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Merging man and maths

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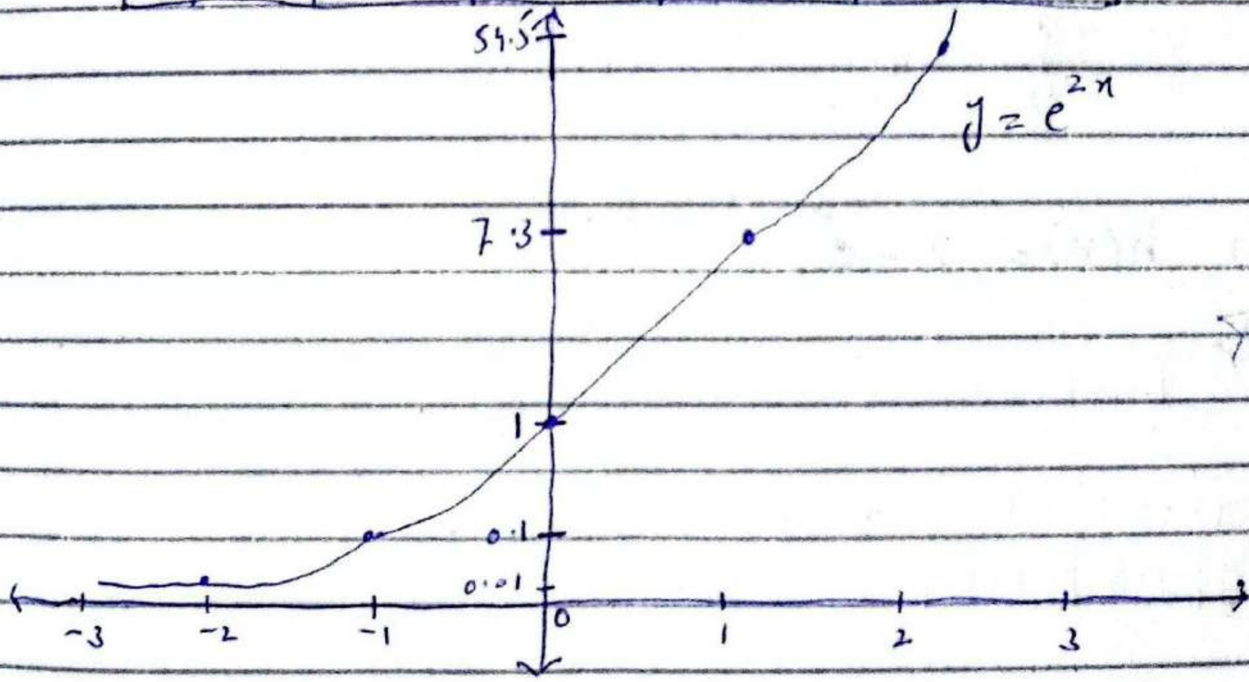
Exc 1.3

Q#1 Draw the graph of Functions.

(i) $F(x) = e^{2x}$

Sol Dom $F(x) = (-\infty, +\infty)$

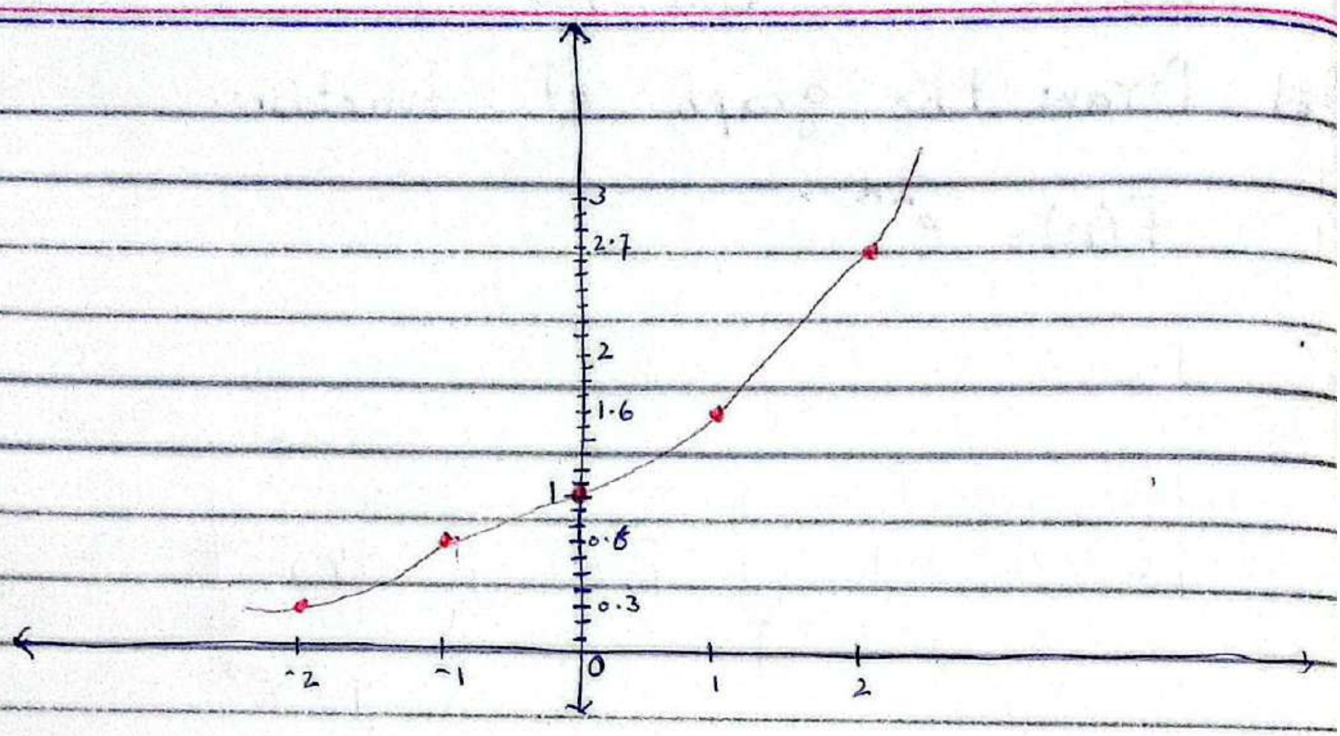
x	-2	-1	0	1	2
y	0.01	0.1	1	7.3	54.5



(ii) $F(x) = e^{0.5x}$

Sol Dom $F(x) = (-\infty, +\infty)$

x	-2	-1	0	1	2
y	0.3	0.6	1	1.6	2.7

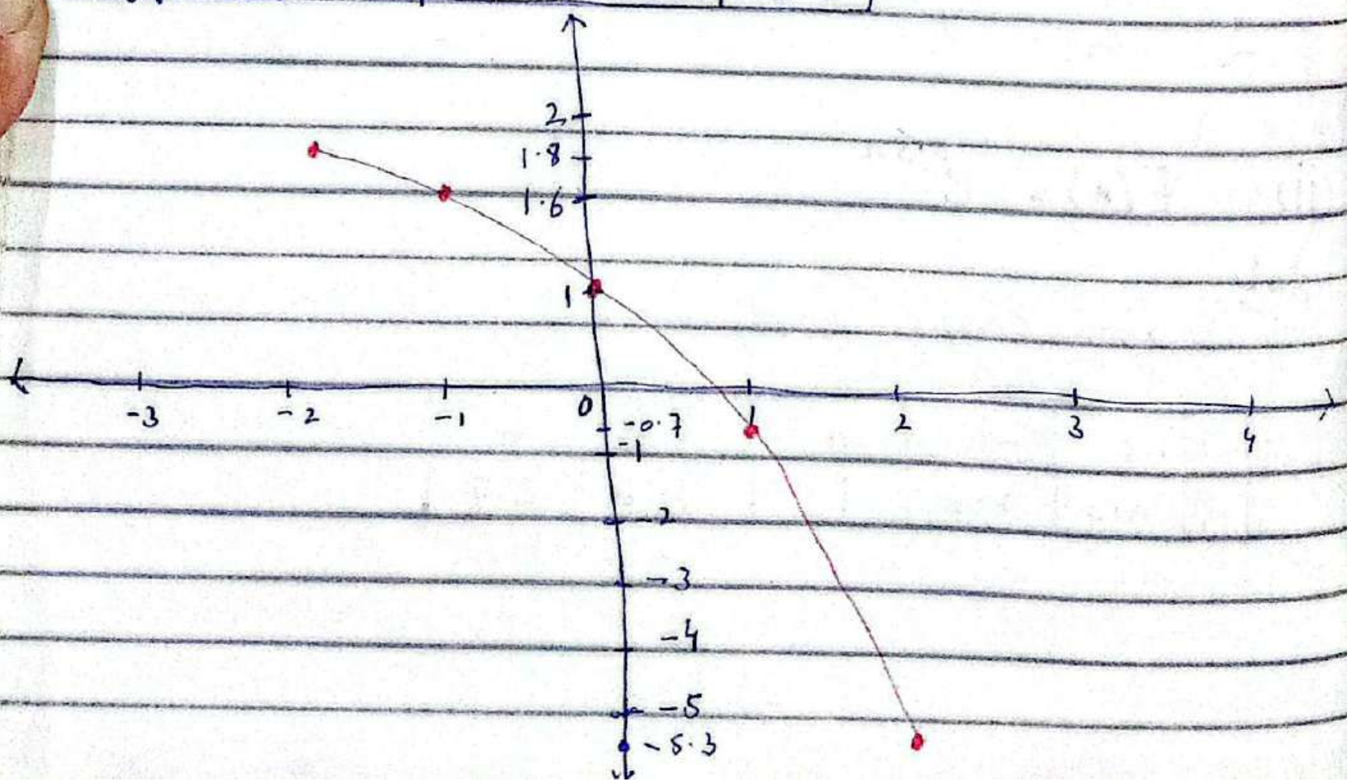


(iii) $h(x) = 2 - e^x$

Sol

Dom $h(x) = (-\infty, +\infty)$

x	-2	-1	0	1	2
y	1.8	1.6	1	-0.7	-5.3

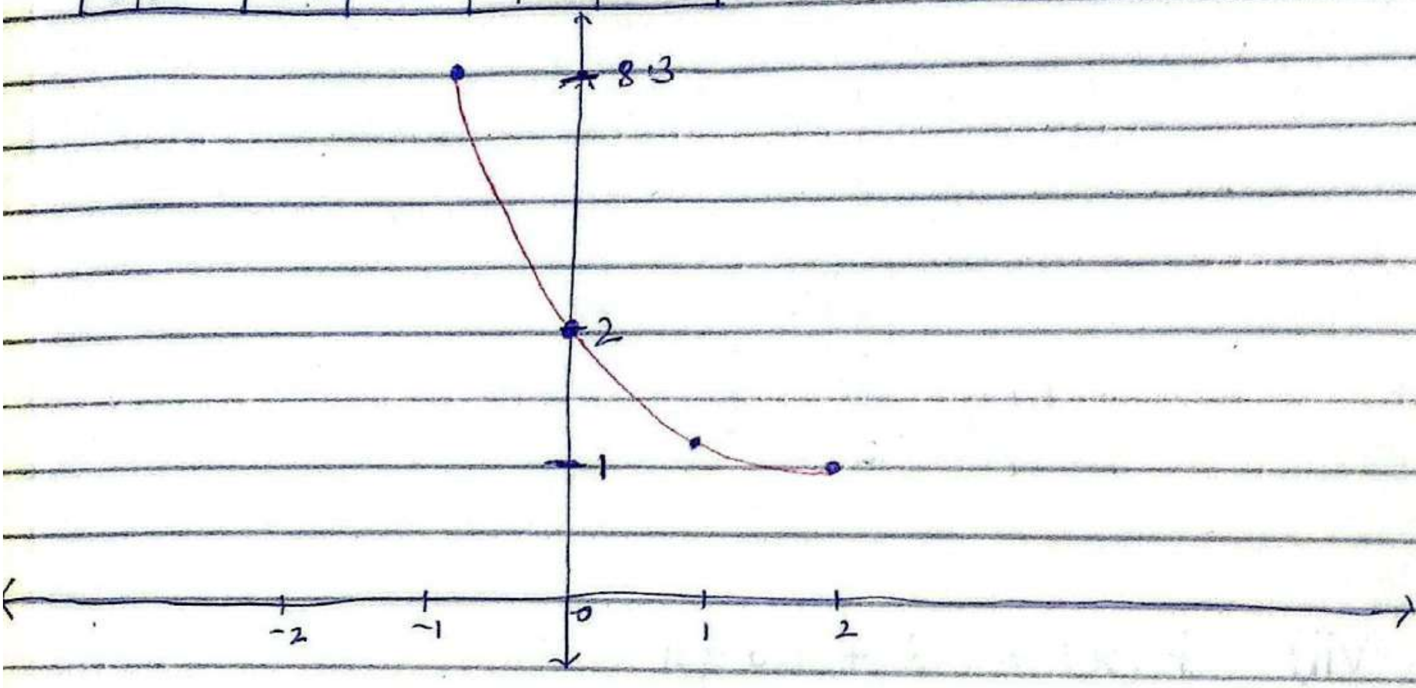


(iv) $h(x) = 1 + e^{-2x}$

Sol

Dom = $(-\infty, +\infty)$

x	-2	-1	0	1	2
y	55.5	8.3	2	1.1	1.0



(V) $F(x) = \ln(2x)$

Sol

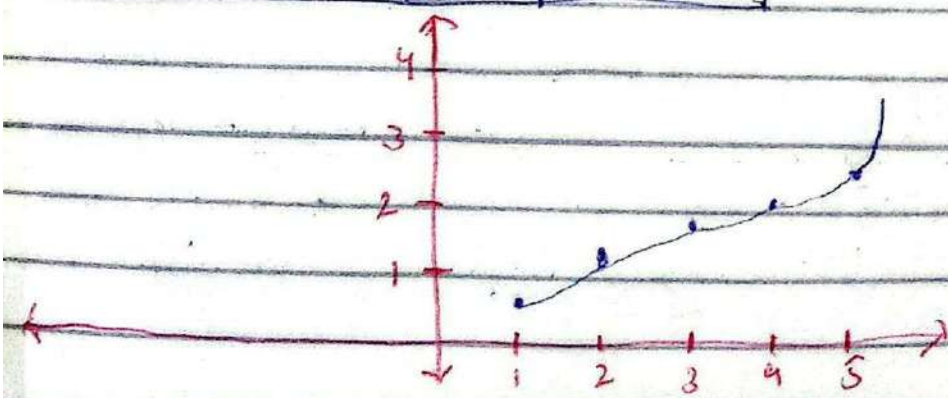
Dom $f(x) = (0, \infty)$ ← Not include

$2x > 0$

$x > \frac{0}{2}$

$\boxed{x > 0}$ Domain

x	1	2	3	4	5
y	0.6	1.3	1.7	2.0	2.3



$\ln(0) = \infty$

$x > 0$

$\ln e$ = Natural

$\log_{10}$ = Common

$\log_2$ = Binary Log

Domain

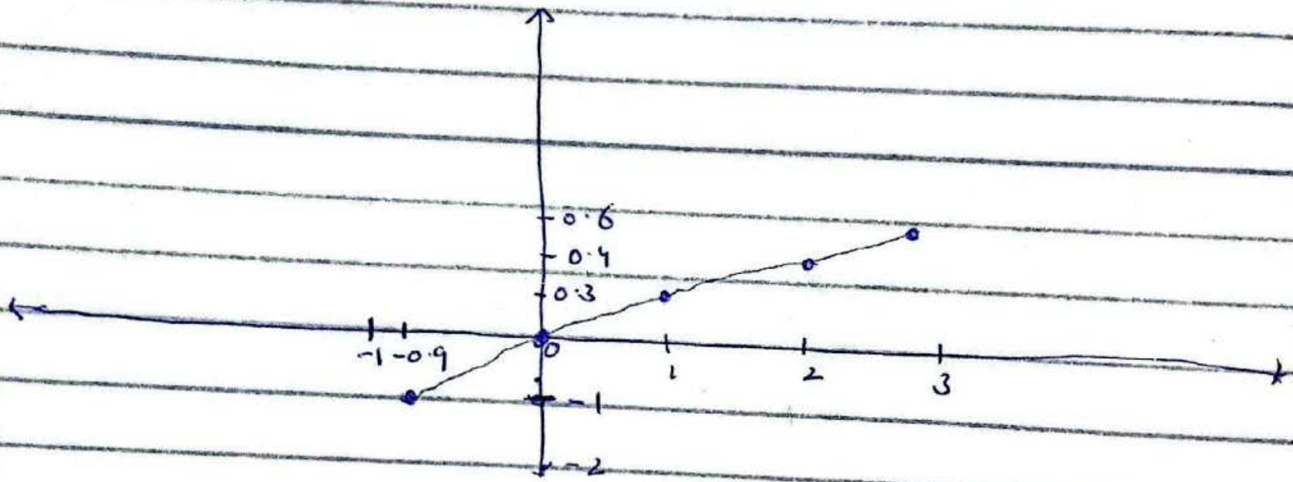
vi) $F(x) = \log_{10}(x+1)$ $\quad x+1 > 0$

$x > -1$

Sol

$\text{Dom } F(x) = (-1, \infty)$

x	-0.9	0	1	2	3
y	-1	0	0.3	0.4	0.6

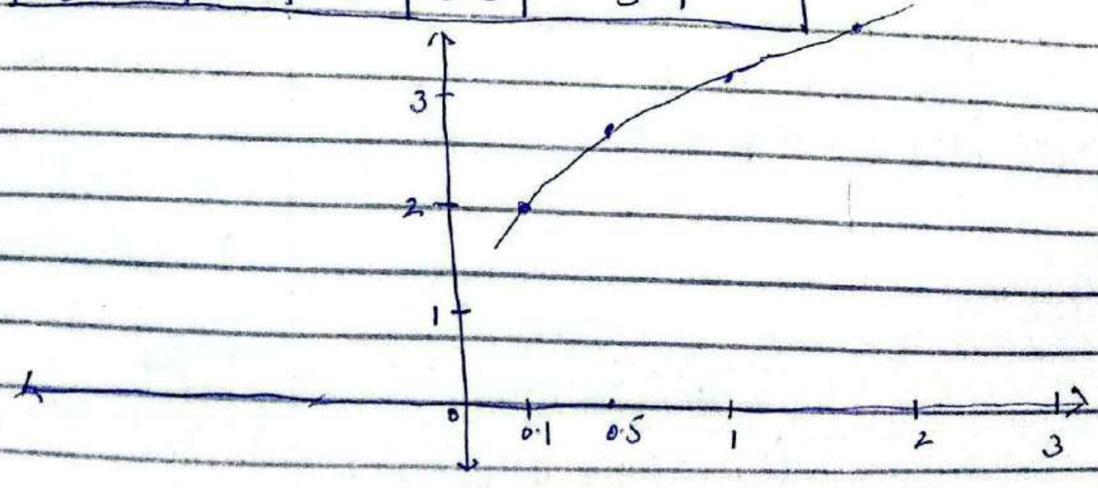


vii) $h(x) = 3 + \log x$

Sol

$\text{Dom } h(x) = (0, +\infty)$ $\quad x > 0$

x	0.1	0.5	1	2	3
y	2	2.7	3	3.3	3.4



(viii) $F(x) = e^{0.6x}$ and $g(x) = Ln(0.6x)$

$0.6x > 0$

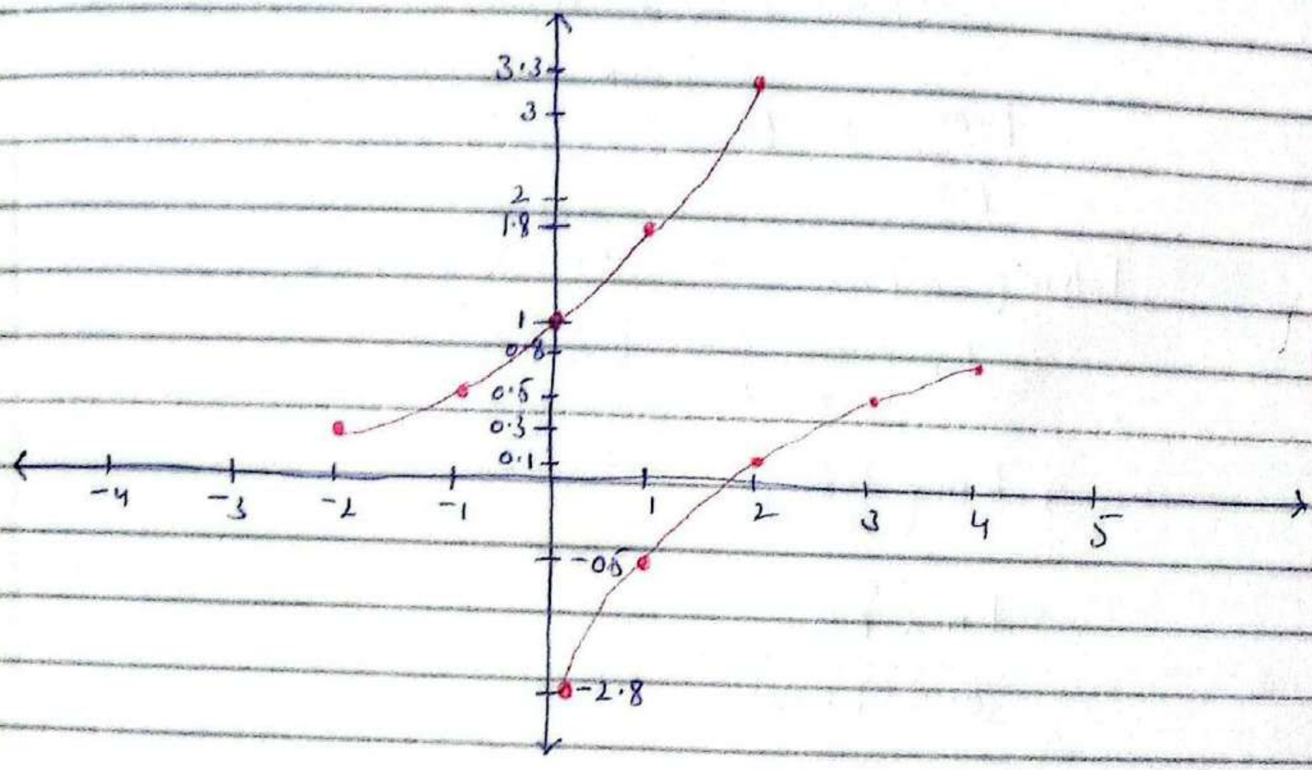
$0.6x > 0$

$x > 0$

Sol Dom: $(-\infty, +\infty)$ Dom: $(0, +\infty)$

x	-2	-1	0	1	2
y	0.3	0.5	1	1.8	3.3

x	0.1	1	2	3	4
y	-2.8	-0.5	0.1	0.5	0.8



Q#2 The number of Compact disc N(in million) Purchased each year increasing exponentially is given by

Sol

Num of dis

$$N(t) = 7.5(6)^{0.5t}$$

Time

in million.

t = 0 in 2024

$$N(0) = 7.5(6)^{0}$$

$N(0) = 7.5$ million

$$\boxed{1 \text{ billion} = 1000 \text{ Million}}$$

$$N(t) = 7.5\,(6)^{0.5t}$$
$$1000 = 7.5\,(6)^{0.5t}$$

$$\frac{1000}{7.5} = (6)^{0.5t}$$

Taking Ln on b. sides.

$$\text{Ln}\left(\frac{1000}{7.5}\right) = \text{Ln}\,(6)^{0.5t}$$

$$\text{Ln}\left(\frac{1000}{7.5}\right) = 0.5t\,\text{Ln}\,(6)$$

$$\frac{\text{Ln}\left(\frac{1000}{7.5}\right)}{(0.5)\,\text{Ln}(6)} = t$$

$$t = \frac{4.8928}{0.8958}$$

$$\boxed{t = 5.4 \text{ Years.}}$$

=> 1 billion Compact disc Sold in $2024 + 5.4 \approx 2029$

(b) What is the doubling time on the Sale of Compact disc.

Double = Num of Compact Disc : N(t)

Initial = 7.5 million.

Double = 7.5x2 = 15 million.

$$N(t) = 7.5(6)^{0.5t}$$

$$15 = 7.5(6)^{0.5t}$$

$$\frac{15}{7.5} = (6)^{0.5t}$$

Taking Ln on b. sides.

$$\ln 2 = 0.5t \ln 6$$

$$\frac{\ln 2}{0.5 \ln 6} = t$$

$$t = 0.77 \text{ Years.}$$

Hence doubling Compact disc sold in 2024.

Q#3 Suppose that Rs. 50,000 is invested at 6% interest compounded annually. After t years - - - -

Sol

$$A(t) = 50,000(1.06)^t$$

(a)

$$450,000 = 50,000(1.06)^t$$

$$\frac{\cancel{450,000}}{\cancel{50,000}} = (1.06)^t$$

$$9 = (1.06)^t$$

Taking Ln on b. sids

$$\ln 9 = \ln (1.06)^t$$

$$\ln 9 = t \ln (1.06)$$

$$\frac{\ln 9}{\ln(1.06)} = t$$

$$t = 37.7 \text{ Years.}$$

(b) Doubling time.

Initial amount at $t=0$

$$A(t) = 50{,}000 \, (1.06)^t$$

$$A(0) = 50{,}000 \, (1.06)^0$$

$$A(0) = 50{,}000$$

$$\text{Double} = 2 \times 50{,}0000$$

$$= 100{,}000$$

$$100000 = 50{,}000 \, (1.06)^t$$

$$\frac{100000}{50000} = (1.06)^t$$

$$2 = (1.06)^t$$

Taking Ln on b. side

$$\ln 2 = \ln (1.06)^t$$

$$\ln 2 = t \ln (1.06)$$

$$\frac{\ln 2}{\ln(1.06)} = t$$

$$t = 11.89 \text{ years.}$$

Q#4 The exponential growth rate of the population of the city is 1% per year. After how many years population will be doubled?

Sol

Let Initial Population = P_0

Let Population at any time = $P(t)$

$$P(t) = P_0 e^{rt}$$

P_0 = Initial Population ; t = time ; r = growth rate.

Given $r = 1\% = \frac{1}{100} = 0.01$ per year growth

Double Population = $P(t) = 2P_0$

$$2P_0 = P_0 e^{(0.01)t}$$

$$\frac{2\cancel{P_0}}{\cancel{P_0}} = e^{(0.01)t}$$

$$2 = e^{0.01t}$$

Taking Ln on b. sides

$$\ln 2 = \ln e^{0.01t}$$

$$\ln 2 = (0.01)t \cdot \ln e$$

$$\frac{\ln 2}{0.01} = t$$

$$t = 69.3 \text{ Years.}$$

Q #5 The Population of the world was 5.2 billion in 1990. The exponential growth rate was 1.6% Per Year.

a) Find the exponential growth Function.

Sol

Initial Population = P_0

Population at any time = $P(t)$

growth Population function

$$P(t) = P_0 e^{rt} \quad r = \text{growth rate.}$$

$P_0 = 5.2$ billion ; $r = 1.6\% = \frac{1.6}{100} = 0.016$

a) $P(t) = 5.2\, e^{0.016t}$

b) time difference =

2000 − 1990 = 10 years

$$P(10) = 5.2\, e^{(0.016)\times 10}$$

$$\boxed{P(10) = 6.1 \text{ billion.}} \quad \text{In } 2000.$$

(C)

$$P(t) = 5.2\, e^{(0.016)t}$$

$$8 = 5.2\, e^{(0.016)t}$$

$$\frac{8}{5.2} = e^{0.016t}$$

Taking Ln on b. sides

$$\ln\left(\frac{8}{5.2}\right) = \ln e^{0.016t}$$

$$0.4307 = 0.016t$$

$$\frac{0.4307}{0.016} = t$$

$$t = 26.9 \text{ Years.}$$

Therefore

$$1990 + 26.9 = 2016.9 \quad \text{Ans}$$

In 2016 in this year population will be 8 billion.

Q #6 Students in a Mathematics class took a final exam in monthly intervals there after. The average score
$S(t) = 68 - 20 \log(t+1) \; ; \; t > 0$

Sol (a)

$$S(t) = 68 - 20 \log(t+1) \; ; \; t \geq 0$$

initial point Put $t = 0$

$$S(0) = 68 - 20 \log(0+1)$$

$$S(0) = 68 - 20(0)$$

$$\boxed{S(0) = 68}$$

b) i) After 4 months

Put $t = 4$

$$S(4) = 68 - 20 \log(4+1)$$

$$\boxed{S(4) = 54.02}$$

ii) After 24 months.

$$S(24) = 68 - 20 \log(24+1)$$

$$S(24) = 68 - 20 \log(25)$$

$$\boxed{S(24) = 40}$$

(c) Graph.

Points

(0, 68) (4, 54) (24, 40)

Put $t = 9$ (Any number)

$S(9) = 68 - 20 \operatorname{Log}(9+1)$

$S(9) = 48$

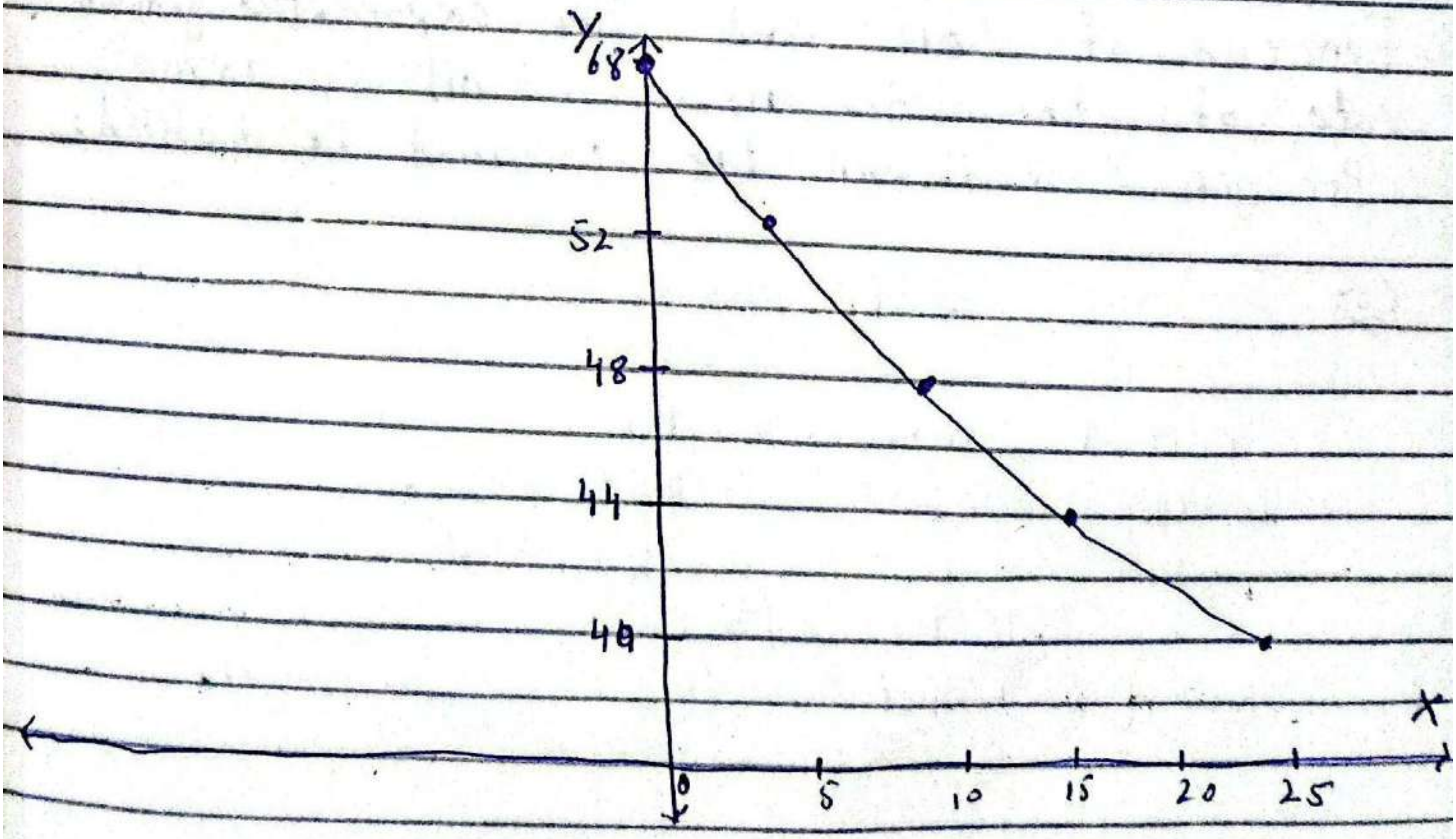
$(9, 48)$

Put $t = 15$ (Any number)

$S(15) = 68 - 20 \operatorname{Log}(15+1)$

$S(15) = 44$

$(15, 44)$



d) time = ? Score = 50

$$50 = 68 - 20\operatorname{Log}(t+1)$$

$$\frac{-18}{-20} = \operatorname{Log} t+1$$

Taking Antilog on b.side

$$\operatorname{Antilog}\left(\frac{18}{20}\right) = t+1$$

$$\operatorname{Antilog}\left(\frac{18}{20}\right) - 1 = t$$

$$t = 6.94 \quad \text{Ans.}$$

Q#7 If $P(t) = P_o e^{kt}$ denotes the growth Function of oil and the exponential growth rate of the demand for oil is 10% per year, When will the demand be doubled?

Sol

$$P(t) = P_o e^{kt}$$

Initial demand = P_o

Double " = $P(t) = 2P_o$

$$P(t) = P_o e^{kt}$$

k = growth rate = $10\% = \frac{10}{100} = 0.01$

$$2P_0 = P_0 e^{0.1t}$$

$$\frac{2\cancel{P_0}}{\cancel{P_0}} = e^{0.1t}$$

$$2 = e^{0.1t}$$

Taking Ln on b.sides

$$\ln 2 = \ln e^{0.1t}$$

$$\ln 2 = 0.1t \cdot \ln e$$

$$\frac{\ln 2}{0.1} = t$$

$t = 6.9$ years. Ans.

Q #8

Approximately two third of all Aluminum Can distributed are recycled each year.

$$N(t) = 250{,}000 \left(\frac{2}{3}\right)^t$$

a) After how many years will 60,000 Cans be in use.

Sol

$N(t)$ = Number of use Cans

Given Model

$$N(t) = 250{,}000 \left(\frac{2}{3}\right)^t$$

(a) $N(t) = 60{,}000 \qquad t = ?$

$$60{,}000 = 250{,}000\left(\frac{2}{3}\right)^t$$

$$\frac{60{,}000}{250{,}000} = \left(\frac{2}{3}\right)^t$$

Taking Ln on b. sides

$$\ln\left(\frac{6}{25}\right) = \ln\left(\frac{2}{3}\right)^t$$

$$\ln\left(\frac{6}{25}\right) = t\ln\left(\frac{2}{3}\right)$$

$$\frac{\ln(6/25)}{\ln(2/3)} = t$$

$$t \approx 3.5 \text{ Years}$$

b) time will only 1000 can be in use.

$N(t) = 1000 \qquad t = ?$

$$1000 = 250{,}000\left(\frac{2}{3}\right)^t$$

$$\frac{1000}{250000} = \left(\frac{2}{3}\right)^t$$

$$\frac{1}{250} = \left(\frac{2}{3}\right)^t$$

Taking Ln on b. sides

$$\ln\left(\frac{1}{250}\right) = \ln\left(\frac{2}{3}\right)^t$$

(Complete)

$$\ln(1/250) = t\ln(2/3)$$

$$\frac{\ln(1/250)}{\ln(2/3)} = t$$

$$t = 13.6 \text{ Years.}$$