Exc 1.4

Q#1 Find the Domain and Range of the function graphically.

(i) $f(x) = 1 \cdot \sin\left(\frac{x}{2}\right)$

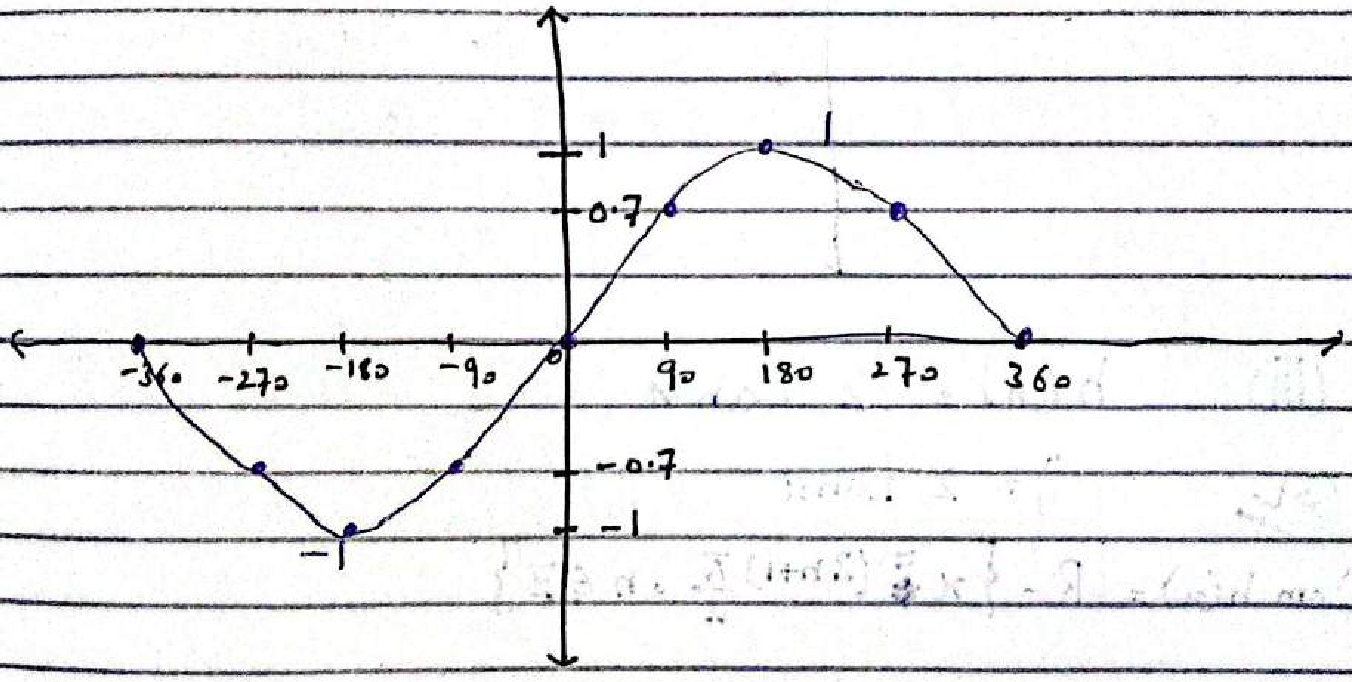
Sol Let $y = 1 \cdot \sin\left(\frac{x}{2}\right)$

Domain $f(x) = (-\infty, +\infty)$
Range $f(x) = [-1, 1]$

$y = a\sin(Bx+C)+D$

$\frac{2\pi}{1/2} = \frac{2\pi}{1} \times \frac{2}{1} = \frac{4\pi}{4} = \pi$

x	-360	-270	-180	-90	0	90	180	270	360
y	0	-0.7	-1	-0.7	0	0.7	1	0.7	0



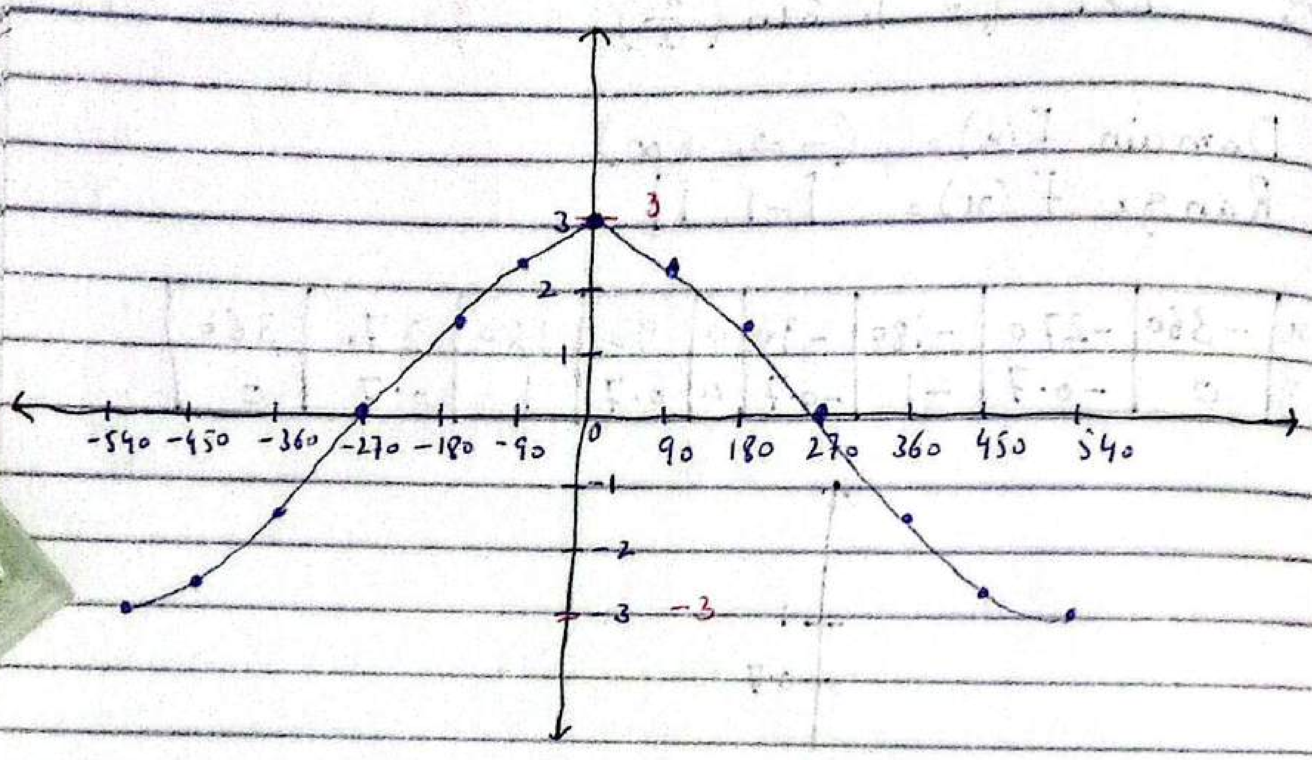
(ii) $g(x) = 3\cos\left(\frac{x}{3}\right)$

Sol Let $y = 3\cos\left(\frac{x}{3}\right)$

Dom $g(x) = (-\infty, +\infty)$

Range $g(x) = [-3, 3]$

x	-540	-450	-360	-270	-180	-90	0	90	180	270°	360	450	540
y	-3	-2.6	-1.5	0	1.5	2.6	3	2.6	1.5	0	-1.5	-2.6	-3



(iii) $h(x) = 2\tan x$

Sol $y = 2\tan x$

Dom $h(x) = R - \{x = (2n+1)\frac{\pi}{2}, n \in Z\}$

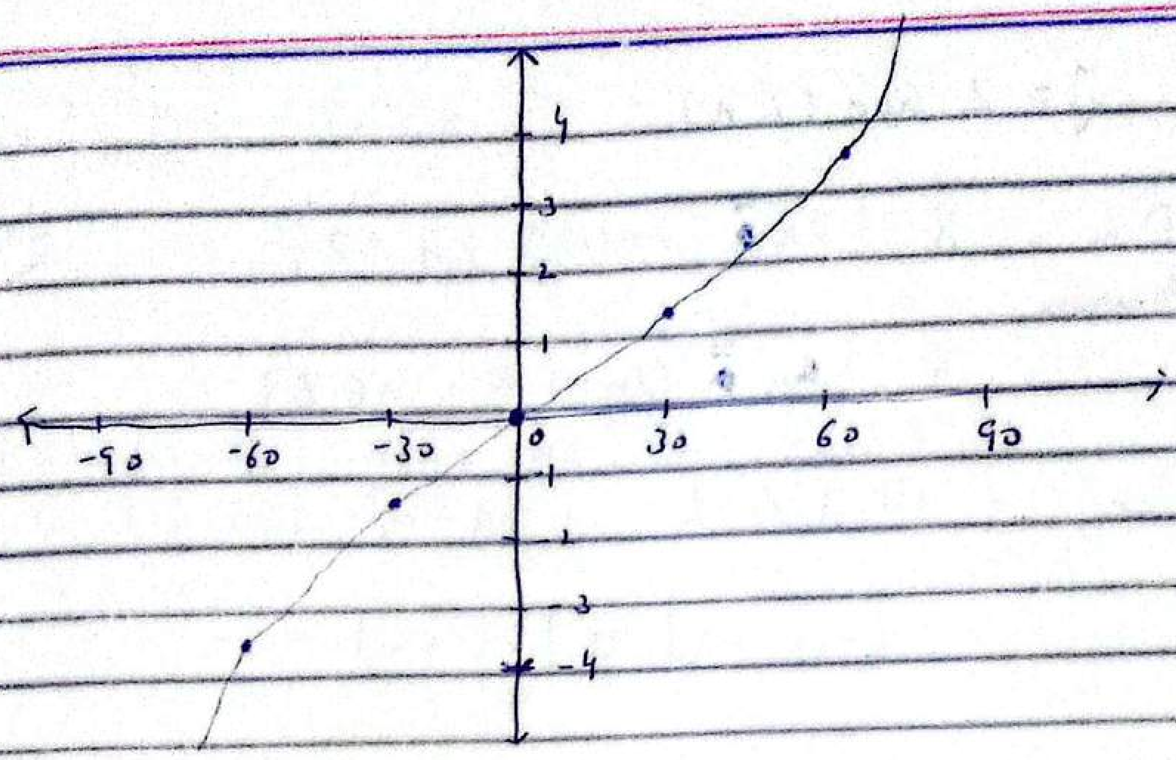
Range $h(x) = (-\infty, +\infty)$

x	-90	-60°	-30	0	30	60	90
y	∞	-3.5	-1.2	0	1.2	3.5	∞

$= 2 \cdot \frac{\sin x}{\cos x}$

$= 2 \frac{\sin x}{\cos(90)} = \frac{2\sin x}{0}$

$= 2 \frac{\sin x}{\cos(3\pi/2)} = \frac{2\sin x}{0}$



(IV) $y = \cot\left(\frac{x}{4}\right)$

Sol

$$\text{Dom } F(x) = R - \left\{ \frac{x}{4} = n\pi \; ; \; n \in Z \right\}$$

or

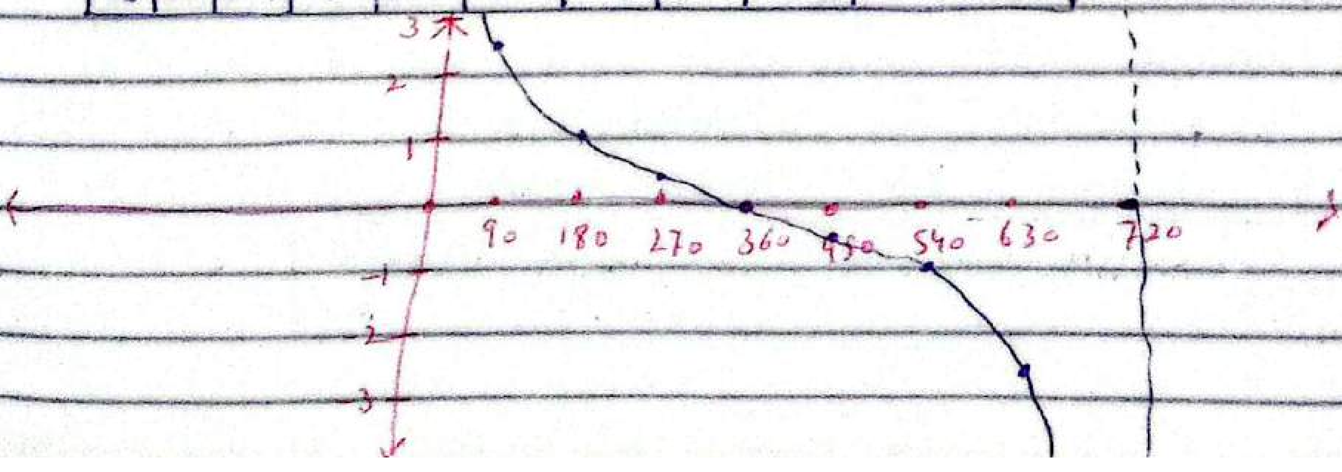
$$\text{Dom } F(x) = R - \{ x = 4n\pi \; ; \; n \in Z \}$$

$$\text{Range } F(x) = (-\infty, +\infty)$$

$\frac{\cos x/4}{\sin x/4}$

$0, \pi, 2\pi, 3\pi$

x	0	90	180	270	360	450	540	630	720
D	∞	2.4	1	0.4	0	-0.4	-1	-2.4	∞



(v) $y = 2\sec(2x)$

$\frac{2}{\cos 2\pi}$

Sol Dom $= R - \{2x \neq (2n+1)\frac{\pi}{2}, n \in Z\}$

or

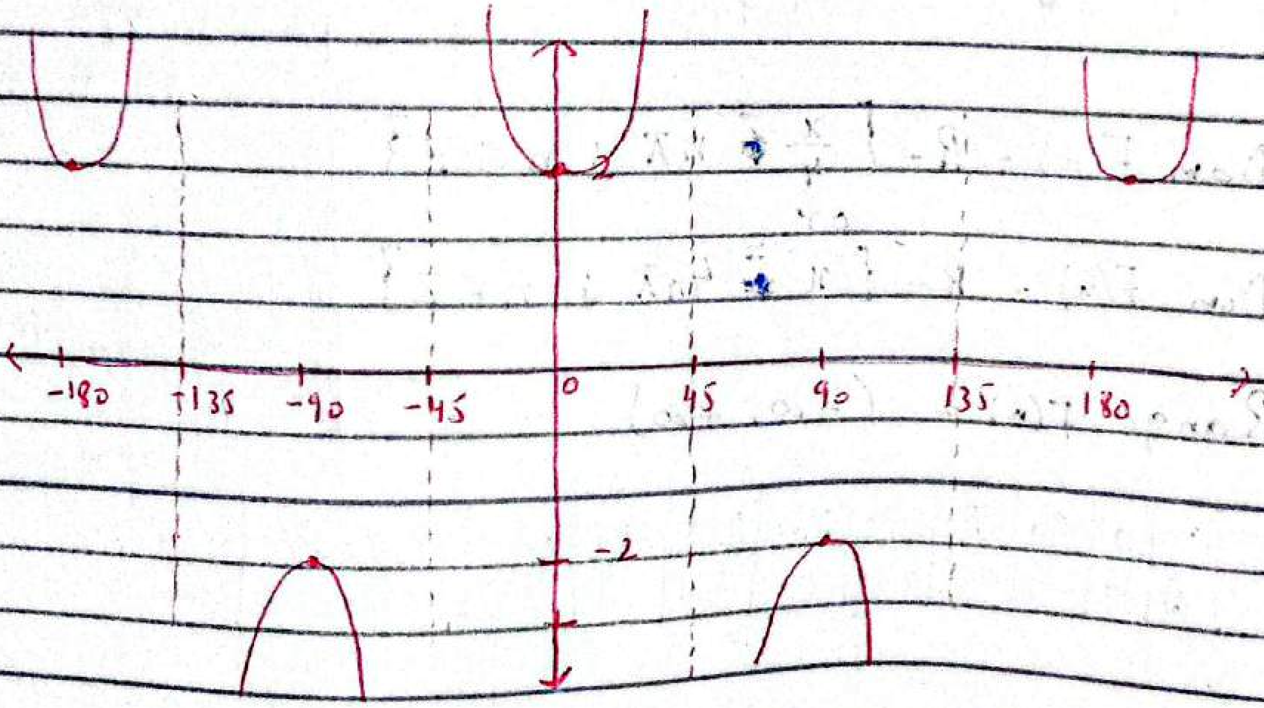
$R - \{x \neq (2n+1)\frac{\pi}{4}, n \in Z\}$

Range $=$ $y \leq -2$ $y \geq 2$

or

$(-\infty, -2] \cup [2, +\infty)$

x	-180	-135	-90	-45	0	45	90	135	180
y	2	∞	-2	∞	2	∞	-2	∞	2



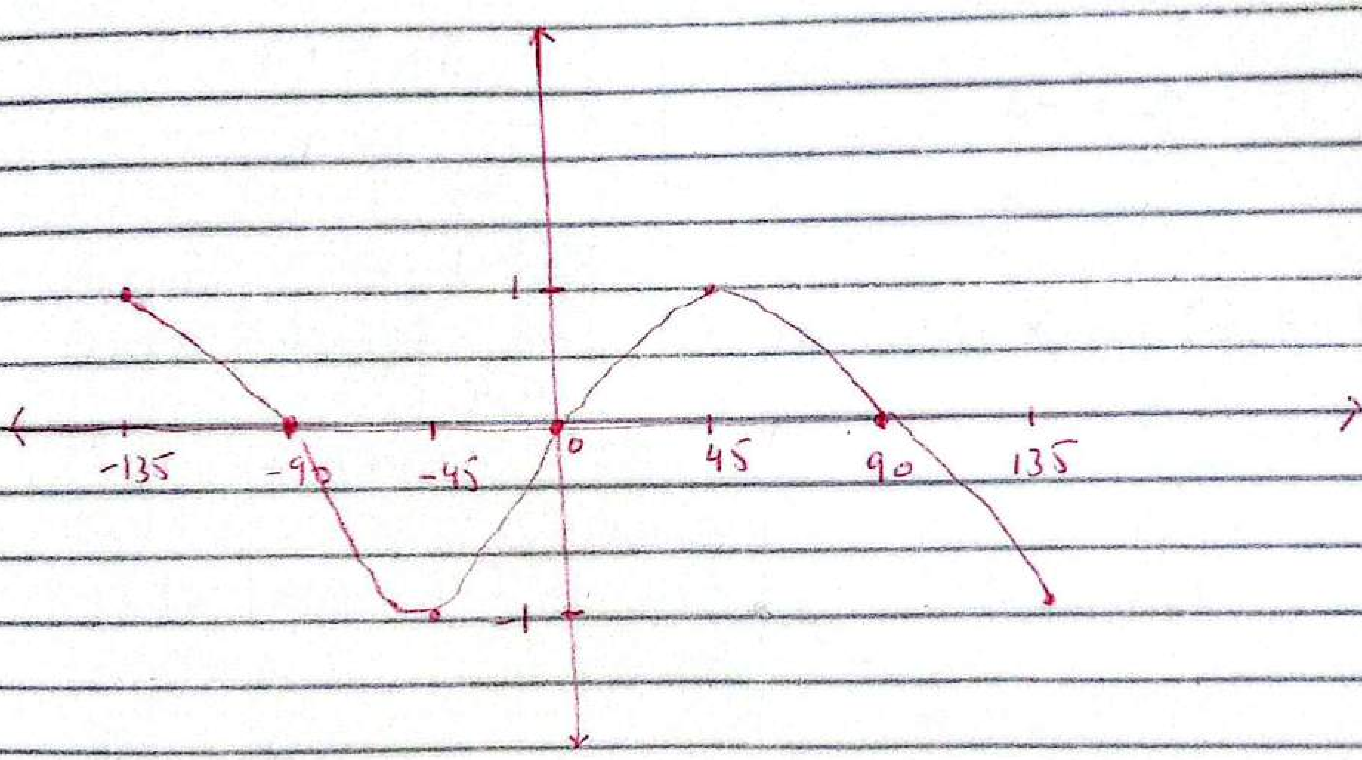
(vi) $y = \sin 2x$

Sol

Dom $h(x) = (-\infty, +\infty)$

Range $h(x) = [-1, 1]$

x	-135	-90°	-45	0	45	90°	135°
y	1	0	-1	0	1	0	-1



Q#2 Determine whether the given Function is one to one by examining its graph. If the Function is one to one then Find the inverse. Also draw the graph of inverse Function.

i) $F(x) = \frac{1}{3}x + 3$

Sol. Let $y = \frac{1}{3}x + 3$

<u>X-intercept</u>

Put $y = 0$

$\frac{x}{3} + 3 = 0$

$\frac{x}{3} = -3$

$x = -9$

$(-9, 0)$ is X intercept

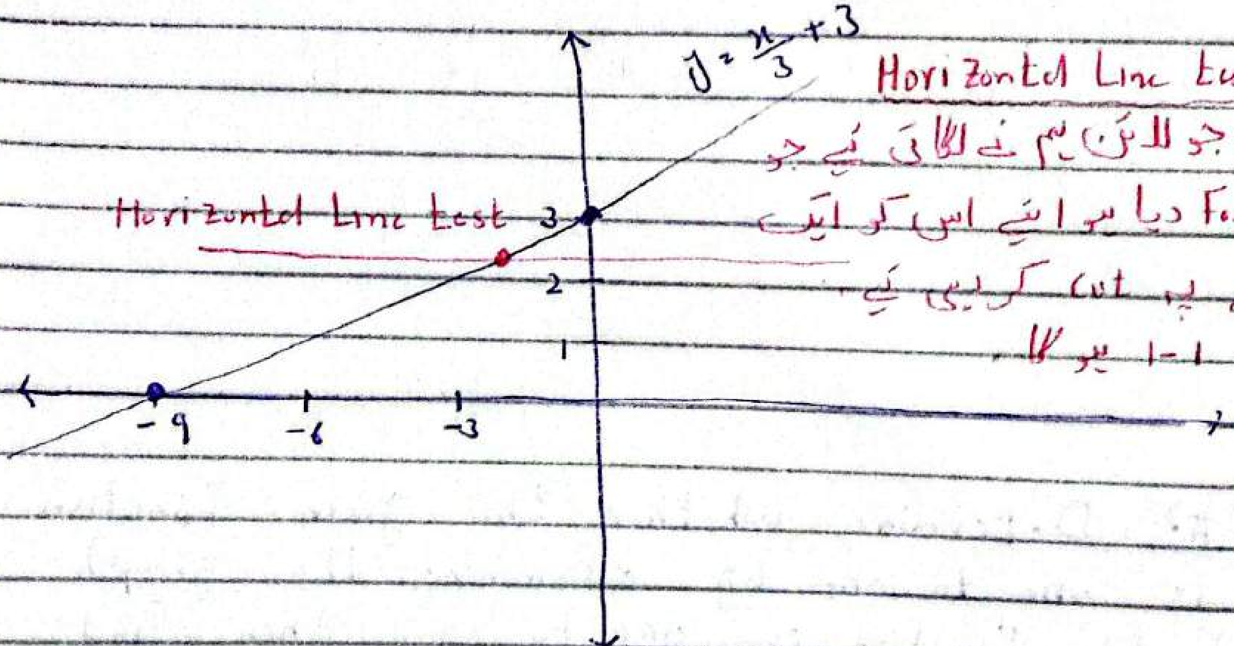
<u>Y-intercept</u>

Put $x = 0$

$y = 0 + 3$

$y = 3$

$(0, 3)$ is Y-intercept



Horizontal Line test

یہ جو لائن ہم نے لگائی ہے جو
Function دیا ہوا ہے اس کو ایک
جگہ پہ cut کر رہی ہے
یہ 1-1 ہوا

Hence given Function is one to one.

Now we have to find its inverse

Let $F(x) = y \Rightarrow x = F^{-1}(y)$

Now

$$y = \frac{x}{3} + 3$$

$$y - 3 = \frac{x}{3}$$

$$3y - 9 = x$$

But $x = f^{-1}(y)$

$$f^{-1}(y) = 3y - 9$$

replace y by x:

$$\boxed{f^{-1}(x) = 3x - 9}$$

$$y = 3x - 9$$

Draw Graph of Inverse Function.

x	No need to do this just Find X-intercept and Y-intercept.
y	

X-intercept

Put $y = 0$

$3x - 9 = 0$

$3x = 9$

$x = 3$

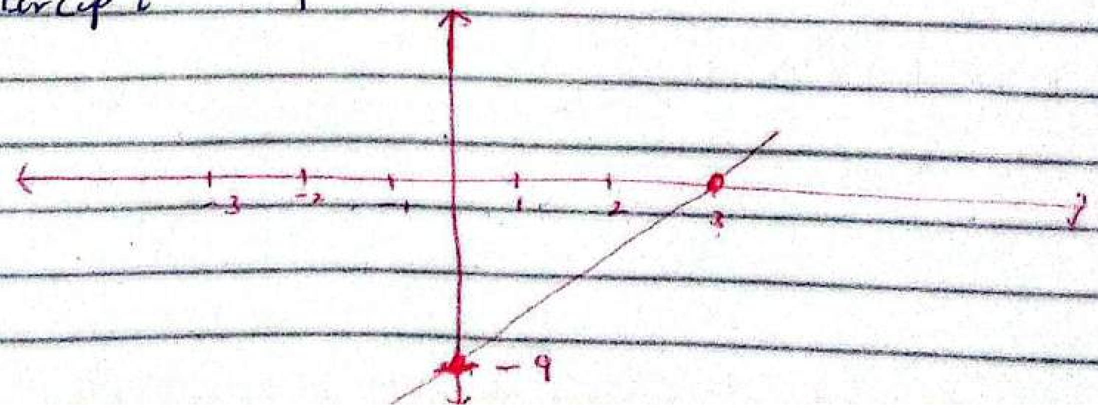
$(3, 0)$ is X-intercept

Y-intercept

Put $x = 0$

$y = -9$

$(0, -9)$ is Y intercept



(iii) $F(x) = x(x-5)$

Sol.

$F(x) = x^2 - 5x$

$y = x^2 - 5x$

$a > 1$

Parabola UPward

X intercept

Put $y=0$

$x(x-5)=0$

$x=0, x-5=0$

$x=0 \quad x=5$

$(0,0)$ and $(5,0)$ are xintercepts

Y-intercept

Put $x=0$

$y=0-0$

$(0,0)$ is the Y-intercept

Vertex

$$x = -\frac{b}{2a} = \frac{-(-5)}{2(1)} = \frac{5}{2} = 2.5$$

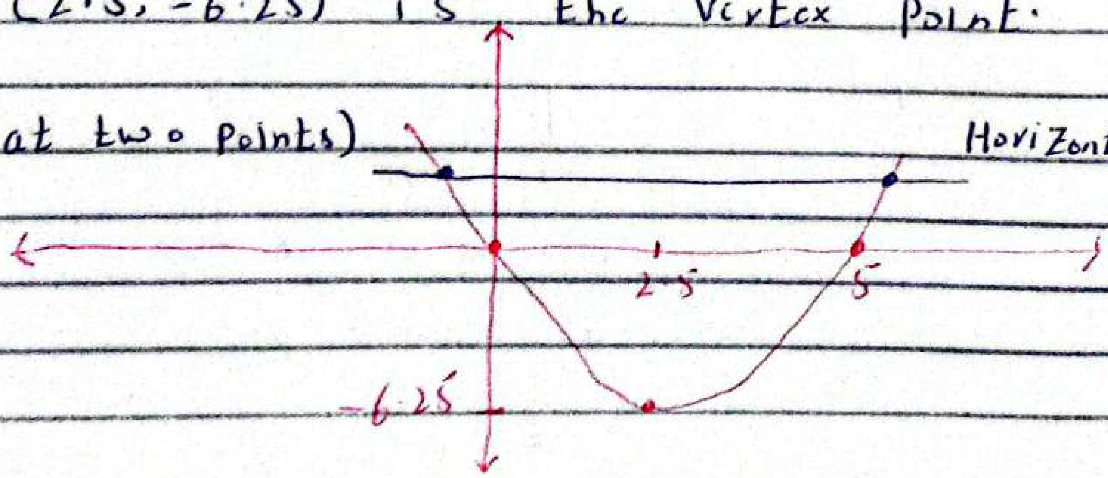
Put $x=2.5$ in given function.

$$y = (2.5)^2 - 5(2.5)$$

$$y = -6.25$$

$(2.5, -6.25)$ is the vertex point.

(cut at two points)



Horizontal Line test

Hence given Function not one-one. We cannot Find its inverse.

(iii) $y = (x+1)^2$

Sol/ $y = x^2 + 2x + 1$ Here $a=1$, $b=2$, $c=1$

X-intercept

put $y=0$

$(x+1)^2 = 0$

$x+1=0 \quad x+1=0$

$x=-1$

$(-1,0)$ is the x-intercept

Y-intercept

Put $x=0$

$y = 0+0+1$

$y=1$

$(0,1)$ is the Y-intercept

Vertex

$$x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$$

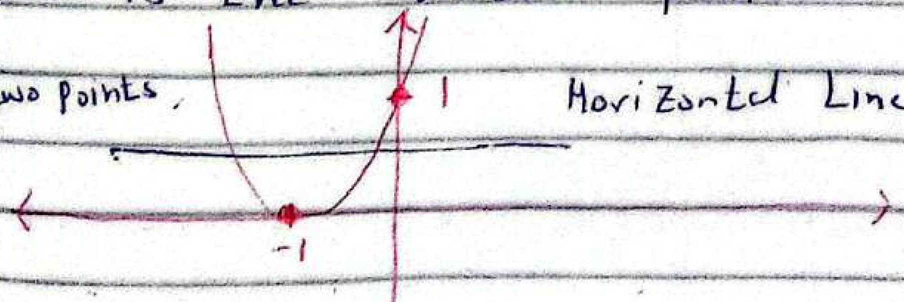
Put $x=-1$ in given function.

$$y = (-1)^2 + 2(-1) + 1$$

$$y = 0$$

$(-1,0)$ is the vertex point.

Cut at two points. Horizontal Line test.



Hence given Function is not one to one, cannot Find its inverse.

(IV) $F(x) = x^3 - 8$

Sol

$y = x^3 - 8$

X-intercept

put $y = 0$

$x^3 - 8 = 0$

$x^3 = 8$

$x = 2$

$(2,0)$ is X-intercept

Y-intercept

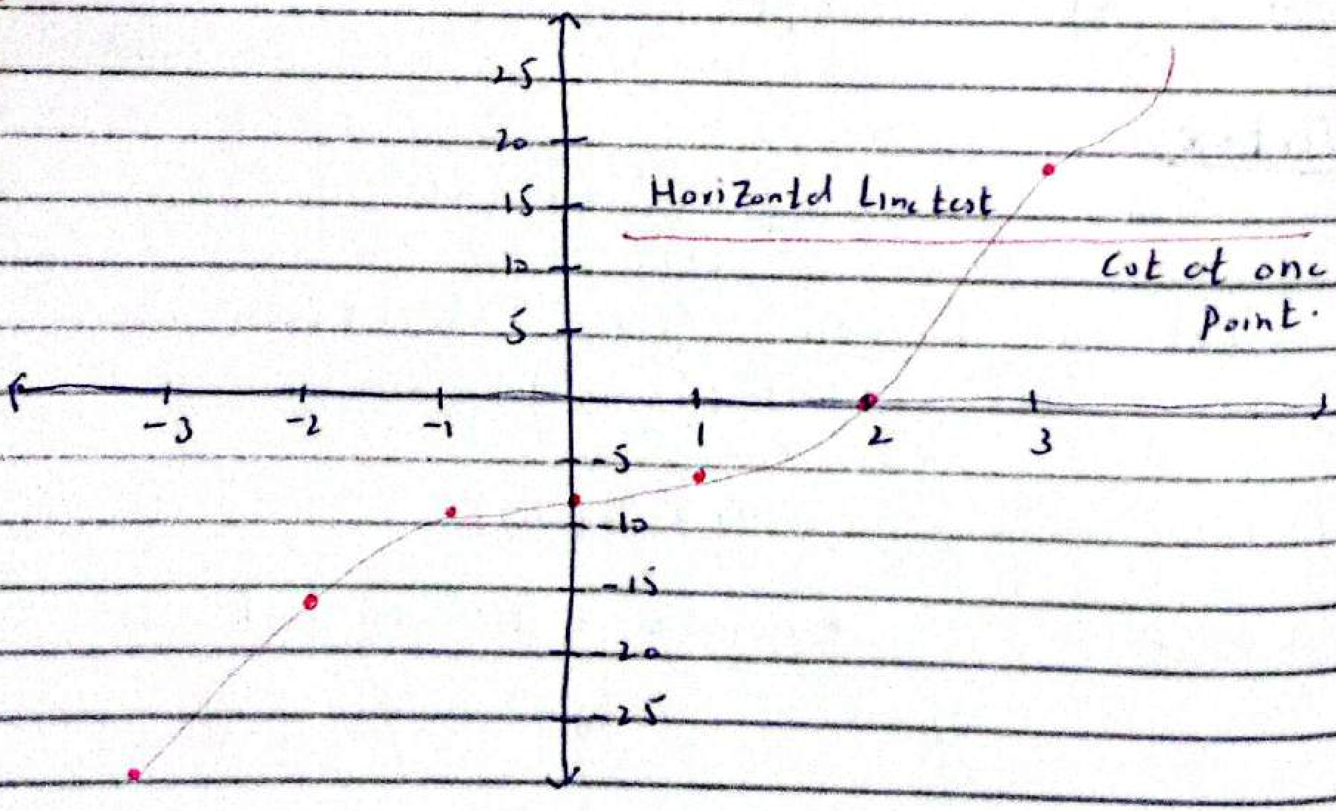
put $x = 0$

$y = 0 - 8$

$y = -8$

$(0,-8)$ is Y-intercept

x	-3	-2	-1	0	1	2	3
y	-35	-16	-9	-8	-7	0	19



Hence given function is one-one

Now we have to Find its inverse

Let $F(x) = y$ $\quad x = F^{-1}(y)$

$$y = x^3 - 8$$

$$y + 8 = x^3$$

$$x = (y+8)^{1/3}$$

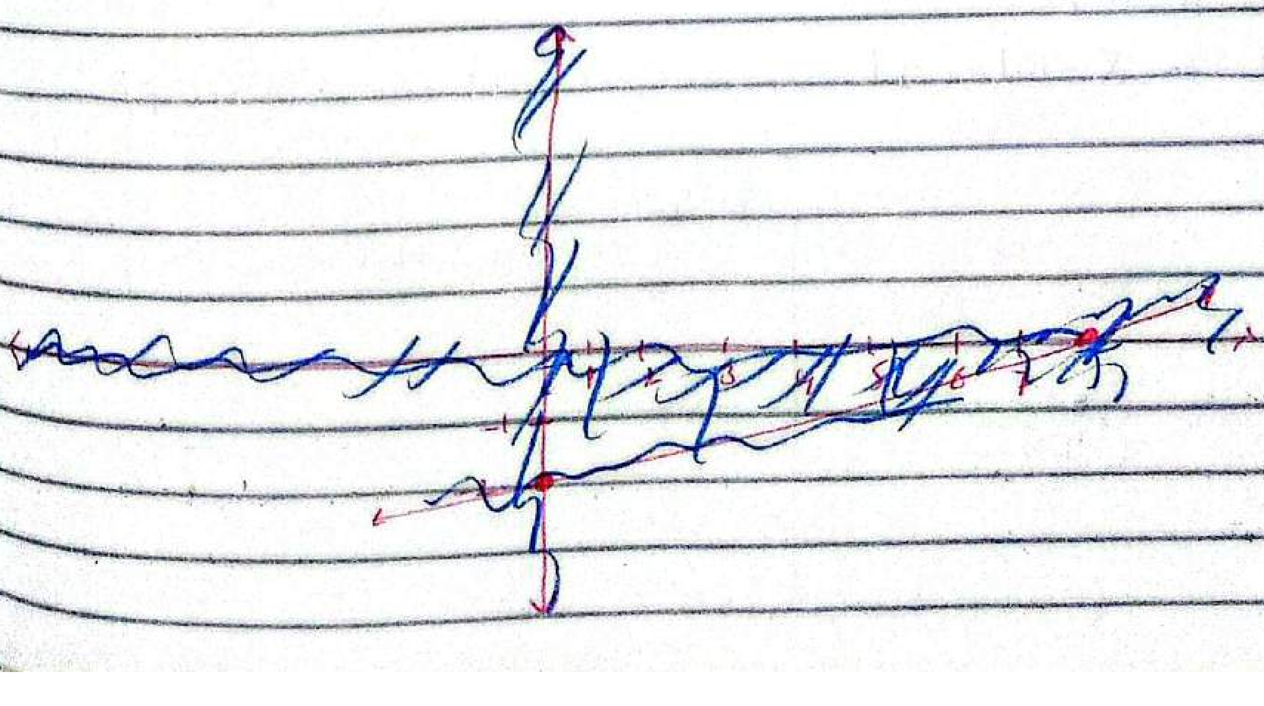
$$F^{-1}(y) = (y+8)^{1/3}$$

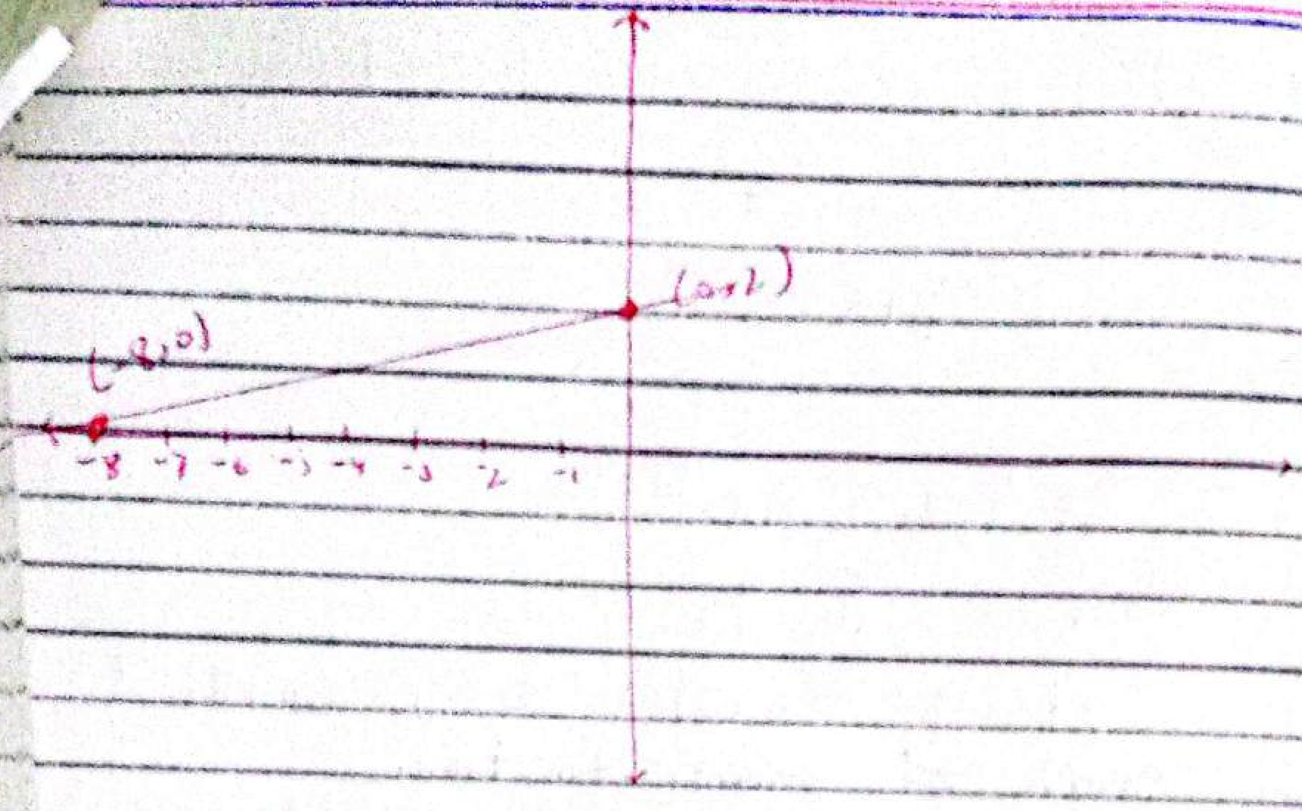
replace y by x.

$$F^{-1}(x) = (x+8)^{1/3} \Rightarrow y = (x+8)^{1/3}$$

Draw graph of inverse function.

X-intercept	Y-intercept
put $y = 0$	put $x = 0$
$(x+8)^{1/3} = 0$	$y = (0+8)^{1/3}$
$x + 8 = 0$	$y = (+2^3)^{1/3}$
$x = -8$	$y = +2$
$(-8, 0)$ is the X-intercept	$(0, +2)$ is the Y-intercept





(v) $g(x) = 4 + x$

Sol. Let $y = 4 + x$

X-intercept

Put $y = 0$

$4 + x = 0$

$x = -4$

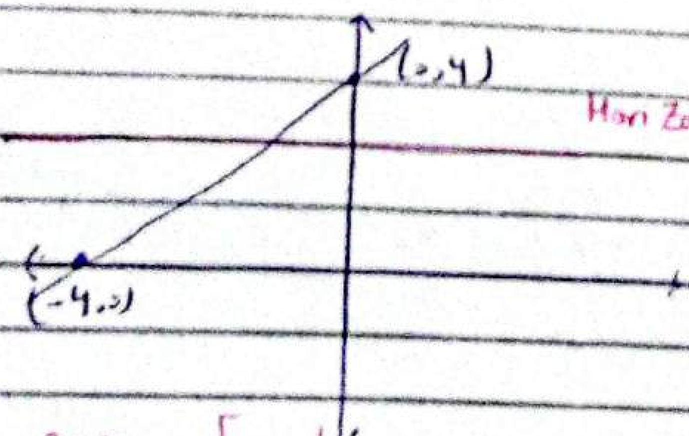
$(-4, 0)$ is X-intercept

Y-intercept

Put $x = 0$

$y = 4$

$(0, 4)$ is Y intercept



Horizontal Line Test

Hence given function is one-one

As $F(x) = y$ $\qquad$ $x = F^{-1}(y)$

$$y = 4 + x$$
$$y - 4 = x$$
$$x = y - 4$$
$$F^{-1}(y) = y - 4$$

replace y by x

$$F^{-1}(x) = x - 4$$

Graph of inverse Function:

Let $y = x - 4$

<u>X-intercept</u>

put $y = 0$

$x - 4 = 0$

$x = 4$

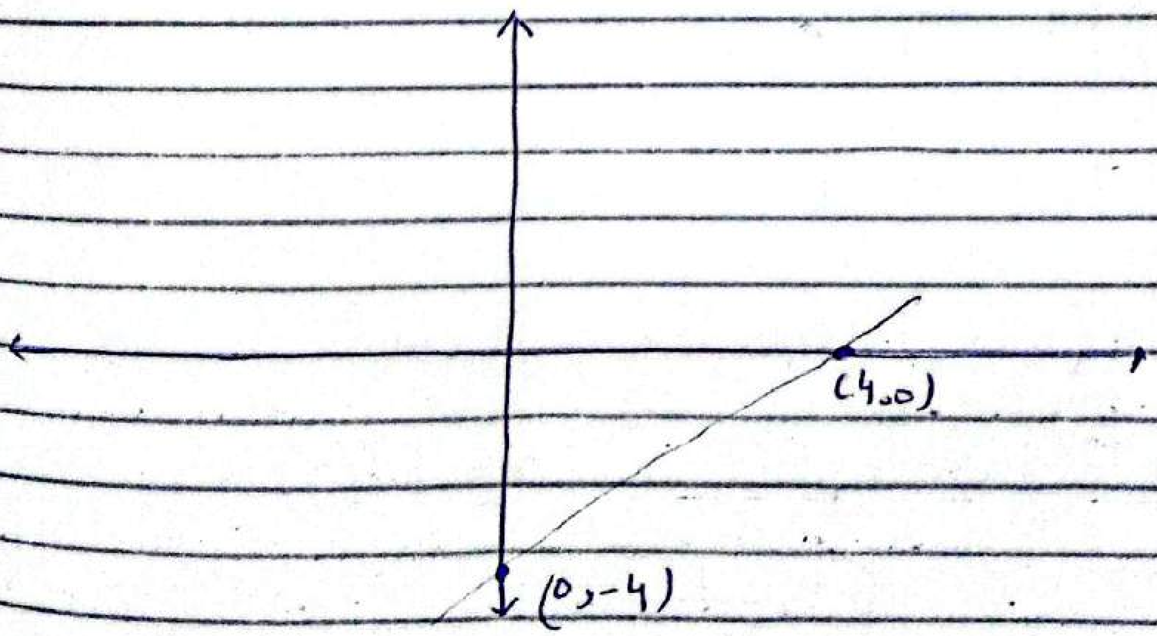
$(4, 0)$ is X-intercept

<u>Y-intercept</u>

put $x = 0$

$y = -4$

$(0, -4)$ is Y-intercept



(vi) $h(x) = \dfrac{1}{3x+5}$

put $3x+5=0$

$x = -5/3$

$x = -1.66$

Sol $y = \dfrac{1}{3x+5}$

X-intercept

put $y=0$

$\dfrac{1}{3x+5} = 0$

$1 = 0$

Not possible

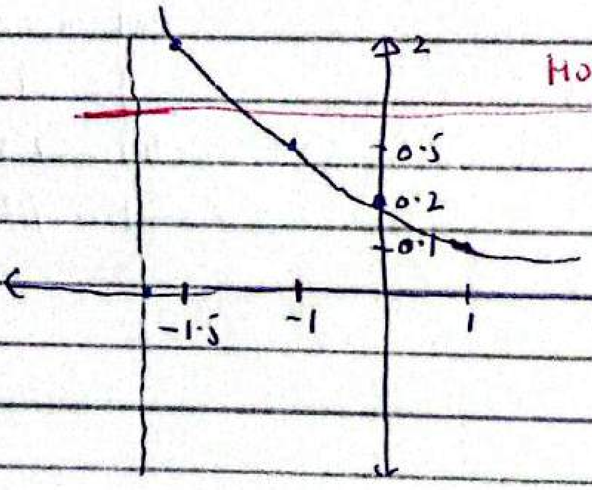
Y-intercept

put $x=0$

$y = \dfrac{1}{5}$

$(0, \frac{1}{5})$ is Y-intercept

x	-1.5	-1	0	1
y	2	0.5	0.2	0.1



Horizontal Line test

Cut at one point

Hence given Function is 1-1.

As $F(x) = y$ $\qquad x = F^{-1}(y)$

$y = \dfrac{1}{3x+5}$

$3xy + 5y = 1$

$3xy = 1 - 5y$

$x = \frac{1-5y}{3y}$

$F^{-1}(y) = \frac{1-5y}{3y}$

replace y by x.

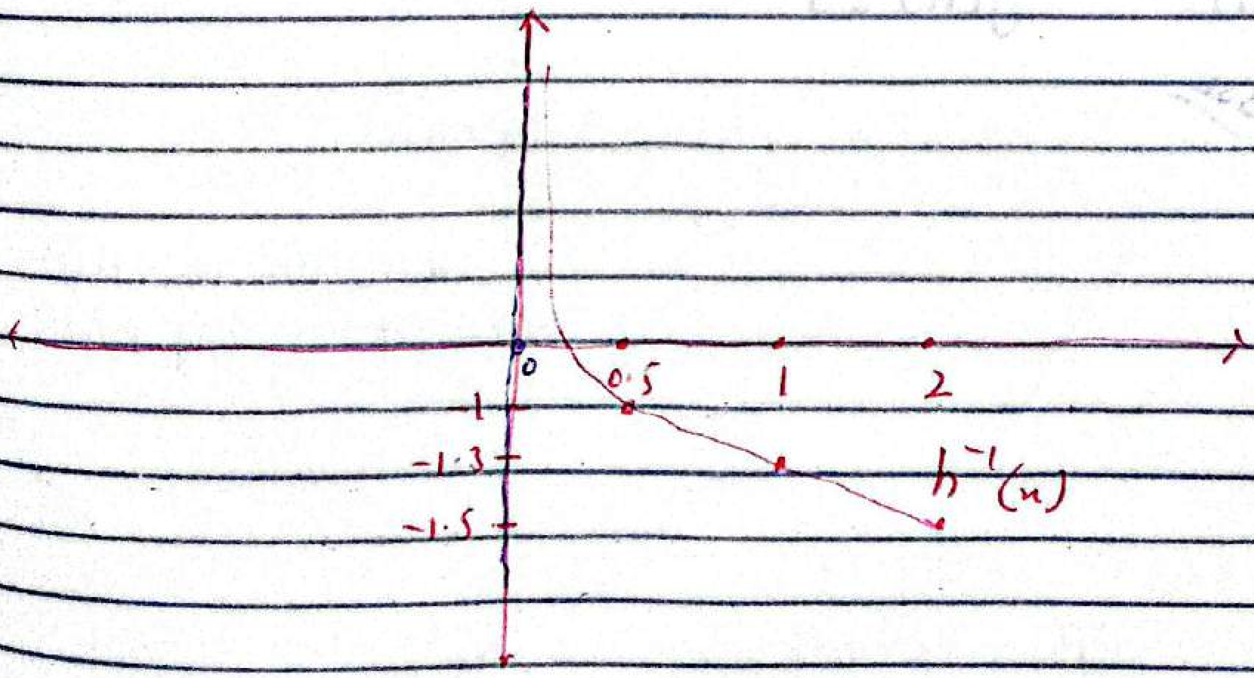
$F^{-1}(x) = \frac{1-5x}{3x}$

Graph of Inverse Function

Let $y = \frac{1-5x}{3x}$

For $x = 0$ Function is undefined

x	0.5	1	2
y	-1	-1.3	-1.5

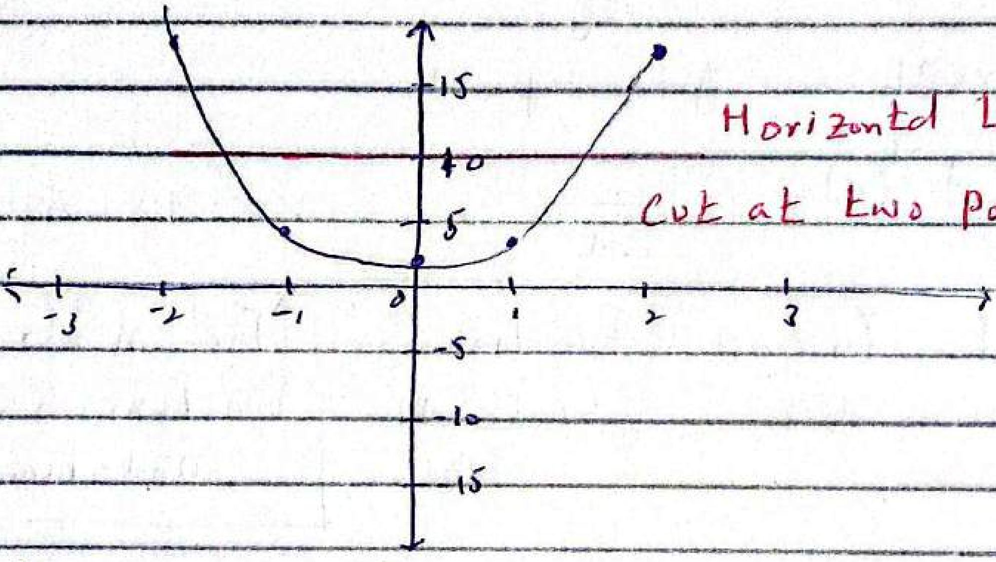


(vii) $F(x) = x^4 + 2$

Sol

$y = x^4 + 2$

x	-2	-1	0	1	2
y	18	3	2	3	18



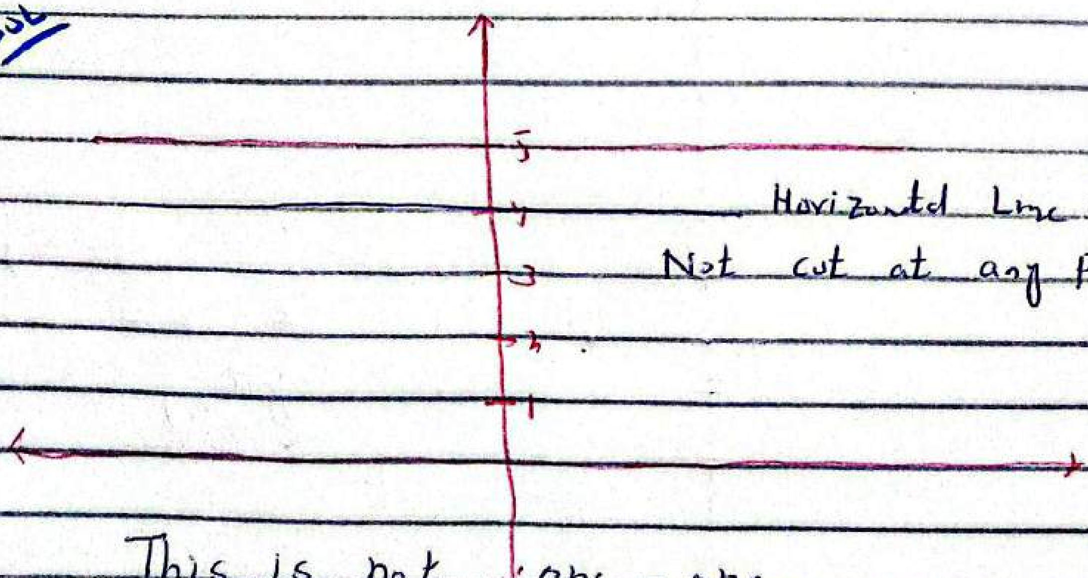
Horizontal Line test

Cut at two Points

Hence given function is not one-one.

(viii) $g(x) = 5 \Rightarrow y = 5$

Sol



Horizontal Line test.

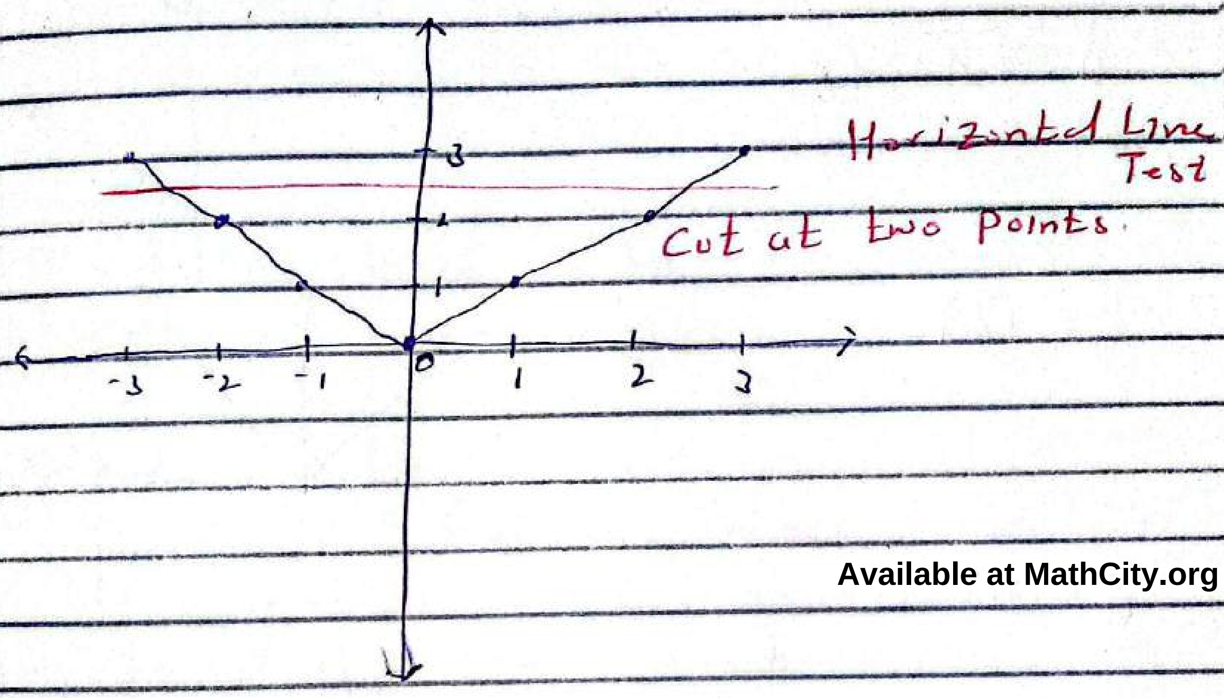
Not cut at any Point

This is not one-one

(ix) $h(x) = |x|$

Sol.

x	-3	-2	-1	0	1	2	3
y	3	2	1	0	1	2	3



Hence given Function is not one-one.
So its inverse doesnot exist.

Complete.