$$A = x^2$$

$$\frac{dA}{dx} = \frac{d}{dx} x^2$$

$$\frac{dA}{dx} = 2x$$

$$dA = 2x\,dx$$

$$dA = 2(10)(\pm 0.3)$$

Max error in Area $= dA = \pm 6$ cm$^2$

Relative Error $= \dfrac{\text{Max Error}}{\text{exact Value}}$

$$A = x^2$$

$$A = (10)^2$$

$$A = 100 \text{ cm}^2 \text{ (exact)}$$

$$= \frac{6}{100}$$

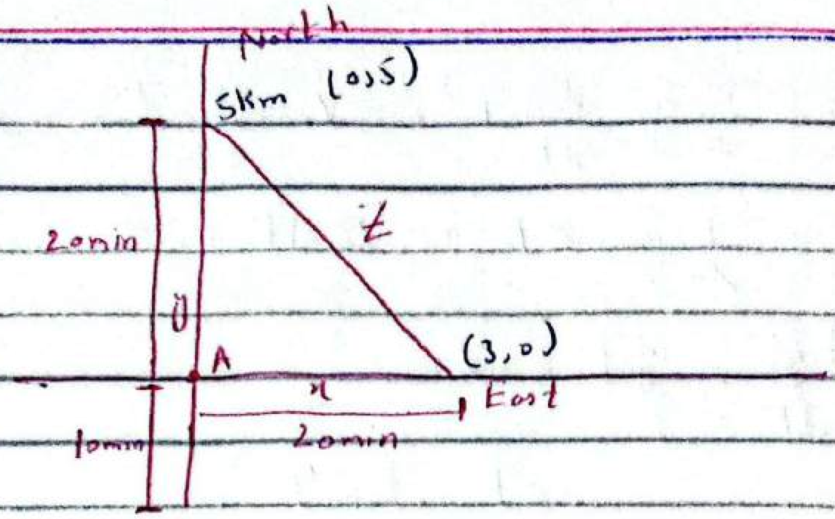
$$A = 0.06$$

Relative error $= 0.06 \times 100\%$

$= 6\%$

CamScanner

Q #6




$$\frac{dz}{dt} = ?$$

Time of Women = 30 min $= \frac{30}{60} = \frac{1}{2}$ hour

Time of Men = 20 min $= \frac{20}{60} = \frac{1}{3}$ hour

Speed = $\frac{\text{Distance}}{\text{time}}$

Distance = Speed × time.

$$y = 10 \times \frac{1}{2} = 5 \text{ Km}.$$

Distance = Speed × time

$$x = 9 \times \frac{1}{3}$$

$$x = 3 \text{ Km}$$

Distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$Z = \sqrt{(0-3)^2 + (5-0)^2}$

$Z = \sqrt{9 + 25}$

$Z = \sqrt{34}$

Using Phy Theorem

$H^2 = P^2 + B^2$

$Z^2 = x^2 + y^2$

Diff w.r.to $t$

$2Z \frac{dZ}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$

Speed

$Z \frac{dZ}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt}$

$Z \frac{dZ}{dt} = 3 \cdot 9 + 5 \cdot 10$

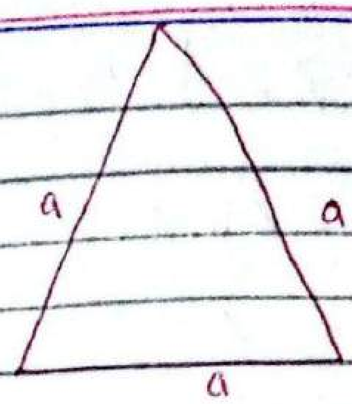
$Z \frac{dZ}{dt} = 77$

$\frac{dZ}{dt} = \frac{77}{Z}$

$\frac{dZ}{dt} = \frac{77}{\sqrt{34}} \approx 13.2$ Ans

CS CamScanner

(1) $\frac{da}{dt} = 2 \text{ cm/hr}$




$a = 8 \text{ cm}$

$\frac{dA}{dt} = ?$

Area of Equilateral Triangle: $\frac{\sqrt{3}}{4} a^2$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2a \cdot \frac{da}{dt}$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{\cancel{4}} \times \cancel{2} \cdot (8)(\cancel{2})$$

$$\frac{dA}{dt} = 8\sqrt{3} \text{ cm}^2/\text{hr}$$

Q#6

$$r = 4\% = \frac{4}{100} = 0.04$$

$$P_0 = 50\$$$

Formula: $P(t) = P_0 e^{rt}$

a) $P(t) = 50 e^{0.04t}$

b) $P(8) = 50\, e^{(0.04)8}$

$P(8) = 68.85$

(c) $\frac{dP}{dt} = P_0 \cdot e^{rt} \cdot r$

At $t = 8$

$= 50\, e^{(0.04)(8)} \cdot (0.04)$

$\frac{dP}{dt} = 2.75$ Ans

17 $C(t) = 500\, e^{0.04t} - 100t$

a) $\frac{dc}{dt} = 500 \frac{d}{dt} e^{0.04t} - 100 \frac{d}{dt} t$

$= 500 \cdot (0.04)\, e^{0.04t} - 100$

$\frac{dc}{dt} = 20 e^{0.04t} - 100$

b) Again Diff w.r.to t

$\frac{d^2c}{dt^2} = 20(0.04)\, e^{0.04t}$

$= \frac{4}{5} e^{0.04t} > 0 \quad \forall t$

Hence cost is minimed.

CS CamScanner

(18) $P(t) = 150(1+0.05t)^2$

Sol

(a) $\dfrac{dP}{dt} = 150 \cdot \dfrac{d}{dt}(1+0.05t)^2$

$= 150\left[2(1+0.05t)^{2-1} \dfrac{d}{dt}(1+0.05t)\right]$

$\dfrac{dP}{dt} = 150\left[2(1+0.05t) \cdot 0.05\right]$

$\dfrac{dP}{dt} = 15\left[1+0.05t\right]$

$\left.\dfrac{dP}{dt}\right|_{t=3} = 15 + 15(0.05)(3)$

$P'(t) = \left.\dfrac{dP}{dt}\right|_{t=3} = 17.25$

(b) Inflation rate

$= \dfrac{P'(3)}{P(3)}$

$P(3) = 150(1+0.05(3))^2$

$P(3) = 198.375$

Inflation rate $= \dfrac{17.25}{198.375} = 0.0869$

Ans

19 $S = d(t) = 15t$

a) Speed $= \frac{ds}{dt}$ or $\frac{dd}{dt}$ $\quad \because S = \frac{d}{t}$

$= \frac{d}{dt} 15t$

$= 15 \longrightarrow$ slope

b) $d(3) = 15(3) = 45$

(c) Here slope is 15 which shows that slope is constant.

20 $S(t) = 5t^2 + 3t$

Sol

(a) Speed(V) $= \frac{ds}{dt} = \frac{d}{dt} 5t^2 + 3t$

$\boxed{V(t) = 10t + 3} \longrightarrow$ slope

(b) $t = 4$

$V(4) = 10(4) + 3$

$\boxed{V(4) = 43}$

(c) Here slope is increase IF increase in time.

10

$y^2 = x+1$ , $\frac{dx}{dt} = 4x+4$

$x = 8$

$$\left.\frac{dx}{dt}\right|_{x=8} = 4(8)+4$$

$$= 32+4$$

$$= 36 \quad \text{An.}$$

12

Let $x$ and $y$ be two non-negative number

$x+y = 60$ — (1) and $P = xy$ — (2)

$\boxed{y = 60-x}$ Put in (II)

$$P(x) = x(60-x)$$
$$P(x) = 60x - x^2$$

$$\frac{dP}{dx} = 60-2x$$

$$\frac{d^2P}{dx^2} = -2 < 0$$

Maximum

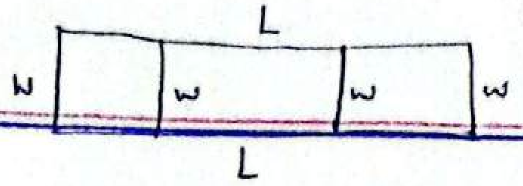
Put $\frac{dP}{dx} = 0$

$$60-2x = 0$$

$$\boxed{x = 30}$$

$$y = 60-30$$

$$\boxed{y = 30}$$




13

Perimeter = Sum of all Sides

$$P = 2L + 4W$$

$$8000 = 2L + 4W$$

$$4000 = L + \cancel{4}^{2}W$$

$$4000 - 2W = L$$

Area of rectangle = $L \times W$

$$A = 4000 - 2W \cdot W$$

$$A = 4000W - 2W^2$$

Diff w.r to W

$$\frac{dA}{dW} = 4000\frac{d}{dW}w - 2\frac{d}{dW}w^2$$

$$\frac{dA}{dW} = 4000 - 4W$$

Again Diff

$$\frac{d^2A}{d^2W} = 0 - 4 < 0$$

maximum area or greatest area.

Put $\frac{dA}{dW} = 0$

$$4000 - 4W = 0$$

$$\boxed{W = 1000}$$

$$L = 4000 - 2(1000)$$

$$\boxed{L = 2000}$$

Q#15

$R(x) = -3x^2 + 970x$ and $C(x) = 2x^2 + 500$

Profit $\Rightarrow P(x) = R(x) - C(x)$

$$P(x) = -3x^2 + 970x - 2x^2 - 500$$

$$\boxed{P(x) = -5x^2 + 970x - 500} \quad \text{--- (A)}$$

$$\frac{dP}{dx} = -10x + 970$$

$$\frac{d^2P}{dx^2} = -10 < 0 \quad \text{maximum profit.}$$

Available at MathCity.org

Put $\frac{dP}{dx} = 0$

$$-10x + 970 = 0$$

$$-10x = -970$$

$$\boxed{x = 97}$$

Put in (A)

$$P(97) = -5(97)^2 + 970(97) - 500$$

$$\boxed{\text{Maximum Profit} = 46545}$$

Average cost $= \frac{C(x)}{x}$

$= \frac{2x^2 + 500}{x}$

$$\boxed{A.C = 2x + 500x^{-1}}$$

Diff w.r.t $x$.

$$\frac{dA}{dx} = 2 - 500x^{-2}$$

$$\frac{d^2A}{dx^2} = +1000x^{-3} > 0$$ Minimum

Put $\frac{dA}{dx} = 0$

$$2 - \frac{500}{x^2} = 0$$

$$2 = \frac{500}{x^2}$$

$$x^2 = 250$$

$$x = \sqrt{250}$$

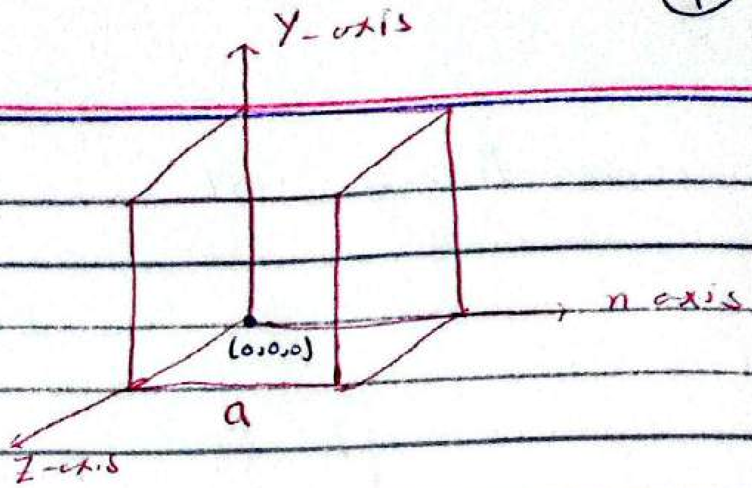
$$A.C = 2\sqrt{250} + \frac{500}{\sqrt{280}}$$

$$A.C = 20\sqrt{10} = 63.24$$

CS CamScanner

9

$\frac{da}{dt} = 5 \text{ cm/hr}$




diagonal.

$\frac{dd}{dt} = ?$

Formula: Diagonal of a cube:

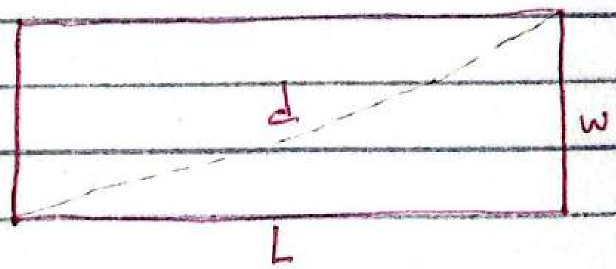
$$\boxed{d = \sqrt{3}\, a}$$

$$\frac{dd}{dt} = \sqrt{3}\, \frac{da}{dt}$$

$$= \sqrt{3} \cdot 5$$

$$\frac{dd}{dt} = 5\sqrt{3} \text{ cm/hr}$$

8




$\frac{dd}{dt} = 1 \text{ in/hr}$

$\frac{dL}{dt} = \frac{1}{4} \text{ in/hr}$

$\frac{dw}{dt} = ?$ $\quad W = 6 \text{in} \quad L = 8 \text{in}$

Using Phy Theore

$$H^2 = P^2 + B^2$$

$$d^2 = L^2 + w^2 \quad \text{--- (1)}$$

$$d^2 = (8)^2 + (6)^2$$

$$\sqrt{d^2} = \sqrt{64 + 36}$$

$$d = \sqrt{100}$$

$$\boxed{d = 10 \text{ inch}}$$

Diff (1) w.r to t

$$2d \frac{dd}{dt} = 2L \frac{dL}{dt} + 2w \frac{dw}{dt}$$

$$2(10) \cdot 1 = 2(8) \cdot \frac{1}{4} + 2 \cdot 6 \cdot \frac{dw}{dt}$$

$$20 = 4 + 12 \frac{dw}{dt}$$

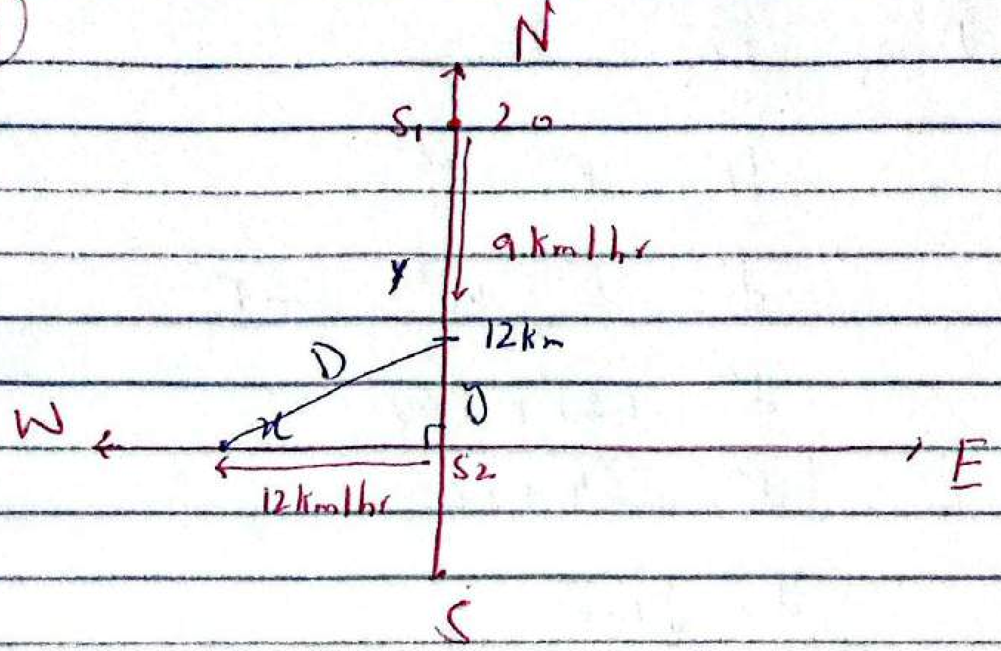
$$20 - 4 = 12 \frac{dw}{dt}$$

$$\frac{\cancel{16}^{4}}{\cancel{12}_{3}} = \frac{dw}{dt}$$

$$\boxed{\frac{dw}{dt} = \frac{4}{3}}$$

CS CamScanner

11




time = 9:20 − 8:00 = 1:20

$= 1 + \frac{20}{60} = 1 + \frac{1}{3} = \frac{4}{3}$ hr

$S_1$(distance) = Speed × time

$= 9 \times \frac{4}{3} = 12$ km

$y = 20 - 12 = 8$ km

$S_2$ (distance) $= x = 12 \times \frac{4}{3} = 16$ km

Using pyg theorem

$D^2 = x^2 + y^2$

$$2D \frac{dD}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

CamScanner

$D = \sqrt{x^2 + y^2}$

$D = \sqrt{16^2 + 8^2}$

$D = 8\sqrt{5}$

because it moves downward towards negative direction

$8\sqrt{5}\,\frac{dD}{dt} = 2(16)\cdot 12 + 2\cdot 8\cdot(-9)$

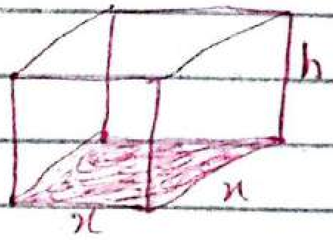
$8\sqrt{5}\,\frac{dD}{dt} = 120$

$\frac{dD}{dt} = \frac{120}{8\sqrt{5}}$

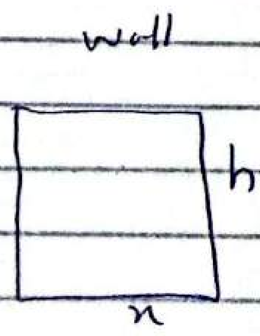
$\frac{dD}{dt} = 6.7$ km/hr.

---

14




wall




Volume $= V = x \times x \times h$

$32000 = x^2 h$

$\boxed{h = \frac{32000}{x^2}}$

Total Surface Area $= x^2 + 4xh$

$A = x^2 + 4x\left(\frac{32000}{x^2}\right)$

$A = x^2 + 128000x^{-1}$

Diff w.r.t $x$

$\frac{dA}{dx} = 2x - 128000\,x^{-2}$

Again Diff

$\frac{d^2A}{dx^2} = 2 + 2 \cdot 128000\,x^{-3}$

$= 2 + \frac{256000}{x^3} > 0$

So Amount is Least or (Minimum).

Put $\frac{dA}{dx} = 0$

Available at MathCity.org

$2x - 128000x^{-2} = 0$

$2x - \frac{128000}{x^2} = 0$

$\frac{2x^3 - 128000}{x^2} = 0$

$2x^3 - 128000 = 0$

$2x^3 = 128000$

$x^3 = \frac{128000}{2}$

CamScanner

$x^3 = 64000$

$x^3 = (40)^3$

Taking cube root

$\boxed{x = 40}$

$h = \dfrac{32000}{(40)^2}$

$h = \dfrac{32000}{1600}$

$\boxed{h = 20}$

Complete

MathCity.org
Merging man and maths

CamScanner