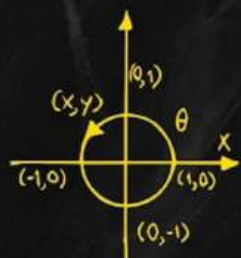


## Learning Outcomes

- Class 11: Mathematics (PECTAA)
- Unit 3: Theory of Quadratic Functions
- Quadratic Functions
- Exercise 3.1: Q1

YouTube Channel: [The Mathematics Outlet](#)

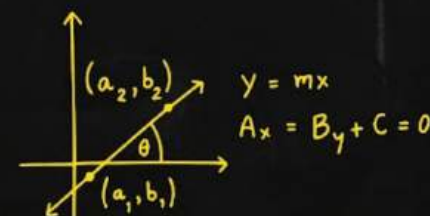


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# Unit 3

# Theory of Quadratic Functions

## 3.1 Quadratic Function

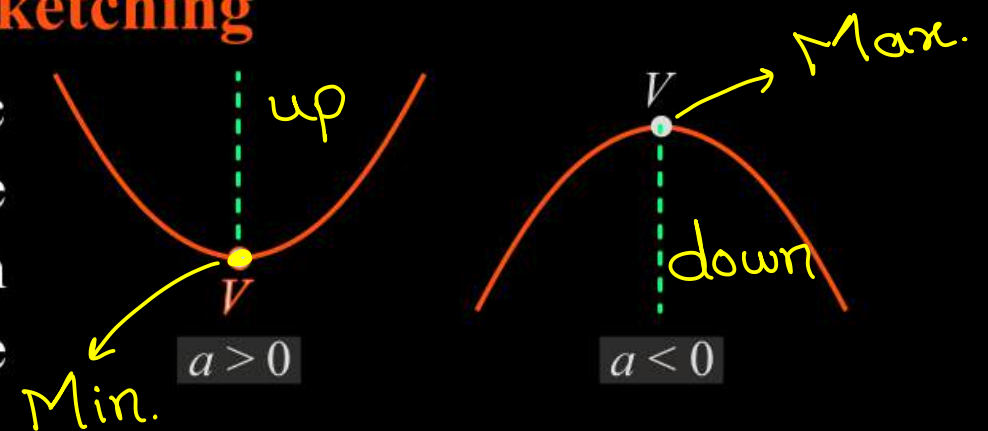
A quadratic function is a polynomial function of degree two. It is typically expressed in the standard form:

$$f(x) = ax^2 + bx + c$$

where  $a$ ,  $b$  and  $c$  are real numbers, and  $a \neq 0$ .

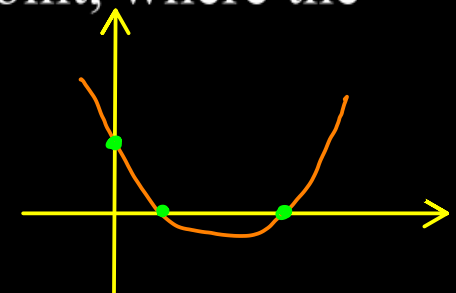
### 3.1.1 Analyzing Quadratic Function by Sketching

As we know shape of the graph of a quadratic function  $f(x) = ax^2 + bx + c$  is a parabola. The parabola opens upward or downward, depending on the sign of the leading coefficient  $a$ , as shown in the given figure.



The tip of the parabola, labeled as **V** in the diagrams above, is known as the vertex having coordinates  $(h, k)$ . The vertical line passing through the vertex serves as the axis of symmetry for the parabola. The vertex represents a turning point, where the graph changes direction.

- If  $a > 0$ , then the vertex is a minimum point.
- If  $a < 0$ , then the vertex is a maximum point.



For sketching the quadratic function, we need to find the  $x$ -intercept,  $y$ -intercept and the vertex. For analyzing the sketch of quadratic function, we find whether the vertex is a minimum or a maximum point and indicate the intervals where the function is increasing or decreasing.



**Example 1:** Sketch and analyze  $y = -x^2 - 2x + 3$ .

**Solution:**  $y = -x^2 - 2x + 3$

The  $y$ -intercept is  $y = -(0)^2 - 2(0) + 3 = 3$

The  $x$ -intercepts are found by solving the equation:

$$-x^2 - 2x + 3 = 0 \quad \text{or} \quad x^2 + 2x - 3 = 0$$

$$x^2 + 3x - x - 3 = 0$$

$$x(x + 3) - 1(x + 3) = 0$$

$$(x + 3)(x - 1) = 0$$

$$x + 3 = 0, x - 1 = 0$$

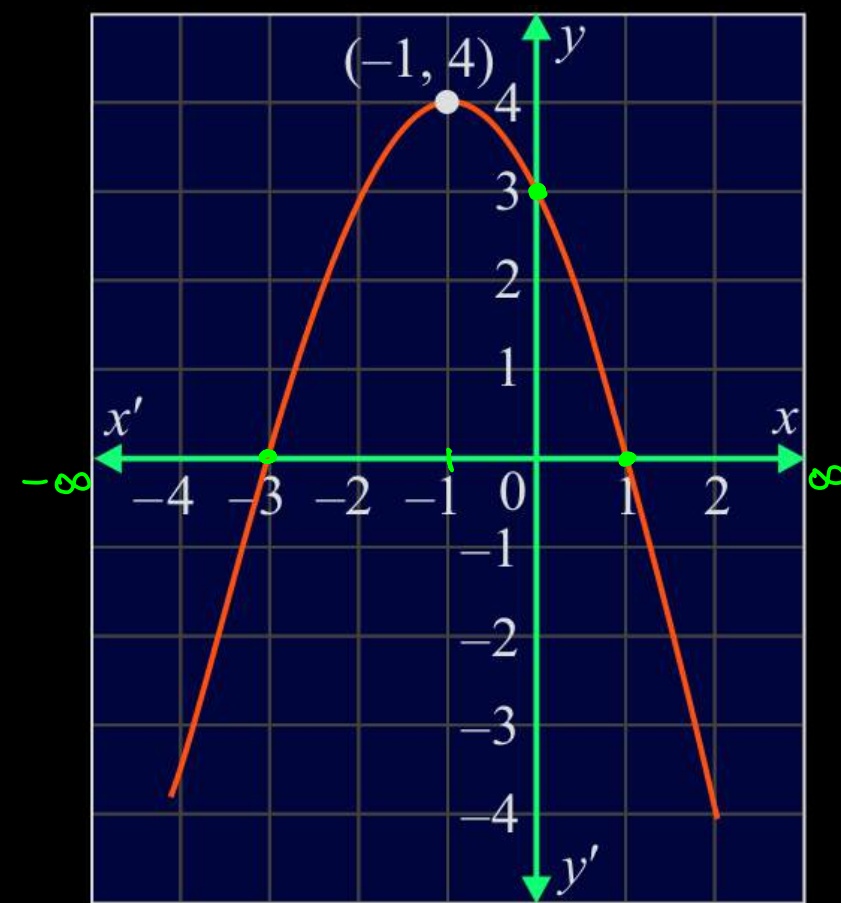
$$x = -3, x = 1$$

Now, we find the vertex

$$h = \frac{-b}{2a} = -\frac{(-2)}{2(-1)} = -1$$

$$k = -(-1)^2 - 2(-1) + 3 = -1 + 2 + 3 = 4$$

So, the vertex  $(-1, 4)$  is a maximum point. The function  $y$  is increasing on  $(-\infty -1)$  and decreasing on  $(-1 \infty)$



### 3.1.2 Finding Maximum and Minimum Values of Quadratic Functions by Completing Square

Completing the square is a technique used to rewrite a quadratic function in the following vertex form:

$$\underline{f(x) = a(x - h)^2 + k}$$

Where vertex =  $(h, k)$ ,  $h = -\frac{b}{2a}$  and  $k = c - \frac{b^2}{4a}$

- If  $a > 0$ , the minimum value of  $f(x)$  at  $x = h$  is  $k$ .
- If  $a < 0$ , the maximum value of  $f(x)$  at  $x = h$  is  $k$ .

**Example 2:** Find the maximum or minimum value of  $f(x) = -2x^2 + 4x + 3$  by completing square.

**Solution:**

$$f(x) = -2(x^2 - 2x) + 3$$

$$f(x) = -2(x^2 - 2x + 1 - 1) + 3$$

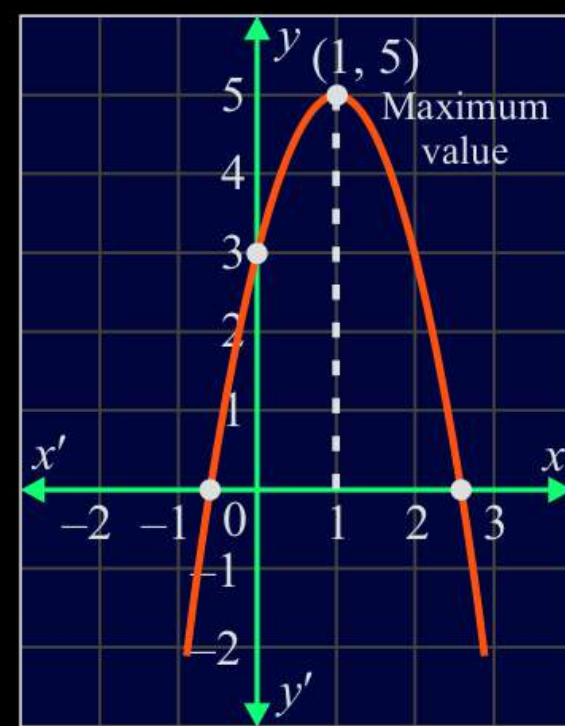
$$f(x) = -2[(x - 1)^2 - 1] + 3$$

$$f(x) = -2(x - 1)^2 + 2 + 3$$

$$\rightarrow f(x) = -2(x - 1)^2 + 5$$

Here  $a = -2 < 0$

Therefore, the maximum value is 5, which occurs when  $x = 1$ .



**Example 3:** Find the maximum or minimum value of  $f(x) = x^2 - 2x - 3$ .

**Solution:** Given that  $f(x) = x^2 - 2x - 3$

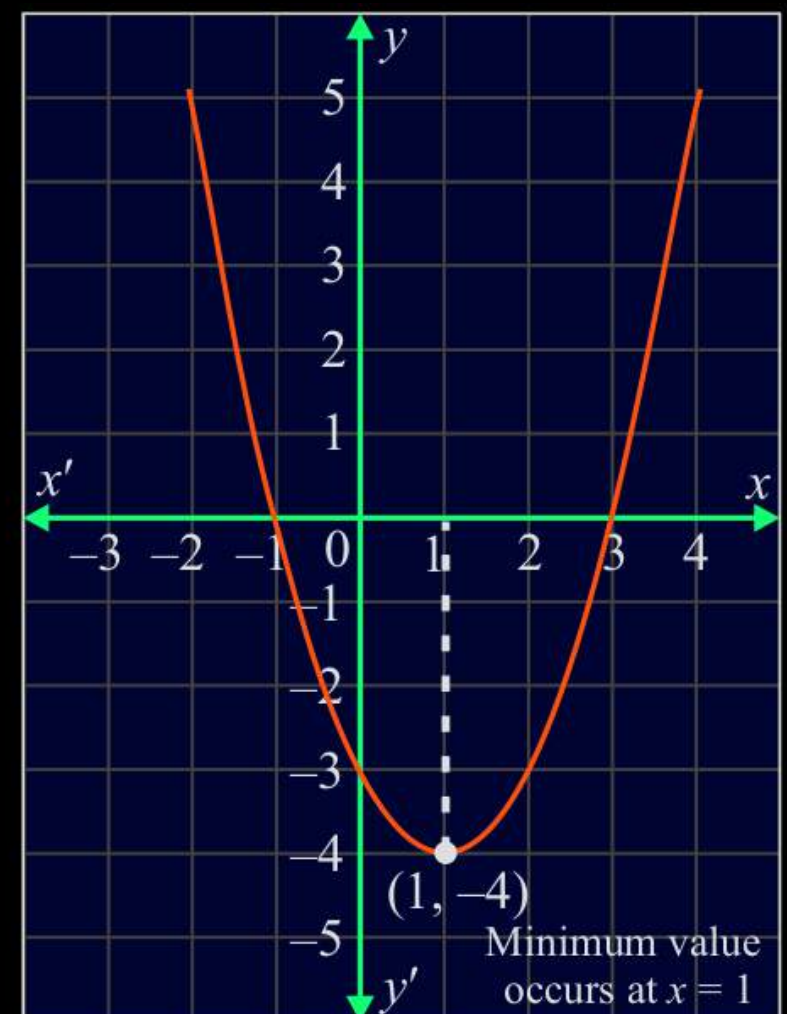
Here  $a = 1$ ,  $b = -2$ ,  $c = -3$

$$h = -\frac{b}{2a} = -\frac{(-2)}{2(1)} = 1$$

and  $k = c - \frac{b^2}{4a} = -3 - \frac{(-2)^2}{4(1)} = -4$

Here  $a = 1 > 0$

Therefore, the minimum value of  $f(x)$  at  $x = 1$  is  $-4$ .





### EXERCISE 3.1

1. Find the maximum or minimum value of the following quadratic functions by completing square:

(i)  $f(x) = x^2 + 6x + 13$

Sol

$$f(x) = x^2 + 6x + 13$$

$$= x^2 + 2(x)(3) + 13$$

Add and Subtract  $(3)^2$ ,

$$f(x) = x^2 + 2(x)(3) + 3^2 - 3^2 + 13$$

$$= (x+3)^2 - 9 + 13$$

$$f(x) = (x+3)^2 + 4$$

Compare it with:

$$f(x) = a(x-h)^2 + k$$

$$a=1, \quad h=-3, \quad k=4$$

Since  $a=1>0$ , so  $(h,k)=(-3,4)$  is minimum point.

$$\therefore (a+b)^2 = a^2 + 2ab + b^2$$

(ii)  $f(x) = x^2 + 4x$

Sol

$$f(x) = x^2 + 4x$$

$$= x^2 + 2(x)(2)$$

Add and Subtract  $(2)^2$ ,

$$f(x) = x^2 + 2(x)(2) + 2^2 - 2^2$$

$$f(x) = (x+2)^2 - 4$$

Compare it with:

$$f(x) = a(x-h)^2 + k$$

$$a=1, \quad h=-2, \quad k=-4$$

Since  $a=1 > 0$

Hence  $f(x)$  is min. at  $x=-2$ .



(iii)  $f(x) = -x^2 + 8x + 13$

Sol

$$\begin{aligned} f(x) &= -(x^2 - 8x - 13) \\ &= -[x^2 - 2(x)(4) - 13] \end{aligned}$$

Add and Subtract  $(4)^2$ ,

$$\begin{aligned} f(x) &= -[x^2 - 2(x)(4) + 4^2 - 4^2 - 13] \\ &= -[(x-4)^2 - 29] \end{aligned}$$

$$f(x) = -(x-4)^2 + 29$$

Compare it with:

$$f(x) = a(x-h)^2 + k$$

$$a = -1 < 0$$

$$h = 4, \quad k = 29$$

Hence,  $f(x)$  is maximum at  $x = 4$ .

Maximum value of  $f(x)$  is 29.

$$\therefore (a-b)^2 = a^2 - 2ab + b^2$$

(iv)  $f(x) = -x^2 - 3x - 5$

Sol

$$f(x) = -[x^2 + 3x + 5]$$

$$= -\left[x^2 + 2(x)\left(\frac{3}{2}\right) + 5\right]$$

Add and Subtract  $\left(\frac{3}{2}\right)^2$ ,

$$= -\left[x^2 + 2(x)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 5\right]$$

$$= -\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + 5\right]$$

$$= -\left[\left(x + \frac{3}{2}\right)^2 + \frac{11}{4}\right]$$

$$f(x) = -\left(x + \frac{3}{2}\right)^2 - \frac{11}{4}$$

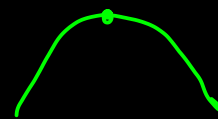
Compare it with:

$$f(x) = a(x-h)^2 + k$$

$$a = -1 < 0$$

$$h = -\frac{3}{2}, \quad k = -\frac{11}{4}$$

Hence Maximum value of  $f(x)$  is  $-\frac{11}{4}$  at  $x = -\frac{3}{2}$ .



(v)  $f(x) = 3x^2 + 6x - 13$

Sol

$$\begin{aligned} f(x) &= 3 \left[ x^2 + 2x - \frac{13}{3} \right] \\ &= 3 \left[ x^2 + 2(x)(1) - \frac{13}{3} \right] \end{aligned}$$

Add and Subtract  $(1)^2$ ,

$$\begin{aligned} f(x) &= 3 \left[ x^2 + 2(x)(1) + \underbrace{1^2 - 1^2}_{- \frac{13}{3}} \right] \\ &= 3 \left[ (x+1)^2 - \frac{16}{3} \right] \end{aligned}$$

$$f(x) = 3(x+1)^2 - 16$$

Compare it with:

$$f(x) = a(x-h)^2 + k$$

$$a = 3 > 0$$

$$h = -1,$$

$$k = -16$$

Hence  $f(x)$  is minimum at  $x = -1$ .





(vi)  $f(x) = -2x^2 - x + 21$

Sol

$$f(x) = -2 \left[ x^2 + \frac{x}{2} - \frac{21}{2} \right]$$

$$= -2 \left[ x^2 + 2(x) \left( \frac{1}{4} \right) - \frac{21}{2} \right]$$

Add and Subtract  $\left( \frac{1}{4} \right)^2$ ,

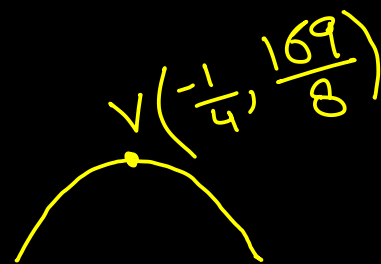
$$= -2 \left[ x^2 + 2(x) \left( \frac{1}{4} \right) + \left( \frac{1}{4} \right)^2 - \left( \frac{1}{4} \right)^2 - \frac{21}{2} \right]$$

$$= -2 \left[ \left( x + \frac{1}{4} \right)^2 - \frac{1}{16} - \frac{21}{2} \right]$$

$$= -2 \left[ \left( x + \frac{1}{4} \right)^2 - \frac{(1 + 168)}{16} \right]$$

$$= -2 \left[ \left( x + \frac{1}{4} \right)^2 - \frac{169}{16} \right]$$

$$f(x) = -2 \left( x + \frac{1}{4} \right)^2 + \frac{169}{8}$$



Compare it with:

$$f(x) = a(x-h)^2 + k$$

$$a = -2, \quad h = -\frac{1}{4}, \quad k = \frac{169}{8}$$

Since  $a = -2 < 0$ , so Max value of  $f(x)$  is  $\frac{169}{8}$  at  $x = -\frac{1}{4}$ .

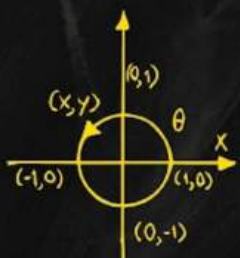
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## Learning Outcomes

- Class 11: Mathematics (PECTAA)
- Unit 3: Theory of Quadratic Functions
- Quadratic Functions
- Exercise 3.1: Q2

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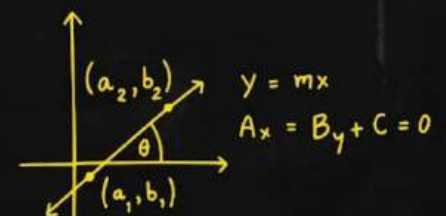


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### EXERCISE 3.1

2. Find the maximum or minimum point by sketching the following quadratic functions. Also find their domain and range:

(i)  $f(x) = x^2 - 4x$

Sol

Let  $y = x^2 - 4x$

For  $x$ -intercepts, put  $y=0$

$$0 = x^2 - 4x$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x=0 \text{ or } x-4=0 \Rightarrow x=4$$

$\Rightarrow$  Parabola intersects  $x$ -axis at  $(0,0)$  and  $(4,0)$ .

For  $y$ -intercept, put  $x=0$

$$y = 0^2 - 4(0)$$

$$y = 0$$

$\Rightarrow$  Parabola intersects  $y$ -axis at  $(0,0)$ .

Lets find the vertex  $(h,k)$ ,

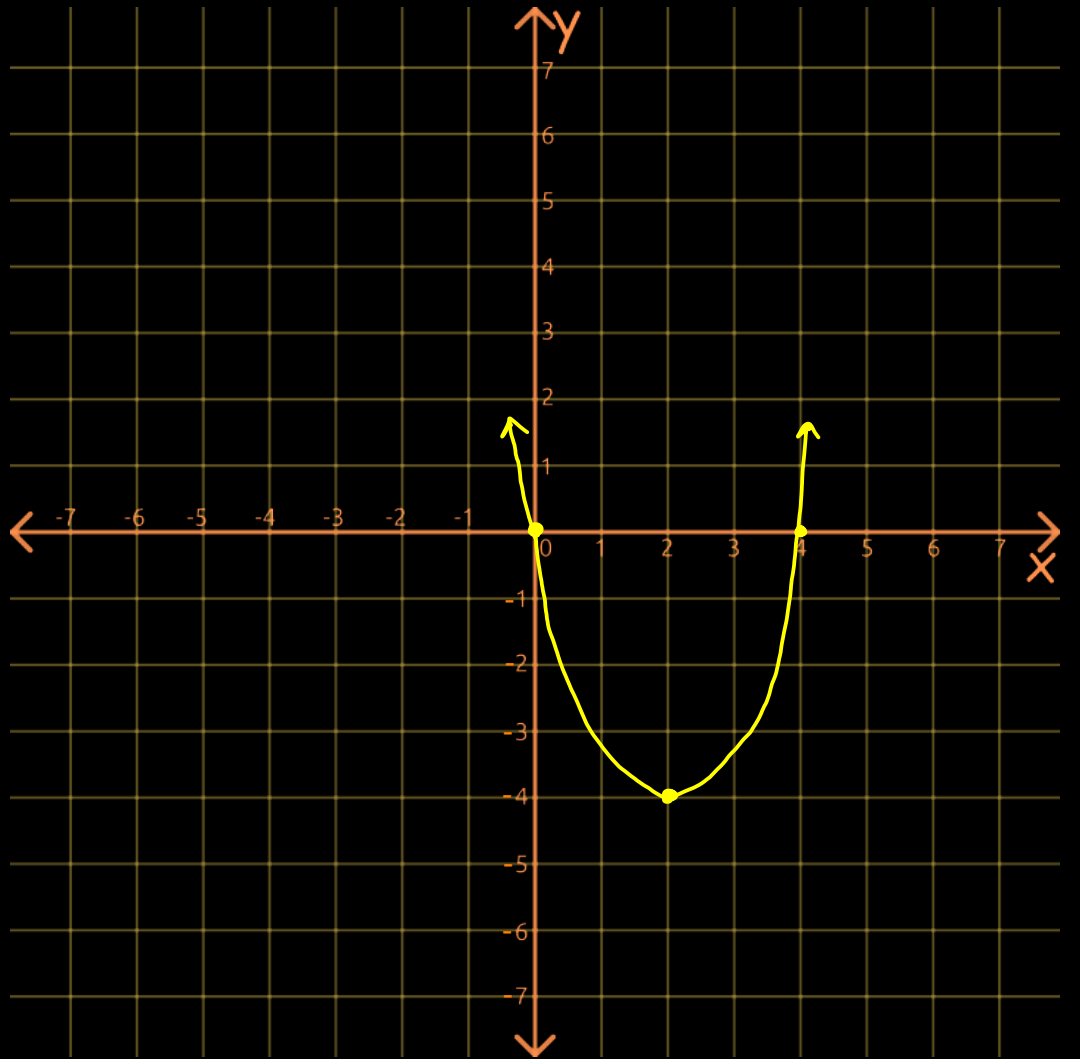
$$f(x) = x^2 - 4x + 0$$

Here  $a=1$ ,  $b=-4$ ,  $c=0$

$$h = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2, \quad k = f(h) = (2)^2 - 4(2) = -4$$

$$\Rightarrow (h,k) = (2,-4)$$

Since  $a=1>0$ , so the vertex  $(2,-4)$  is a minimum point.



Domain  $= (-\infty, \infty)$   
Since vertex is minimum, so  
Range  $= [-4, \infty)$

(ii)  $f(x) = x^2 - 5x + 6$

Sol

Let  $y = x^2 - 5x + 6$

Lets find the vertex  $(h, k)$ ,

$$h = \frac{-b}{2a} = \frac{-(-5)}{2(1)} = \frac{5}{2}$$

$$k = c - \frac{b^2}{4a} = 6 - \frac{(-5)^2}{4(1)} = 6 - \frac{25}{4} = -\frac{1}{4}$$

So,  $(h, k) = \left(\frac{5}{2}, -\frac{1}{4}\right)$

For  $y$ -intercept, put  $x=0$

$$f(0) = 0^2 - 5(0) + 6 = 6$$

$\Rightarrow$  Parabola intersects  $y$ -axis at  $(0, 6)$ .

For  $x$ -intercepts, put  $y=0$ :

$$0 = x^2 - 5x + 6$$

$$x^2 - 2x - 3x + 6 = 0$$

$$x(x-2) - 3(x-2) = 0$$

$$(x-2)(x-3) = 0$$

$$x-2=0$$

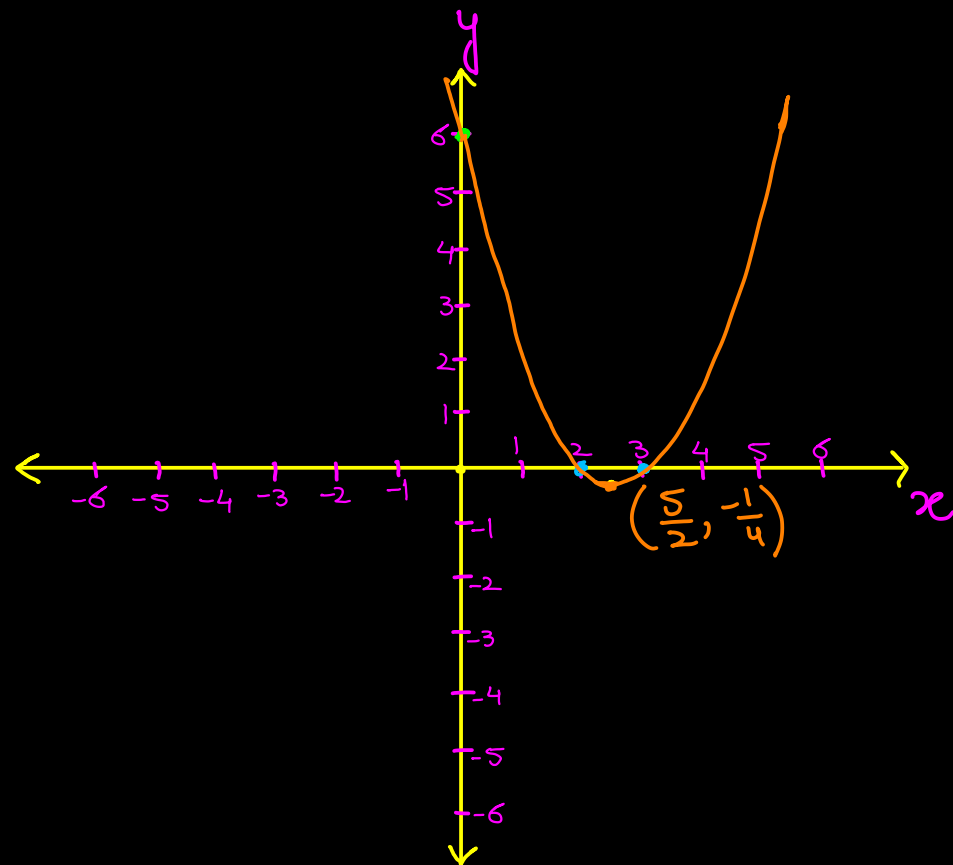
or

$$x-3=0$$

$$x=2$$

$$x=3$$

$\Rightarrow$  Parabola intersects  $x$ -axis at  $(2, 0)$  and  $(3, 0)$ .



$$\text{Domain} = (-\infty, \infty)$$

Since  $a=1>0$ , so vertex  $(h, k) = \left(\frac{5}{2}, -\frac{1}{4}\right)$  is minimum.

$$\Rightarrow \text{Range} = \left[-\frac{1}{4}, \infty\right)$$

(iii)  $f(x) = -x^2 + 2x - 8$

Sol

$$f(x) = -x^2 + 2x - 8$$

Lets find the vertex  $(h, k)$ ,

$$h = \frac{-b}{2a} = \frac{-2}{2(-1)} = 1$$

$$k = f(1) = -1^2 + 2(1) - 8$$

$$k = -1 + 2 - 8 = -7$$

Vertex:  $(h, k) = (1, -7)$

For  $y$ -intercept, put  $x=0$

$$f(0) = -0^2 + 2(0) - 8 = -8$$

$\Rightarrow$  Parabola intersects  $y$ -axis at  $(0, -8)$ .

For  $x$ -intercept, put  $y=0$

$$0 = -x^2 + 2x - 8$$

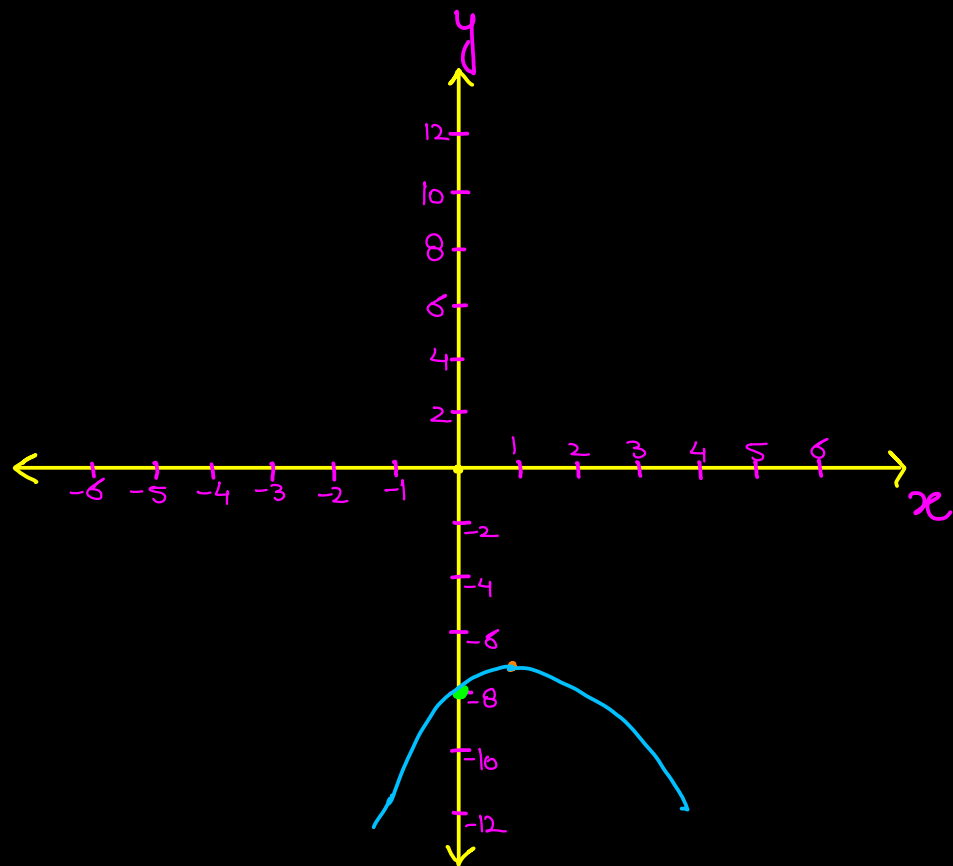
$$x^2 - 2x + 8 = 0$$

$$b^2 - 4ac = (-2)^2 - 4(1)(8)$$

$$= 4 - 32$$

$$= -28 < 0$$

$\Rightarrow$  No  $x$ -intercepts.



$$\text{Domain} = \mathbb{R}$$

Since  $(1, -7)$  is a max value  
 $\Rightarrow \text{Range} = (-\infty, -7]$



(iv)  $f(x) = x^2 - 4x + 4$

Sol

$$y = f(x) = x^2 - 4x + 4$$

Lets find the vertex  $(h, k)$ ,

$$h = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$$

$$k = c - \frac{b^2}{4a} = 4 - \frac{(-4)^2}{4(1)} = 4 - \frac{16}{4} = 0$$

So,  $(h, k) = (2, 0)$

For  $x$ -intercept, put  $y=0$

$$0 = x^2 - 4x + 4$$

$$(x-2)^2 = 0$$

Taking square root,

$$x - 2 = 0$$

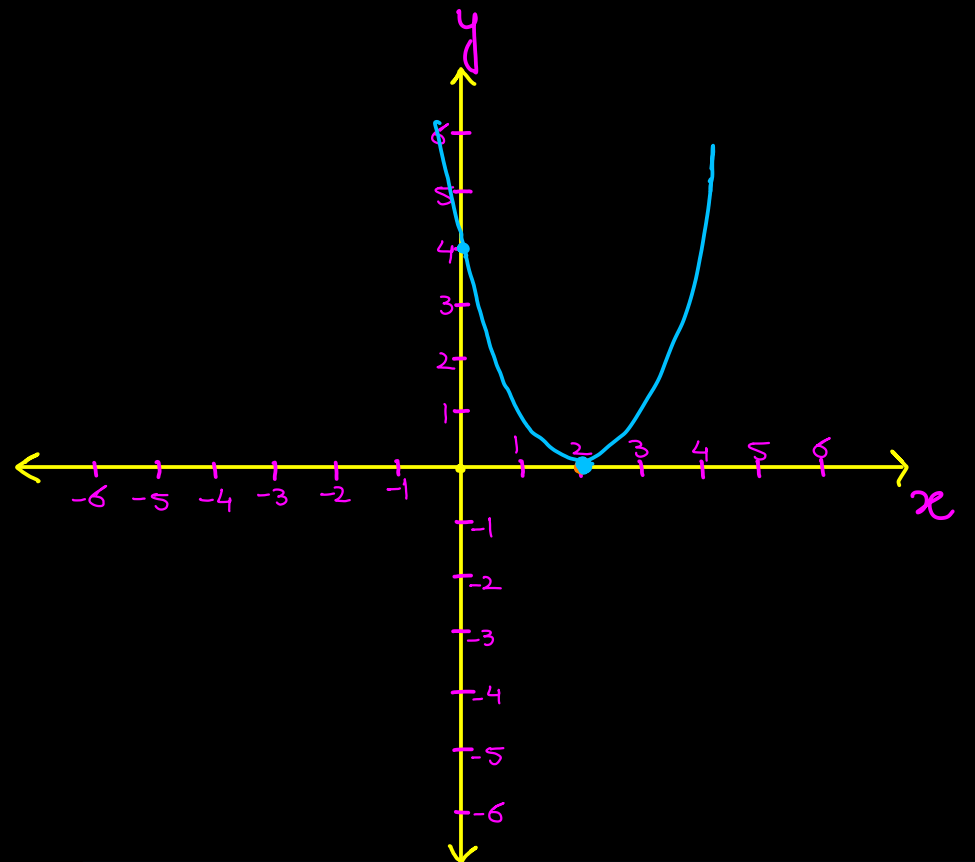
$$x = 2$$

$\Rightarrow$  Parabola intersects  $x$ -axis at  $(2, 0)$ .

For  $y$ -intercept, put  $x=0$

$$f(0) = 0^2 - 4(0) + 4 = 4$$

$\Rightarrow$  Parabola intersects  $y$ -axis at  $(0, 4)$ .



Domain =  $\mathbb{R}$

Since vertex  $(2, 0)$  is the minimum point,

$$\text{Range} = [0, \infty)$$

(v)  $f(x) = x^2 + 2x - 8.3$

Sol

$$f(x) = x^2 + 2x - 8.3$$

Lets find the vertex  $(h, k)$ ,

$$h = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$$

$$k = f(-1) = (-1)^2 + 2(-1) - 8.3 = 1 - 2 - 8.3 = -9.3$$

So, vertex:  $(-1, -9.3)$

For  $x$ -intercept, put  $y=0$

$$0 = x^2 + 2x - 8.3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = -4.05,$$

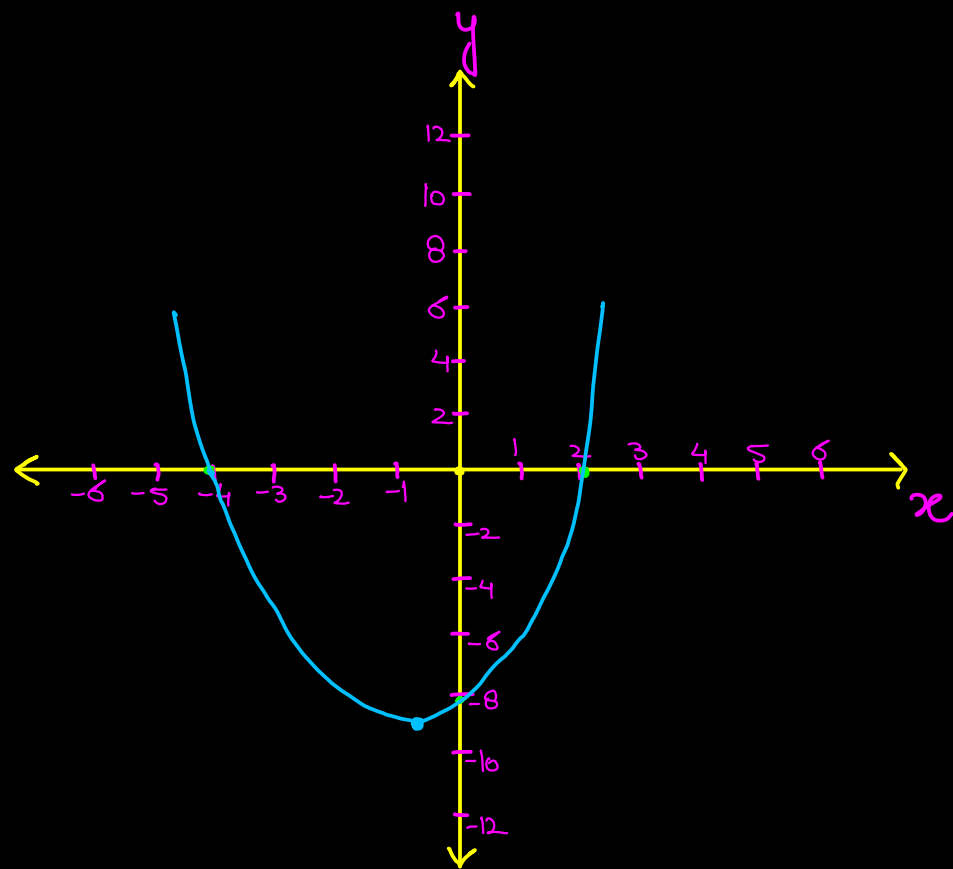
$$x = 2.05$$

$\Rightarrow$  Parabola intersects  $x$ -axis at  $(-4.05, 0)$  and  $(2.05, 0)$ .

For  $y$ -intercept, put  $x=0$

$$f(0) = 0^2 + 2(0) - 8.3 = -8.3$$

$\Rightarrow$  Parabola intersects  $y$ -axis at  $(0, -8.3)$ .



Domain =  $\mathbb{R}$

Since  $a > 0$ , so vertex is min.  
 $\Rightarrow$  Range =  $[-9.3, \infty)$

(vi)  $f(x) = 6 - x - x^2$

Sol

$$y = f(x) = -x^2 - x + 6$$

Lets find the vertex  $(h, k)$ ,

$$h = \frac{-(-1)}{2(-1)} = -\frac{1}{2}$$

$$k = c - \frac{b^2}{4a} = 6 - \frac{(-1)^2}{4(-1)} = 6 + \frac{1}{4} = \frac{25}{4}$$

Vertex:  $(h, k) = \left(-\frac{1}{2}, \frac{25}{4}\right)$

For  $x$ -intercept, put  $y=0$

$$0 = -x^2 - x + 6$$

$$-0 = x^2 + x - 6$$

$$x^2 + x - 6 = 0$$

$$x^2 + 3x - 2x - 6 = 0$$

$$x(x+3) - 2(x+3) = 0$$

$$(x+3)(x-2) = 0$$

$$x+3 = 0$$

$$x = -3$$

or

$$x-2 = 0$$

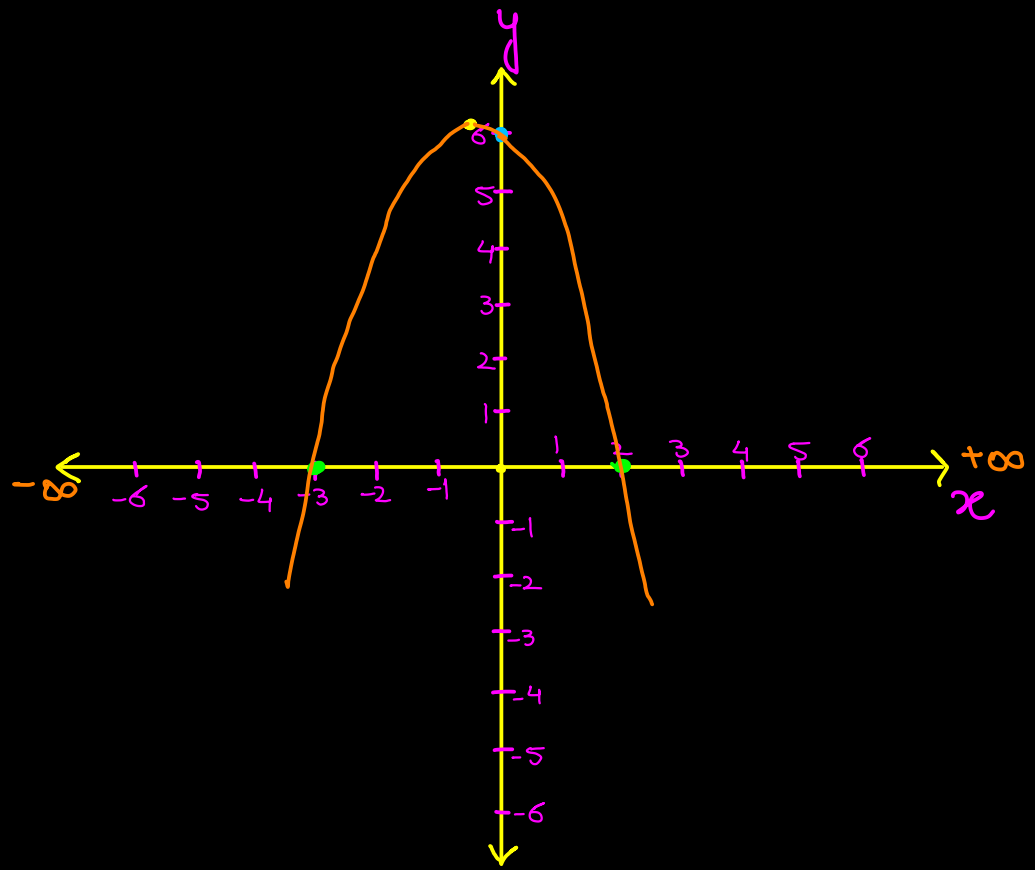
$$x = 2$$

$\Rightarrow$  Parabola intersects  $x$ -axis at  $(-3, 0)$  and  $(2, 0)$ .

For  $y$ -intercept, put  $x=0$

$$f(0) = -0^2 - 0 + 6 = 6$$

$\Rightarrow$  Parabola intersects  $y$ -axis at  $(0, 6)$ .



$$\text{Domain} = (-\infty, \infty)$$

Since  $a = -1 < 0$ , so vertex  $(h, k) = \left(-\frac{1}{2}, \frac{25}{4}\right)$  is a maximum point.

$$\Rightarrow \text{Range} = \left(-\infty, \frac{25}{4}\right]$$

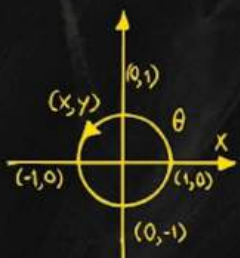
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## Learning Outcomes

- Class 11: Mathematics (PECTAA)
- Unit 3: Theory of Quadratic Functions
- Inverse of Quadratic Functions
- Exercise 3.1: Q3

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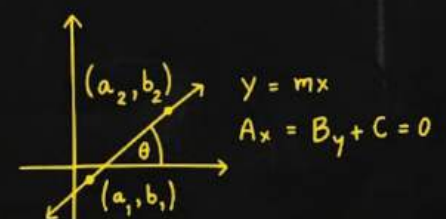


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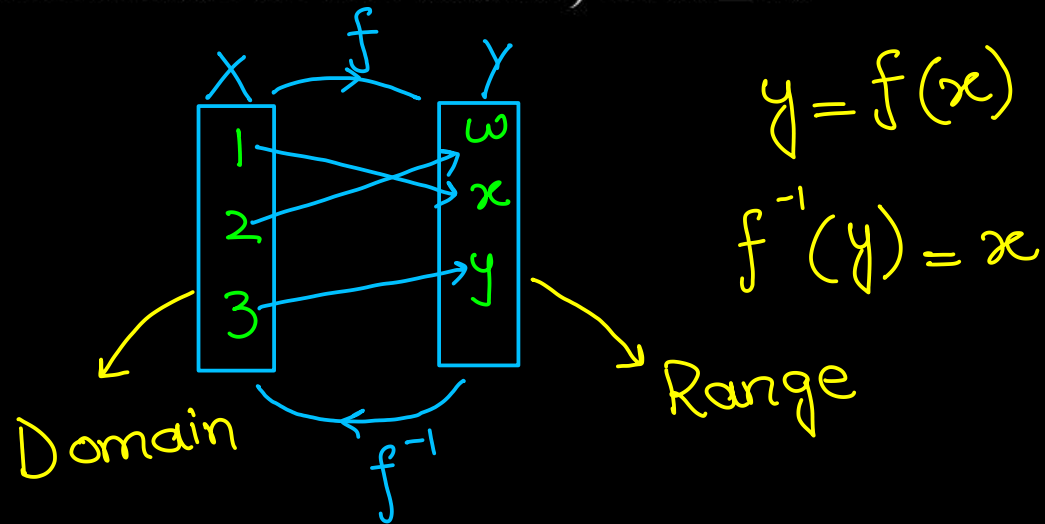
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## 3.2 Inverse of Quadratic Function

Quadratic functions are typically not one-to-one over their entire domain. To find an inverse for a quadratic function, we must restrict its domain to a portion where it is one-to-one. Commonly, we restrict the domain to either  $x \geq h$  (where  $h$  is the  $x$ -coordinate of the vertex) or  $x \leq h$ .

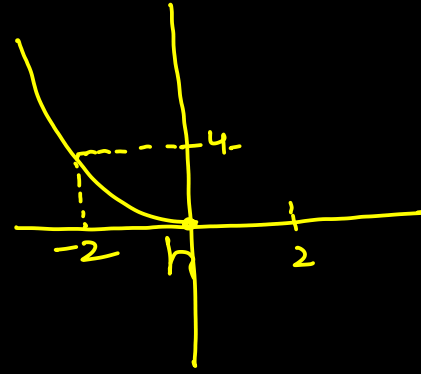


$$y = f(x)$$

$$f^{-1}(y) = x$$

$$\text{Domain}(f) = \text{Range}(f^{-1})$$

$$\text{Range}(f) = \text{Domain}(f^{-1})$$



**Example 4:** Find the inverse of  $f(x) = x^2 + 4x + 3, x \geq -2$ . Also find its domain and range.

$$\rightarrow D(f) = R(f^{-1})$$

**Solution:**  $f(x) = x^2 + 4x + 3, x \geq -2$

$$\rightarrow y = x^2 + 4x + 3$$

$$x = y^2 + 4y + 3$$

$$y^2 + 4y + \underline{3 - x} = 0$$

$$y = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(3 - x)}}{2(1)}$$

$$y = \frac{-4 \pm \sqrt{16 - 12 + 4x}}{2}$$

$$y = \frac{-4 \pm \sqrt{4 + 4x}}{2}$$

$$y = \frac{-4 \pm 2\sqrt{1 + x}}{2}$$

$$f^{-1}(x) = -2 \pm \sqrt{1 + x}$$

(Interchange  $x$  and  $y$ )

(Using the quadratic formula)

$$f^{-1}(x) = -2 + \sqrt{1 + x} \quad \leftarrow [-2, \infty)$$

$$-2 - \sqrt{1 + x} \quad \times$$

$$x + 1 \geq 0$$

$$x \geq -1$$

(Replace  $y$  with  $f^{-1}(x)$ )

The above inverse function has both a positive and a negative component. To determine which is the inverse, we find domain and range of the given function.

$$\text{Domain } f = [-2, \infty)$$



To find range, we proceed as

Since  $x \geq -2$

$$x^2 \geq +4$$

$$4x \geq -8$$

$$x^2 + 4x \geq -4$$

$$x^2 + 4x + 3 \geq -4 + 3$$

$$\Rightarrow f(x) \geq -1$$

As  $f(x) = x^2 + 4x + 3$

$$\Rightarrow f(x) = (x + 2)^2 - 1$$

Therefore, minimum value of  $f(x)$  is  $-1$  and hence

$$\text{Range } f = [-1, \infty)$$

$$\text{Domain } f^{-1} = [-1, \infty), \text{ Range } f^{-1} = [-2, \infty)$$

Now, we substitute any value of  $x$  that falls within the domain. We choose the value  $x = 0$ .

$$f^{-1}(0) = -2 + \sqrt{1+0} = -1$$

$$f^{-1}(0) = +2 - \sqrt{1-0} = -3$$

We notice only  $-1$  lies in the range of  $f$ . Therefore, we discard negative component.

Hence  $f^{-1}(x) = -2 + \sqrt{1+x}$

### EXERCISE 3.1

3. Find the inverse of the following quadratic functions. Also find their domain and range:

(i)  $f(x) = x^2 - 3, \quad x \leq 0$

Sol

$$f(x) = x^2 - 3,$$

$$y = x^2 - 3$$

$$y + 3 = x^2$$

$$x^2 = y + 3$$

$$\sqrt{x^2} = \pm \sqrt{y + 3}$$

$$x = \pm \sqrt{y + 3}$$

$$f^{-1}(y) = \pm \sqrt{y + 3}$$

Replace  $y$  with  $x$

$$f^{-1}(x) = \pm \sqrt{x + 3}$$

Using the fact that

$$\text{Domain}(f) = \text{Range}(f^{-1})$$

Since  $\text{Domain}(f) = (-\infty, 0] \Rightarrow$

$$\boxed{\text{Range}(f^{-1}) = (-\infty, 0]}$$

$$\Rightarrow \boxed{f^{-1}(x) = -\sqrt{x + 3}}$$

Now,

To find domain of  $f^{-1}$ ,

$$x + 3 \geq 0$$

$$x \geq -3$$

$$\Rightarrow \boxed{\text{Domain}(f^{-1}) = [-3, \infty)}$$

(ii)  $f(x) = x^2 + 6x + 4, \quad x < -3$

Sol

$$f(x) = x^2 + 6x + 4,$$

$$\text{Domain}(f) = (-\infty, -3)$$

$$y = x^2 + 6x + 4$$

$$\therefore y = f(x)$$

$$x^2 + 6x + 4 = y$$

$$x^2 + 2(x)(3) + 3^2 - 3^2 + 4 = y$$

$$(x+3)^2 - 9 + 4 = y$$

$$(x+3)^2 - 5 = y$$

$$(x+3)^2 = y + 5$$

$$\sqrt{(x+3)^2} = \pm \sqrt{y+5}$$

$$x+3 = \pm \sqrt{y+5}$$

$$x = -3 \pm \sqrt{y+5}$$

$$f^{-1}(y) = -3 \pm \sqrt{y+5}$$

$$\therefore x = f^{-1}(y)$$

Replace  $y$  with  $x$ ,

$$f^{-1}(x) = -3 \pm \sqrt{x+5}$$

To decide the inverse function, We use the fact that

$$\text{Domain}(f) = \text{Range}(f^{-1})$$

$$\text{Since } \text{Domain}(f) = (-\infty, -3) \Rightarrow \text{Range}(f^{-1}) = (-\infty, -3)$$

$$\Rightarrow f^{-1}(x) = -3 - \sqrt{x+5}$$

$$\text{To find the domain: } x+5 \geq 0 \Rightarrow x \geq -5$$

$$\text{Domain}(f^{-1}) = (-5, \infty)$$

$$(iii) \quad f(x) = 2x^2 - 8x + 11, \quad x \geq 2$$

Sol

$$f(x) = 2x^2 - 8x + 11$$

$$y = 2x^2 - 8x + 11$$

$$2x^2 - 8x + 11 - y = 0$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(11-y)}}{2(2)}$$

$$= \frac{8 \pm \sqrt{64 - 88 + 8y}}{4}$$

$$x = \frac{8 \pm \sqrt{8y - 24}}{4}$$

$$= \frac{8 \pm \sqrt{8} \sqrt{y-3}}{4}$$

$$= \frac{8 \pm 2\sqrt{2} \sqrt{y-3}}{4}$$

$$x = \frac{2(4 \pm \sqrt{2} \sqrt{y-3})}{4}$$

$$= \frac{4}{2} \pm \frac{\sqrt{2} \sqrt{y-3}}{2}$$

$$= 2 \pm \frac{\sqrt{y-3}}{\sqrt{2}}$$

$$x = 2 \pm \sqrt{\frac{y-3}{2}}$$

$$f^{-1}(y) = 2 \pm \sqrt{\frac{y-3}{2}}$$

$$f^{-1}(x) = 2 \pm \sqrt{\frac{x-3}{2}}$$

$$\therefore \frac{\sqrt{2}}{\sqrt{2} \sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\text{Range}(f^{-1}) = [2, \infty)$$

$$f^{-1}(x) = 2 + \sqrt{\frac{x-3}{2}}$$

For domain,

$$\frac{x-3}{2} \geq 0$$

$$x-3 \geq 0$$

$$x \geq 3$$

$$\text{Domain}(f^{-1}) = [3, \infty)$$



$$(iv) f(x) = 3x^2 - 2x + 6, \quad x \geq 5$$

Sol

$$y = 3x^2 - 2x + 6, \quad x \geq 5$$

$$0 = 3x^2 - 2x + 6 - y$$

$$3x^2 - 2x + (6 - y) = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(6-y)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{4 - 72 + 12y}}{6}$$

$$x = \frac{2 \pm \sqrt{12y - 68}}{6}$$

$$= \frac{2 \pm \sqrt{4(3y - 17)}}{6}$$

$$x = \frac{2 \pm 2\sqrt{3y - 17}}{6}$$

$$x = \frac{2(1 \pm \sqrt{3y - 17})}{6}$$

$$\rightarrow x = \frac{1 \pm \sqrt{3y - 17}}{3}$$

$$f^{-1}(y) = \frac{1 \pm \sqrt{3y - 17}}{3}$$

$$f^{-1}(x) = \frac{1 \pm \sqrt{3x - 17}}{3}$$

$$f^{-1}(x) = \frac{1}{3} \pm \frac{\sqrt{3x - 17}}{3}$$

$$h = \frac{-b}{2a}$$

$$= \frac{-(-2)}{2(3)}$$

$$h = \frac{1}{3}$$

$$\therefore x = f^{-1}(y)$$

$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

To decide the inverse function, We use the fact that

$$\text{Domain}(f) = \text{Range}(f^{-1})$$

Since  $\text{Domain}(f) = [5, \infty) \Rightarrow \text{Range}(f^{-1}) = [5, \infty)$

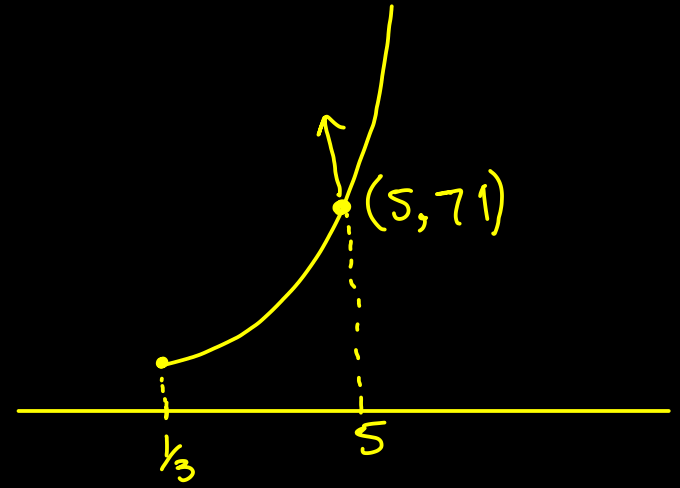
$$\Rightarrow f^{-1}(x) = \frac{1}{3} + \frac{\sqrt{3x-17}}{3}$$

To find the domain of  $f^{-1}$ :

$$f(5) = 3(5)^2 - 2(5) + 6$$

$$f(5) = 71$$

$$\Rightarrow \text{Range}(f^{-1}) = [71, \infty)$$



(v)  $f(x) = 2(x-3)^2 + 1, \quad x \geq 3$

sol

$$y = 2(x-3)^2 + 1,$$

$$x \geq 3$$

$$x \geq h$$

$$y-1 = 2(x-3)^2$$

$$\frac{y-1}{2} = (x-3)^2$$

$$(x-3)^2 = \frac{y-1}{2}$$

$$\sqrt{(x-3)^2} = \pm \sqrt{\frac{y-1}{2}}$$

$$x-3 = \pm \sqrt{\frac{y-1}{2}}$$

$$x = 3 \pm \sqrt{\frac{y-1}{2}}$$

$$f^{-1}(y) = 3 \pm \sqrt{\frac{y-1}{2}}$$

$$f^{-1}(x) = 3 \pm \sqrt{\frac{x-1}{2}}$$

$$\text{Range } f^{-1} = [3, \infty)$$

$$\Rightarrow f^{-1}(x) = 3 + \sqrt{\frac{x-1}{2}}$$

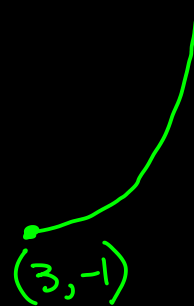
For domain  $f^{-1}$

$$\frac{x-1}{2} \geq 0$$

$$x-1 \geq 0$$

$$x \geq 1$$

$$\text{Domain}(f^{-1}) = [1, \infty)$$



(vi)  $f(x) = -3(x+4)^2 - 5, \quad x < -4$

Sol

$$y = -3(x+4)^2 - 5, \quad x < -4$$

$$y+5 = -3(x+4)^2$$

$$\frac{y+5}{-3} = (x+4)^2$$

$$\pm \sqrt{\frac{-(y+5)}{3}} = x+4$$

$$x = -4 \pm \sqrt{\frac{-(y+5)}{3}}$$

$$f^{-1}(y) = -4 \pm \sqrt{\frac{-(y+5)}{3}}$$

$$f^{-1}(x) = -4 \pm \sqrt{\frac{-(x+5)}{3}}$$

$$\text{Range}(f^{-1}) = (-\infty, -4)$$

$$\Rightarrow f^{-1}(x) = -4 - \sqrt{\frac{-(x+5)}{3}}$$

For domain:

$$-\frac{(x+5)}{3} \geq 0$$

$$-(x+5) \geq 0$$

$$x+5 \leq 0$$

$$x \leq -5$$

$$\text{Domain}(f^{-1}) = (-\infty, -5)$$

(In the Name of Allah, the Most Compassionate, the Most Merciful)

## Learning Outcomes

- Class 11: Mathematics (PECTAA)
- Unit 3: Theory of Quadratic Functions
- Absolute Value Quadratic Equations and Inequalities
- Exercise 3.1: Q4 (i - v)

YouTube Channel: [The Mathematics Outlet](#)

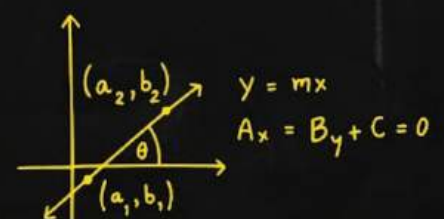


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### 3.3 Absolute Value

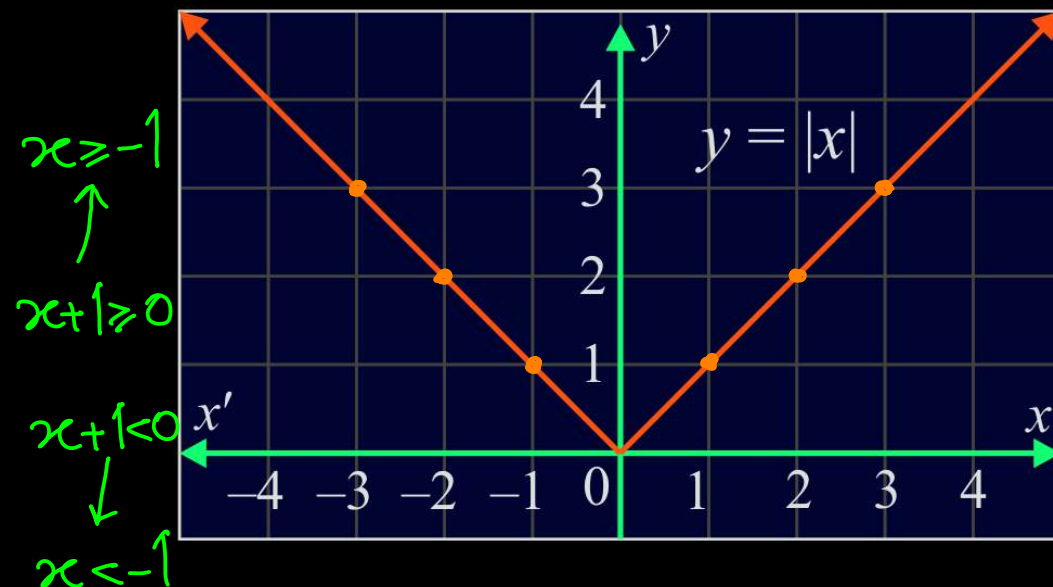
The absolute value of  $x$ , is defined as

$$|x| = \begin{cases} x & , \quad x \geq 0 \\ -x & , \quad x < 0 \end{cases}$$

$$|2| = 2$$

$$|-2| = -(-2) = 2$$

$$|x+1| = \begin{cases} x+1, \\ -(x+1), \end{cases}$$



#### 3.3.1 Absolute Value Quadratic Equations

To solve the absolute value quadratic equations, all answers must be substituted back into the original equation to verify whether they are valid or not. Sometimes, "extraneous" solutions may appear which are not valid and must be eliminated from the final answer.

**Example 5:** Solve  $|x^2 - 4| = 5$

**Solution:**

$$|x^2 - 4| = 5$$

$$\rightarrow x^2 - 4 = \pm 5$$

$$\underline{x^2 - 4 = 5} \quad \text{or} \quad \underline{x^2 - 4 = -5}$$

$$x^2 = 9 \quad \text{or} \quad x^2 = -1$$

$$x = \pm 3 \quad \text{or} \quad x = \pm \sqrt{-1} = \text{imaginary}$$

**Check:** For  $x = 3$

$$|3^2 - 4| = 5$$

$$|5| = 5$$

$$5 = 5$$

For  $x = -3$

$$|(-3)^2 - 4| = 5$$

$$|5| = 5$$

$$5 = 5$$

Hence solution set =  $\{\pm 3\}$



### 3.3.2 Absolute Value Quadratic Inequalities

Absolute value quadratic inequalities are inequalities that involve a quadratic expression within absolute value bars. They are generally of the following form:

$$|ax^2 + bx + c| < d, |ax^2 + bx + c| > d, |ax^2 + bx + c| \leq d, |ax^2 + bx + c| \geq d$$

Solution set is  $(-\infty, -0.16) \cup (6.16, \infty)$

**Example 6:** Solve  $|x^2 - 6x - 4| < 3$

**Solution:**  $|x^2 - 6x - 4| < 3$

$$-3 < x^2 - 6x - 4 < 3$$

$$\begin{array}{ll} -3 < x^2 - 6x - 4 & \text{or} \quad x^2 - 6x - 4 < 3 \\ x^2 - 6x - 4 + 3 > 0 & \text{or} \quad x^2 - 6x - 4 - 3 < 0 \end{array}$$

$$x^2 - 6x - 1 > 0 \quad (\text{i}), \quad x^2 - 6x - 7 < 0 \quad (\text{ii})$$

Here we solve  $x^2 - 6x - 1 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 + 4}}{2}$$

$$x = \frac{6 \pm \sqrt{40}}{2}$$

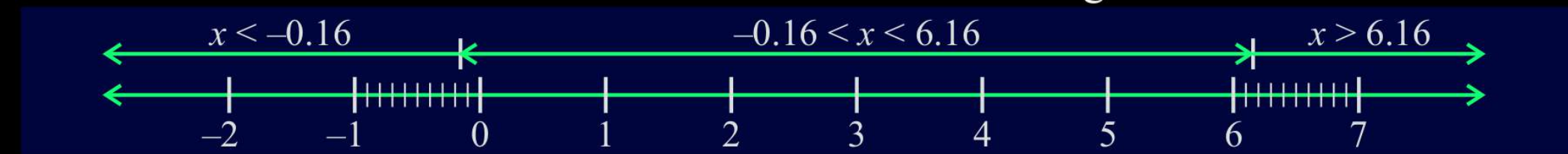
$$x = \frac{6 \pm 2\sqrt{10}}{2}$$

$$x = 3 \pm \sqrt{10}$$

$$x = 3 - \sqrt{10}, \quad 3 + \sqrt{10}$$

$$x = -0.16, \quad 6.16$$

Hence critical numbers divide the number line into three regions.



Test  $x = -1$  in (i), we have

$$(-1)^2 - 6(-1) - 1 > 0 \Rightarrow +6 > 0 \quad (\text{True})$$

$$\begin{array}{l} |x| \leq c \\ \rightarrow -c \leq x \leq c \end{array}$$

$$\begin{array}{l} |x| \geq c \\ \rightarrow x \leq -c \quad \text{or} \quad x \geq c \end{array}$$

Test  $x = 0$  in (i), we have

$$(0)^2 - 6(0) - 1 > 0 \Rightarrow -1 > 0 \quad (\text{False})$$

Test  $x = 7$  in (i), we have

$$(7)^2 - 6(7) - 1 > 0 \Rightarrow 6 > 0 \quad (\text{True})$$

Solution set is  $(-\infty, -0.16) \cup (6.16, \infty)$

Now, we take (ii) and solve

$$x^2 - 6x - 7 = 0$$

$$x^2 + x - 7x - 7 = 0$$

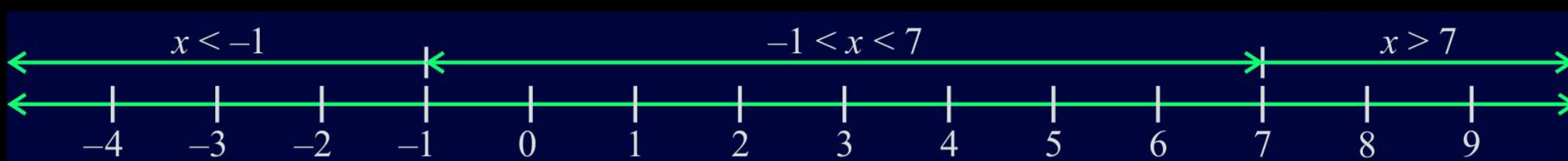
$$x(x + 1) - 7(x + 1) = 0$$

$$(x + 1)(x - 7) = 0$$

$$x + 1 = 0 \quad , \quad x - 7 = 0$$

$$x = -1 \quad , \quad x = 7$$

These critical numbers divide the number line into three regions.



Test  $x = -2$  ,  $x = 0$  and  $x = 10$  in (ii), we have

$$(-2)^2 - 6(-2) - 7 < 0 \Rightarrow 9 < 0 \quad (\text{False})$$

$$(0)^2 - 6(0) - 7 < 0 \Rightarrow -7 < 0 \quad (\text{True})$$

$$(10)^2 - 6(10) - 7 < 0 \Rightarrow 33 < 0 \quad (\text{False})$$

Solution set is  $(-1, 7)$

Hence the solution set of the given absolute value quadratic inequality is

$$(-\infty, -0.16) \cup (6.16, \infty) \cap \{(-1, 7)\} = (-1, -0.16) \cup (6.16, 7)$$

### EXERCISE 3.1

4. Solve the following absolute value quadratic equations and inequalities:

(i)  $|x^2 + 1| = 5$

Sol

$$|x^2 + 1| = 5$$

$$x^2 + 1 = \pm 5$$

$$x^2 + 1 = 5$$

or

$$x^2 + 1 = -5$$

$$x^2 = 5 - 1$$

$$x^2 = -5 - 1$$

$$x^2 = 4$$

$$x^2 = -6$$

Square root,

$$\sqrt{x^2} = \pm \sqrt{4}$$

$$x = \pm 2$$

(Imaginary)

Check

For  $x = 2$

$$|2^2 + 1| = |4 + 1|$$

$$= |5|$$

$$= 5 \quad (\text{True})$$

For  $x = -2$

$$|(-2)^2 + 1| = |4 + 1|$$

$$= |5|$$

$$= 5 \quad (\text{True})$$

$$\text{S.S.} = \{\pm 2\}$$

$$(ii) \quad |x^2 + 5x + 4| = 0$$

Sol

$$|x^2 + 5x + 4| = 0$$

$$x^2 + 5x + 4 = 0$$

$$x^2 + x + 4x + 4 = 0$$

$$x(x+1) + 4(x+1) = 0$$

$$(x+1)(x+4) = 0$$

$$x+1=0$$

$$x = -1$$

$$x+4=0$$

$$x = -4$$

Check

For  $x = -1$

$$\begin{aligned} |(-1)^2 + 5(-1) + 4| &= |1 - 5 + 4| \\ &= |0| \\ &= 0 \text{ (True)} \end{aligned}$$

For  $x = -4$

$$\begin{aligned} |(-4)^2 + 5(-4) + 4| &= |16 - 20 + 4| \\ &= |0| \\ &= 0 \text{ (True)} \end{aligned}$$

$$S.S. = \{-4, -1\}$$

$$(iii) \quad |x^2 - 6x + 8| = 4$$

Sol

$$|x^2 - 6x + 8| = 4$$

$$x^2 - 6x + 8 = \pm 4$$

$$x^2 - 6x + 8 = 4$$

$$x^2 - 6x + 8 - 4 = 0$$

$$x^2 - 6x + 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{20}}{2}$$

$$= \frac{6 \pm \sqrt{2^2 \times 5}}{2}$$

$$x = \frac{6 \pm 2\sqrt{5}}{2}$$

$$x = \frac{6}{2} \pm \frac{2\sqrt{5}}{2}$$

$$x = 3 \pm \sqrt{5}$$

$$x^2 - 6x + 8 = -4$$

$$x^2 - 6x + 8 + 4 = 0$$

$$x^2 - 6x + 12 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(12)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{-12}}{2}$$

(Imaginary)

Check

For  $x = 3 + \sqrt{5}$

$$|(3 + \sqrt{5})^2 - 6(3 + \sqrt{5}) + 8| = |9 + 6\cancel{\sqrt{5}} + 5 - 18 - 6\cancel{\sqrt{5}} + 8|$$

$$= |4| = 4 \quad (\text{True})$$





$$(iv) \quad |3x^2 - 7x + 2| = x^2 - x + 1$$

Sol

$$3x^2 - 7x + 2 = \pm (x^2 - x + 1)$$

$$3x^2 - 7x + 2 = x^2 - x + 1$$

$$3x^2 - 7x + 2 - x^2 + x - 1 = 0$$

$$2x^2 - 6x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{28}}{4}$$

$$= \frac{6 \pm \sqrt{2^2 \times 7}}{4}$$

$$x = \frac{6 \pm 2\sqrt{7}}{4}$$

$$= \frac{2(3 \pm \sqrt{7})}{4}$$

$$x = \frac{3 \pm \sqrt{7}}{2}$$

$$3x^2 - 7x + 2 = -(x^2 - x + 1)$$

$$3x^2 - 7x + 2 + x^2 - x + 1 = 0$$

$$4x^2 - 8x + 3 = 0$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(3)}}{2(4)}$$

$$= \frac{8 \pm \sqrt{16}}{8}$$

$$x = \frac{8 \pm 4}{8}$$

$$x = \frac{8-4}{8};$$

$$x = \frac{8+4}{8}$$

$$x = \frac{1}{2}$$

$$x = \frac{3}{2}$$

Check

$$\text{For } x = \frac{1}{2}$$

$$\text{L.H.S.} = \left| 3\left(\frac{1}{2}\right)^2 - 7\left(\frac{1}{2}\right) + 2 \right|$$

$$= \left| \frac{3}{4} - \frac{7}{2} + 2 \right|$$

$$\text{R.H.S.} = \left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1$$

$$= \frac{1}{4} - \frac{2}{4} + \frac{4}{4}$$

$$= \left| \frac{3}{4} - \frac{14}{4} + \frac{8}{4} \right|$$

$$= \left| \frac{3-14+8}{4} \right|$$

$$= \left| \frac{-3}{4} \right|$$

$$= \frac{3}{4}$$

$$= \frac{1-2+4}{4}$$

$$= \frac{3}{4}$$

Hence

$$\text{L.H.S.} = \text{R.H.S}$$

$$\text{S.S.} = \left\{ \frac{3 \pm \sqrt{7}}{2}, \frac{1}{2}, \frac{3}{2} \right\}$$

$$(v) \quad |x^2 - 4| < 5$$

Sol

$$|x^2 - 4| < 5$$

$$-5 < x^2 - 4 < 5$$

Add 4,

$$-5 + 4 < x^2 - 4 + 4 < 5 + 4$$

$$-1 < x^2 < 9$$

$$-1 < x^2$$

$$x^2 > -1$$

(Always true)

$$x \in \mathbb{R}$$

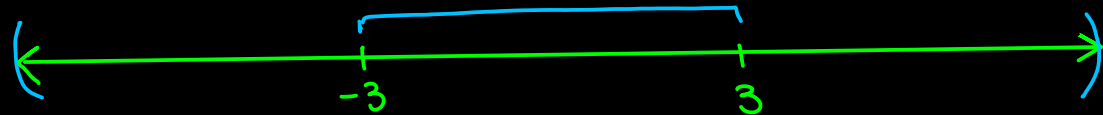
$$x^2 < 9$$

Taking square root

$$\sqrt{x^2} < \sqrt{9}$$

$$|x| < 3$$

$$-3 < x < 3$$



$$\text{S.S.} = (-\infty, \infty) \cap (-3, 3)$$

$$= (-3, 3)$$

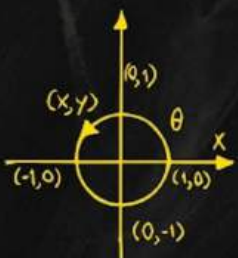
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## Learning Outcomes

- Class 11: Mathematics (PECTAA)
- Unit 3: Theory of Quadratic Functions
- Absolute Value Quadratic Equations and Inequalities
- Exercise 3.1: Q4 (vi - viii)

YouTube Channel: [The Mathematics Outlet](#)

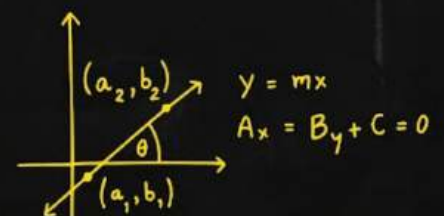


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# EXERCISE 3.1

4. Solve the following absolute value quadratic equations and inequalities:

(vi)  $|x^2 - 3x + 2| > 4$

Sol

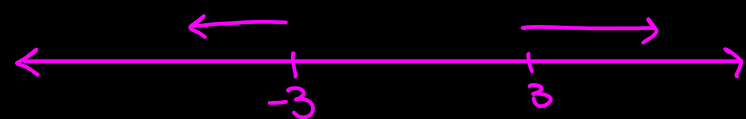
$$|x^2 - 3x + 2| > 4$$

$$\therefore |x| > c$$

$$x < -c \quad \text{or} \quad x > c$$

$$x^2 - 3x + 2 < -4$$

$$\text{or} \quad x^2 - 3x + 2 > 4$$



$$x^2 - 3x + 6 < 0$$

$$x^2 - 3x - 2 > 0$$

Associated equations are:

$$x^2 - 3x + 6 = 0$$

$$b^2 - 4ac = (-3)^2 - 4(1)(6)$$

$$= 9 - 24$$

$$= -15 < 0$$

(Imaginary)

$$x^2 - 3x - 2 = 0$$

$$b^2 - 4ac = (-3)^2 - 4(1)(-2)$$

$$= 9 + 8$$

$$= 17 > 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

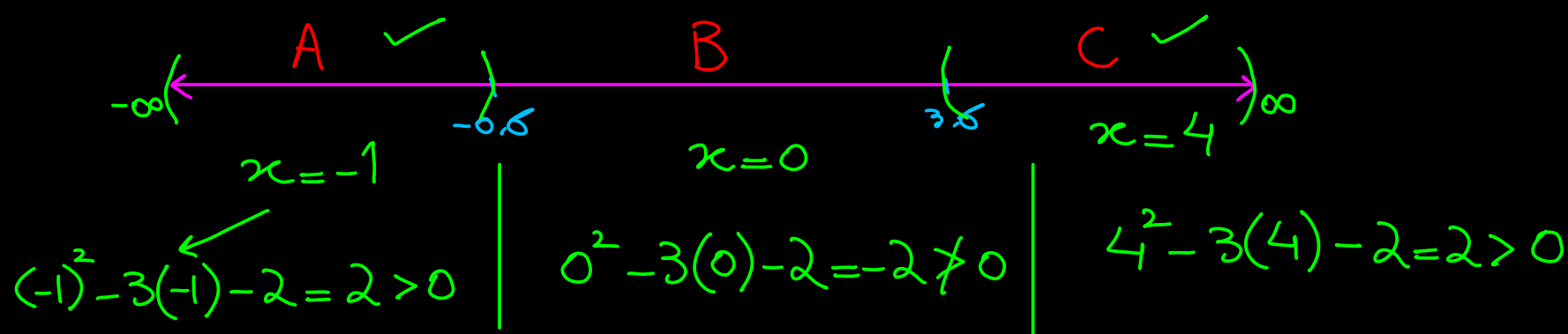
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9 - (-8)}}{2}$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$

$$\frac{3 - \sqrt{17}}{2} = -0.6$$

$$\frac{3 + \sqrt{17}}{2} = 3.6$$



Hence

$$\text{S.S.} = \left(-\infty, \frac{3 - \sqrt{17}}{2}\right) \cup \left(\frac{3 + \sqrt{17}}{2}, \infty\right)$$

$$(vii) |x^2 - 5x + 6| \leq x + 2$$

Sol

$$|x^2 - 5x + 6| \leq x + 2$$

$$-(x+2) \leq x^2 - 5x + 6 \leq x+2$$

$$-x-2 \leq x^2 - 5x + 6$$

$$0 \leq x^2 - 5x + 6 + x + 2$$

$$x^2 - 4x + 8 \geq 0$$

Associated equations are:

$$x^2 - 4x + 8 = 0$$

$$b^2 - 4ac = (-4)^2 - 4(1)(8)$$

$$= 16 - 32$$

$$= -16 < 0$$

So,

$$x \in (-\infty, \infty)$$

$$\therefore |x| \leq c$$

$$-c \leq x \leq c$$



$$x^2 - 5x + 6 \leq x + 2$$

$$x^2 - 5x + 6 - x - 2 \leq 0$$

$$x^2 - 6x + 4 \leq 0$$

$$x^2 - 6x + 4 = 0$$

$$b^2 - 4ac = (-6)^2 - 4(1)(4)$$

$$= 36 - 16$$

$$= 20 > 0$$

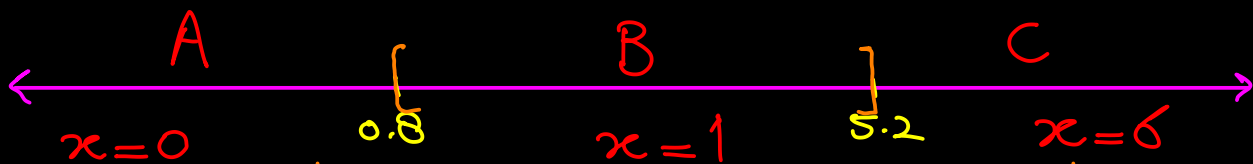
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{20}}{2}$$

$$= \frac{6 \pm 2\sqrt{5}}{2}$$

$$x = \frac{6}{2} \pm \frac{2\sqrt{5}}{2}$$

$$x = 3 \pm \sqrt{5} \begin{matrix} \nearrow 0.8 \\ \searrow 5.2 \end{matrix}$$



$$0^2 - 6(0) + 4 = 4 \neq 0$$

$$1^2 - 6(1) + 4 = -1 \leq 0$$

$$6^2 - 6(6) + 4 = 4 \neq 0$$

$$x \in [3 - \sqrt{5}, 3 + \sqrt{5}]$$

Hence

$$S.S. = (-\infty, \infty) \cap [3 - \sqrt{5}, 3 + \sqrt{5}]$$

$$= [3 - \sqrt{5}, 3 + \sqrt{5}]$$



$$(viii) |2x^2 - 3x - 5| < 4$$

Sol

$$|2x^2 - 3x - 5| < 4$$

$$-4 < 2x^2 - 3x - 5 < 4$$

$$-4 < 2x^2 - 3x - 5$$

$$2x^2 - 3x - 1 > 0$$

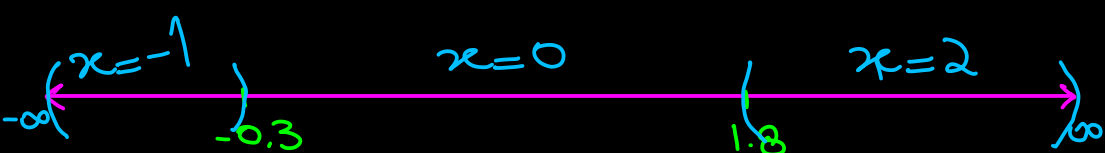
Associated equations are:

$$2x^2 - 3x - 1 = 0$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

$$x \approx -0.3, 1.8$$



$$x \in \left(-\infty, \frac{3 - \sqrt{17}}{4}\right) \cup \left(\frac{3 + \sqrt{17}}{4}, \infty\right)$$

$$2x^2 - 3x - 5 < 4$$

$$2x^2 - 3x - 9 < 0$$

$$2x^2 - 3x - 9 = 0$$

$$2x^2 - 6x + 3x - 9 = 0$$

$$2x(x-3) + 3(x-3) = 0$$

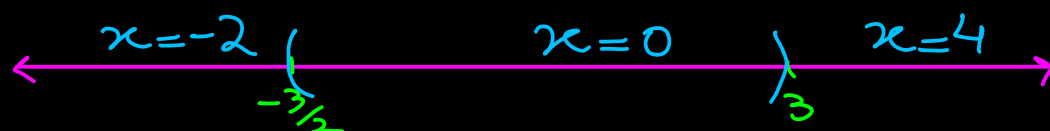
$$(x-3)(2x+3) = 0$$

$$x-3=0$$

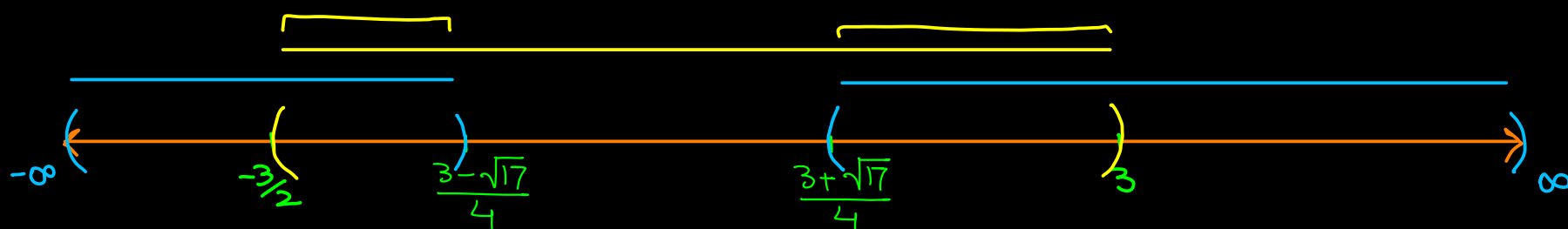
$$x=3$$

$$2x+3=0$$

$$x = -\frac{3}{2}$$



$$x \in \left(-\frac{3}{2}, 3\right)$$



Hence,

$$S.S = \left(-\frac{3}{2}, \frac{3 - \sqrt{17}}{4}\right) \cup \left(\frac{3 + \sqrt{17}}{4}, 3\right)$$



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## Learning Outcomes

- Class 11: Mathematics (PECTAA)
- Unit 3: Theory of Quadratic Functions
- Rational Equations Reducible to the Quadratic Equation
- Exercise 3.2: Q1 (i - iii)

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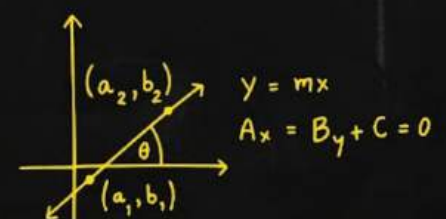


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### 3.4 Solution of Equations Reducible to the Quadratic Equation

There are certain types of equations, which do not look to be of degree 2, but they can be reduced to the quadratic equation. We shall discuss the solutions of the rational, radical and exponential equations.

#### 3.4.1 Rational Equations Reducible to the Quadratic Equation

A rational equation is an equation containing one or more rational expressions, where rational expressions typically contain a variable in the denominator.

**Example 7:** Solve  $\frac{1}{x} + \frac{2}{x+1} = 1$ ,  $x \neq 0$ ,  $x \neq -1$

**Solution:**  $\frac{1}{x} + \frac{2}{x+1} = 1$   $x \cdot (x+1)$

Multiplying both sides by  $x(x+1)$ , we have

$$(x+1) + 2x = x(x+1)$$

$$x+1+2x = x^2+x$$

$$3x+1 = x^2+x$$

$$x^2+x-3x-1=0$$

$$x^2-2x-1=0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = \frac{2 \pm \sqrt{4+4}}{2}$$

$$x = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

Hence, Solution Set =  $\{1 \pm \sqrt{2}\}$

## EXERCISE 3.2

1. Solve the following equations:

(i)  $\frac{1}{3x} + \frac{4x}{6} = 1, x \neq 0$

Sol

$$\frac{1}{3x} + \frac{4x}{6} = 1$$

$$\text{lcm}(3x, 6) = 6x$$

Multiply by  $6x$ ,

$$6x \left( \frac{1}{3x} + \frac{4x}{6} \right) = 6x \times 1$$

$$2 + 4x^2 = 6x$$

$$4x^2 - 6x + 2 = 0$$

$$2(2x^2 - 3x + 1) = 0$$

Divide by 2,

$$2x^2 - 3x + 1 = 0$$

$$2x^2 - x - 2x + 1 = 0$$

$$x(2x-1) - 1(2x-1) = 0$$

$$(2x-1) \cdot (x-1) = 0$$

$$2x-1=0$$

or

$$x-1=0$$

$$x = \frac{1}{2}$$

$$x = 1$$

$$\text{S.S.} = \left\{ \frac{1}{2}, 1 \right\}$$

$$(ii) \quad \frac{x}{x+1} + \frac{x+1}{x} = \frac{5}{2}; x \neq -1, 0$$

Sol

$$\frac{x}{x+1} + \frac{x+1}{x} = \frac{5}{2}$$

$$\frac{x^2 + (x+1)^2}{(x+1)x} = \frac{5}{2}$$

$$x^2 + x^2 + 2x + 1 = \frac{5}{2} x(x+1)$$

$$2x^2 + 2x + 1 = \frac{5}{2} (x^2 + x)$$

$$4x^2 + 4x + 2 = 5x^2 + 5x$$

$$0 = 5x^2 + 5x - 4x^2 - 4x - 2$$

$$x^2 + x - 2 = 0$$

$$x^2 - x + 2x - 2 = 0$$

$$x(x-1) + 2(x-1) = 0$$

$$(x-1)(x+2) = 0$$

$$x-1=0 \quad \text{or} \quad x+2=0$$

$$x=1$$

$$x=-2$$

$$\text{Hence} \quad \text{S.S.} = \{-2, 1\}$$

$$(iii) \quad \frac{1}{x+1} + \frac{2}{x+2} = \frac{7}{x+5}; x \neq -1, -2, -5$$

Sol

$$\frac{x+2+2(x+1)}{(x+1)(x+2)} = \frac{7}{x+5}$$

$$\frac{x+2+2x+2}{(x+1)(x+2)} = \frac{7}{x+5}$$

$$\frac{3x+4}{x^2+2x+x+2} = \frac{7}{x+5}$$

$$(3x+4)(x+5) = 7(x^2+3x+2)$$

$$3x^2+15x+4x+20 = 7x^2+21x+14$$

$$3x^2+19x+20 = 7x^2+21x+14$$

$$0 = 7x^2+21x+14-3x^2-19x-20$$

$$4x^2+2x-6 = 0$$

$$2(2x^2+x-3) = 0$$

Divide by 2,

$$2x^2+x-3 = 0$$

$$2x^2-2x+3x-3 = 0$$

$$2x(x-1)+3(x-1) = 0$$

$$(x-1)(2x+3) = 0$$

$$x-1=0$$

or

$$2x+3=0$$

$$x=1$$

$$x = -\frac{3}{2}$$

Hence S.S =  $\left\{-\frac{3}{2}, 1\right\}$

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

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## Learning Outcomes

- Class 11: Mathematics (PECTAA)
- Unit 3: Theory of Quadratic Functions
- Radical Equations Reducible to the Quadratic Equation
- Exercise 3.2: Q1 (iv - vii)

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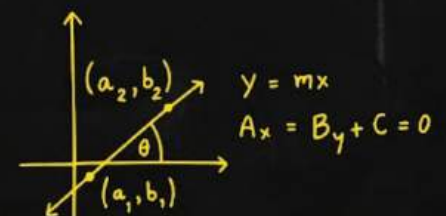


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### 3.4.2 Radical Equations Reducible to the Quadratic Equation

Equations involving radical expressions of the variable are called radical equations. To solve a radical equation, we first obtain an equation free from radicals. Every solution of radical equation is also a solution of the radical-free equation but the new equation has solutions that are not solutions of the original radical equation. Such extra solutions (roots) are called extraneous roots.

**Example 8:** Solve  $\sqrt{x+8} + \sqrt{x+3} = \sqrt{12x+13}$

**Solution:**  $\sqrt{x+8} + \sqrt{x+3} = \sqrt{12x+13}$

Squaring both sides, we get

$$x + 8 + x + 3 + 2\sqrt{x+8}\sqrt{x+3} = 12x + 13$$

$$2\sqrt{x+8}\sqrt{x+3} = 10x + 2$$

$$\Rightarrow \sqrt{(x+8)(x+3)} = 5x + 1$$

Squaring again, we have

$$x^2 + 11x + 24 = 25x^2 + 10x + 1$$

$$\Rightarrow 24x^2 - x - 23 = 0$$

$$\Rightarrow (24x + 23)(x - 1) = 0$$

$$x = -\frac{23}{24} \text{ or } x = 1$$

On checking we find that  $-\frac{23}{24}$  is an extraneous root. Hence solution set =  $\{1\}$



**EXERCISE 3.2**

1. Solve the following equations:

$$(iv) \quad \frac{a}{ax-1} + \frac{b}{bx-1} = a+b; x \neq \frac{1}{a}, \frac{1}{b}$$

Sol

$$\frac{a}{ax-1} + \frac{b}{bx-1} = a+b$$

$$\frac{a}{ax-1} - b + \frac{b}{bx-1} - a = 0$$

$$\frac{a-b(ax-1)}{ax-1} + \frac{b-a(bx-1)}{bx-1} = 0$$

$$\frac{a-abx+b}{ax-1} + \frac{b-abx+a}{bx-1} = 0$$

$$(a+b-abx) \left[ \frac{1}{ax-1} + \frac{1}{bx-1} \right] = 0$$

$$(a+b-abx) \left[ \frac{bx-1+ax-1}{(ax-1)(bx-1)} \right] = 0$$

Multiply by  $(ax-1)(bx-1)$ ,

$$(a+b-abx)(ax+bx-2) = 0$$

$$a+b-abx = 0$$

$$a+b = abx$$

$$\frac{a+b}{ab} = x$$

$$ax+bx-2=0$$

$$(a+b)x = 2$$

$$x = \frac{2}{a+b}$$

$$S.S. = \left\{ \frac{a+b}{ab}, \frac{2}{a+b} \right\}$$

$$(v) \quad \frac{x+1}{x-1} + \frac{x-1}{x+1} = 2, x \neq 1, x \neq -1$$

Sol

$$\frac{x+1}{x-1} + \frac{x-1}{x+1} = 2$$

Multiply by  $(x-1)(x+1)$ ,

$$(x-1)(x+1) \left[ \frac{x+1}{x-1} + \frac{x-1}{x+1} \right] = (x-1)(x+1) \times 2$$

$$(x+1)^2 + (x-1)^2 = 2(x^2 - 1^2)$$

$$x^2 + 2\cancel{x} + 1 + x^2 - 2\cancel{x} + 1 = 2x^2 - 2$$

$$2\cancel{x} + 2 = 2\cancel{x} - 2$$

$$2 = -2$$

(Not possible)

$\Rightarrow$  No solution.

$$(vi) \quad 3x^2 + 15x - 2\sqrt{x^2 + 5x + 1} = 2$$

Sol

$$3(x^2 + 5x) - 2\sqrt{x^2 + 5x + 1} = 2 \quad \text{--- ①}$$

$$\text{Let } y = x^2 + 5x \quad \text{--- ②}$$

$$3y - 2\sqrt{y+1} = 2$$

$$3y - 2 = 2\sqrt{y+1}$$

Squaring both sides,

$$(3y - 2)^2 = (2\sqrt{y+1})^2$$

$$9y^2 - 12y + 4 = 4(y+1)$$

$$9y^2 - 12y + \cancel{4} = 4y + \cancel{4}$$

$$9y^2 - 12y - 4y = 0$$

$$9y^2 - 16y = 0$$

$$y(9y - 16) = 0$$

$$y = 0$$

or

$$9y - 16 = 0$$

$$9y = 16$$

$$y = \frac{16}{9}$$

From ②

$$x^2 + 5x = 0$$

$$x(x+5) = 0$$

$$x = 0 \quad \text{or} \quad x + 5 = 0$$

$$x = -5$$

$$x^2 + 5x = \frac{16}{9}$$

$$9x^2 + 45x = 16$$

$$9x^2 + 45x - 16 = 0$$

Check for Extraneous roots,

Put  $x=0$  into ①

$$\begin{aligned} 3(0^2) + 15(0) - 2\sqrt{0^2 + 5(0) + 1} &= 0 - 2\sqrt{1} \\ &= -2 \\ &(\neq 2) \end{aligned}$$

$$\begin{aligned} 9x^2 + 48x - 3x - 16 &= 0 \\ 3x(3x+16) - 1(3x+16) &= 0 \end{aligned}$$

$$(3x+16)(3x-1) = 0$$

$$\begin{aligned} 3x+16 &= 0, & 3x-1 &= 0 \\ x &= -\frac{16}{3} & x &= \frac{1}{3} \end{aligned}$$

Put  $x=-5$  into ①

$$\begin{aligned} 3(-5)^2 + 15(-5) - 2\sqrt{(-5)^2 + 5(-5) + 1} &= 3(25) - 75 - 2\sqrt{25 - 25 + 1} \\ &= 75 - 75 - 2\sqrt{1} \\ &= -2 \\ &(\neq 2) \end{aligned}$$

Now, check for  $x = -\frac{16}{3}, \frac{1}{3}$

DIY

⋮

$$\text{S.S.} = \left\{ -\frac{16}{3}, \frac{1}{3} \right\}$$

$$(vii) \sqrt{2x+8} + \sqrt{x+5} = 7$$

Sol

$$\sqrt{2x+8} + \sqrt{x+5} = 7 \quad \text{--- ①}$$

Squaring,

$$\left(\sqrt{2x+8} + \sqrt{x+5}\right)^2 = 7^2$$

$$2x+8 + x+5 + 2\sqrt{2x+8}\sqrt{x+5} = 49$$

$$3x+13 + 2\sqrt{(2x+8)(x+5)} = 49$$

$$2\sqrt{2x^2+10x+8x+40} = 49-3x-13$$

$$2\sqrt{2x^2+18x+40} = 36-3x$$

Squaring again,

$$\left(2\sqrt{2x^2+18x+40}\right)^2 = (36-3x)^2$$

$$4(2x^2+18x+40) = 1296 + 9x^2 - 216x$$

$$8x^2 + 72x + 160 = 1296 + 9x^2 - 216x$$

$$0 = 1296 + 9x^2 - 216x - 8x^2 - 72x - 160$$

$$x^2 - 288x + 1136 = 0$$

$$x^2 - 284x - 4x + 1136 = 0$$

$$x(x-284) - 4(x-284) = 0$$

$$(x-284)(x-4) = 0$$

$$x-284=0$$

$$\text{or}$$

$$x-4=0$$

$$x = 284$$

$$x = 4$$

Checking for Extraneous roots

Put  $x = 284$  into ①

$$\begin{aligned}\sqrt{2(284)+8} + \sqrt{284+5} &= \sqrt{568} + \sqrt{289} \\ &= 24 + 17 \\ &= 41 (\neq 7)\end{aligned}$$

Extraneous  
root

Put  $x = 4$  into ①

$$\begin{aligned}\sqrt{2(4)+8} + \sqrt{4+5} &= \sqrt{16} + \sqrt{9} \\ &= 4 + 3 \\ &= 7\end{aligned}$$

Hence,

$$\text{S.S.} = \{4\}$$

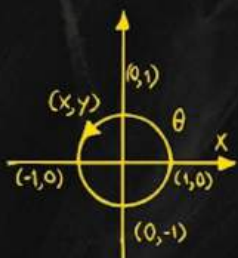


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## Learning Outcomes

- Class 11: Mathematics (PECTAA)
- Unit 3: Theory of Quadratic Functions
- Radical Equations Reducible to the Quadratic Equation
- Exercise 3.2: Q1 (viii - x)

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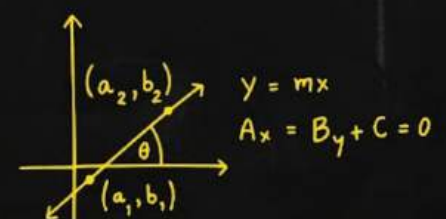


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## EXERCISE 3.2

1. Solve the following equations:

(viii)  $\sqrt{3x+4} = 2 + \sqrt{2x-4}$

Sol

$$\sqrt{3x+4} = 2 + \sqrt{2x-4}$$

Squaring,

$$(\sqrt{3x+4})^2 = (2 + \sqrt{2x-4})^2$$

$$3x+4 = 2^2 + (\sqrt{2x-4})^2 + 2(2)\sqrt{2x-4}$$

$$3x+4 = \cancel{4} + 2x - \cancel{4} + 4\sqrt{2x-4}$$

$$3x+4-2x = 4\sqrt{2x-4}$$

$$x+4 = 4\sqrt{2x-4}$$

Squaring again,

$$(x+4)^2 = (4\sqrt{2x-4})^2$$

$$x^2 + 4^2 + 8x = 16(2x-4)$$

$$x^2 + 16 + 8x = 32x - 64$$

$$x^2 + 8x + 16 - 32x + 64 = 0$$

$$x^2 - 24x + 80 = 0$$

$$x^2 - 20x - 4x + 80 = 0$$

$$x(x-20) - 4(x-20) = 0$$

$$(x-20)(x-4) = 0$$

$$x-20=0$$

or

$$x-4=0$$

$$x=20$$

$$x=4$$

Checking for Extraneous roots:

For  $x=4$

$$\begin{aligned}\text{L.H.S.} &= \sqrt{3(4)+4} \\ &= \sqrt{12+4} \\ &= 4\end{aligned}$$

$$\begin{aligned}\text{R.H.S.} &= 2 + \sqrt{2(4)-4} \\ &= 2 + \sqrt{8-4} \\ &= 2 + \sqrt{4} \\ &= 4\end{aligned}$$

$$\text{L.H.S.} = \text{R.H.S.}$$

For  $x=20$

$$\begin{aligned}\text{L.H.S.} &= \sqrt{3(20)+4} \\ &= \sqrt{60+4} \\ &= 8\end{aligned}$$

$$\begin{aligned}\text{R.H.S.} &= 2 + \sqrt{2(20)-4} \\ &= 2 + \sqrt{36} \\ &= 2 + 6 \\ &= 8\end{aligned}$$

$$\text{L.H.S.} = \text{R.H.S.}$$

Hence

$$\text{S.S.} = \{4, 20\}$$

$$(ix) \quad \sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$$

Sol

$$\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$$

Squaring,

$$\left(\sqrt{x+7} + \sqrt{x+2}\right)^2 = \left(\sqrt{6x+13}\right)^2$$

$$\left(\sqrt{x+7}\right)^2 + \left(\sqrt{x+2}\right)^2 + 2\sqrt{x+7}\sqrt{x+2} = 6x+13$$

$$x+7 + x+2 + 2\sqrt{(x+7)(x+2)} = 6x+13$$

$$2x+9 + 2\sqrt{x^2+2x+7x+14} = 6x+13$$

$$2\sqrt{x^2+9x+14} = 6x+13-2x-9$$

$$2\sqrt{x^2+9x+14} = 4x+4$$

$$\frac{2\sqrt{x^2+9x+14}}{2} = \frac{4x+4}{2}$$

$$\sqrt{x^2+9x+14} = 2x+2$$

Squaring again,

$$\left(\sqrt{x^2+9x+14}\right)^2 = (2x+2)^2$$

$$x^2+9x+14 = 4x^2+4+8x$$

$$0 = 4x^2+4+8x-x^2-9x-14$$

$$3x^2-x-10=0$$

$$3x^2+5x-6x-10=0$$

$$x(3x+5)-2(3x+5)=0$$

$$(3x+5)(x-2)=0$$

$$3x+5=0$$

$$x = -\frac{5}{3}$$

or

$$x-2=0$$

$$x=2$$

Checking for Extraneous roots:

For  $x = -\frac{5}{3}$

$$\text{L.H.S.} = \sqrt{-\frac{5}{3}+7} + \sqrt{-\frac{5}{3}+2}$$

$$= \sqrt{\frac{-5+21}{3}} + \sqrt{\frac{-5+6}{3}}$$

$$= \sqrt{\frac{16}{3}} + \sqrt{\frac{1}{3}}$$

$$= \frac{4}{\sqrt{3}} + \frac{1}{\sqrt{3}}$$

$$\text{L.H.S.} = \frac{5}{\sqrt{3}}$$

$$\text{R.H.S.} = \sqrt{6\left(-\frac{5}{3}\right)+13}$$

$$= \sqrt{2(-5)+13}$$

$$\text{R.H.S.} = \sqrt{3}$$

$$\text{L.H.S.} \neq \text{R.H.S.}$$

So,  $x = -\frac{5}{3}$  is an extraneous root.

For  $x=2$ ,

$$\text{L.H.S.} = \sqrt{2+7} + \sqrt{2+2}$$

$$= \sqrt{9} + \sqrt{4}$$

$$= 5$$

$$\text{R.H.S.} = \sqrt{6(2)+13}$$

$$= \sqrt{25}$$

$$= 5$$

$$\text{L.H.S.} = \text{R.H.S.}$$

Hence

$$\text{S.S.} = \{2\}$$

$$(x) \quad \sqrt{x+5} - \sqrt{x-3} = 2$$

Sol

$$\sqrt{x+5} - \sqrt{x-3} = 2$$

Squaring,

$$(\sqrt{x+5} - \sqrt{x-3})^2 = 2^2$$

$$(\sqrt{x+5})^2 + (\sqrt{x-3})^2 - 2\sqrt{x+5}\sqrt{x-3} = 4$$

$$x+5+x-3-2\sqrt{(x+5)(x-3)} = 4$$

$$2x+2-2\sqrt{x^2-3x+5x-15} = 4$$

$$-2\sqrt{x^2+2x-15} = 4-2x-2$$

$$-2\sqrt{x^2+2x-15} = 2-2x$$

Divide by -2,

$$\frac{-2\sqrt{x^2+2x-15}}{-2} = \frac{2-2x}{-2}$$

$$\sqrt{x^2+2x-15} = x-1$$

Squaring again,

$$(\sqrt{x^2+2x-15})^2 = (x-1)^2$$

$$\cancel{x^2} + 2x - 15 = \cancel{x^2} + 1 - 2x$$

$$2x - 15 + 2x - 1 = 0$$

$$4x - 16 = 0$$

$$4x = 16$$

$$x = 4$$



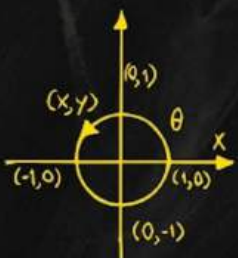


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## Learning Outcomes

- Class 11: Mathematics (PECTAA)
- Unit 3: Theory of Quadratic Functions
- Real World Problems of Quadratic Equations and Inequalities
- Exercise 3.2: Q2 - Q7

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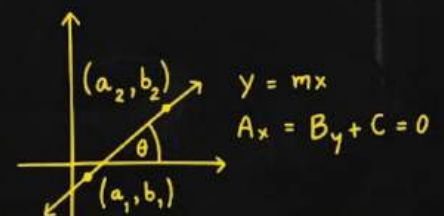


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### 3.5 Real World Problems of Quadratic Equations and Inequalities

We shall now proceed to solve the problems which, when expressed symbolically, lead to quadratic equations in one or two variables.

In order to solve such problems, we must:

- i. Suppose the unknown quantities to be  $x$  or  $y$  etc.
- ii. Translate the problem into symbols and form the equations satisfying the given conditions.

The method of solving the problems will be illustrated through the following examples:

**Example 9:** The length of a room is 3 metres greater than its breadth. If the area of the room is 180 square metres, find length and the breadth of the room.

**Solution:** Let the breadth of room =  $x$  metres  
and the length of room =  $(x + 3)$  metres  
 $\therefore$  Area of the room =  $x(x + 3)$  square metres

By the given condition, we have

$$x(x + 3) = 180 \quad \dots(i)$$

$$\Rightarrow x^2 + 3x - 180 = 0 \quad \dots(ii)$$

$$\Rightarrow (x + 15)(x - 12) = 0$$

$$\therefore x = -15 \text{ or } x = 12$$

As breadth cannot be negative so  $x = -15$  is not admissible.

When  $x = 12$ , we get  $x + 3 = 12 + 3 = 15$

Hence breadth of the room = 12 metres and length of the room = 15 metres.

**Example 10:** A company manufactures laptops and its weekly profit function (in thousands of dollars) is  $P(x) = -x^2 + 2x + 3$ , where  $x$  is the number of laptops produced (in hundreds). Find the range of production levels where the company makes at least \$4,000 profit.

**Solution:** Here  $P(x) \geq 4$

$$-x^2 + 2x + 3 \geq 4$$

$$-x^2 + 2x + 3 - 4 \geq 0$$

$$-x^2 + 2x - 1 \geq 0$$

$$x^2 - 2x + 1 \leq 0$$

$$(x - 1)^2 \leq 0$$

This only holds true when  $(x - 1)^2 = 0 \Rightarrow x = 1$

The company makes exactly \$4,000 profit when 100 laptops are produced (since  $x = 1$  means 100 laptops). There is no production level where profit is more than \$4,000.

## EXERCISE 3.2

- 2 A farmer bought some sheep for Rs. 9000. If he had paid Rs. 100 less for each, he would have got 3 sheep more for the same money. How many sheep did he buy, when the rate in each case is uniform?

Sol

Step 1: Let variables

Let Number of sheep =  $x$

Cost of each sheep =  $\frac{9000}{x}$

Step 2: New situation

If price is Rs. 100 less:

New price per sheep =  $\frac{9000}{x} - 100$

Number of sheep =  $x + 3$

Total money is still Rs. 9000.

So,

$$(x+3) \left( \frac{9000}{x} - 100 \right) = 9000$$

Step 3: Solve

$$9000 - 100x + \frac{27000}{x} - 300 = 9000$$

$$-100x + \frac{27000}{x} - 300 = 0$$

Multiply by 'x' to remove fraction:

$$x \left( -100x + \frac{27000}{x} - 300 \right) = x \times 0$$

$$-100x^2 + 27000 - 300x = 0$$

Divide by -100,

$$\frac{-100x^2 + 27000 - 300x}{-100} = \frac{0}{-100}$$

$$x^2 + 3x - 270 = 0$$

$$x^2 + 18x - 15x - 270 = 0$$

$$x(x+18) - 15(x+18) = 0$$

$$(x+18)(x-15) = 0$$

$$x+18=0$$

or

$$x-15=0$$

$$x = -18$$

$$x = 15$$

(Impossible)

Hence,

Number of sheep = 15

3. A man sold his stock of eggs for Rs. 2400. If he had 2 dozen more, he would have got the same money by selling the whole for Rs. 0.50 per dozen cheaper. How many dozen eggs did he sell?

Sol

Step 1: Let variables

Let

Number of dozen =  $x$

Price per dozen =  $\frac{2400}{x}$

Step 2: New situation

New number of dozens =  $x + 2$

New price per dozen =  $\frac{2400}{x} - 0.5$

Total money remains 2400.

So,

$$(x+2)\left(\frac{2400}{x} - 0.5\right) = 2400$$

Step 3: Solve

$$2400 - 0.5x + \frac{4800}{x} - 1 = 2400$$

$$-\frac{1}{2}x + \frac{4800}{x} - 1 = 0$$

Multiply by  $-2x$ :

$$-2x\left(-\frac{1}{2}x + \frac{4800}{x} - 1\right) = -2x \times 0$$

$$x^2 - 9600 + 2x = 0$$

$$x^2 + 2x - 9600 = 0$$

$$a=1, b=2, c=-9600$$

By using quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-9600)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 + 38400}}{2}$$

$$x = \frac{-2 \pm \sqrt{38404}}{2}$$

$$x = \frac{-2 \pm 196}{2}$$

$$\therefore \sqrt{38404} \approx 195.96$$

$$x = \frac{-2 + 196}{2}$$

or

$$x = \frac{-2 - 196}{2}$$

$$x = 97$$

$$x = -99$$

(Impossible)

Hence

Number of dozen = 97



4. A cyclist travelled 48 km at a uniform speed. If he had travelled 2 km/hour slower, he would have taken 2 hours more to perform the journey. How long did he take to cover 48 km?

Sol

Step 1: Let variables

Let

$$\text{Speed} = x \text{ km/h}$$

$$\therefore x = \frac{d}{t}$$

Time taken:

$$t = \frac{48}{x} \quad \text{--- ①}$$

Step 2: Slower speed

$$\text{New speed} = x - 2$$

$$\text{New time} = \frac{48}{x-2}$$

Given that

$$\frac{48}{x-2} = \frac{48}{x} + 2$$

Step 3: Solve

$$\frac{48}{x-2} - \frac{48}{x} = 2$$

Take LCM:

$$\frac{48x - 48(x-2)}{x(x-2)} = 2$$

$$\frac{48\cancel{x} - 48\cancel{x} + 96}{x(x-2)} = 2$$

$$\frac{96}{x(x-2)} = 2$$

$$96 = 2x(x-2)$$

Divide by 2,

$$48 = x(x-2)$$

$$48 = x^2 - 2x$$

$$0 = x^2 - 2x - 48$$

$$x^2 + 6x - 8x - 48 = 0$$

$$x(x+6) - 8(x+6) = 0$$

$$(x+6)(x-8) = 0$$

$$x+6=0$$

or

$$x-8=0$$

$$x=8 \quad \text{Put in ①}$$

$$x = -6$$

(Impossible)

$$t = \frac{48}{8}$$

$$t = 6 \text{ hours}$$



5. To do a piece of work, Abdullah takes 10 days more than Abdul Hadi. Together they finish the work in 12 days. How long would Abdul Hadi take to finish it alone?

Sol

Step 1: Let variables

Let Time taken by Abdul Hadi =  $x$  days

Then Time taken by Abdullah =  $x + 10$  days

Step 2: Work rates

$$\text{Work rate} = \frac{1}{\text{time}}$$

$$\text{Abdul Hadi's rate} = \frac{1}{x}$$

$$\text{Abdullah's rate} = \frac{1}{x+10}$$

Together:

$$\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}$$

Step 3: Solve

Take LCM:

$$\frac{x+10+x}{x(x+10)} = \frac{1}{12}$$

$$\frac{2x+10}{x(x+10)} = \frac{1}{12}$$

$$12(2x+10) = x(x+10)$$



6. The braking distance (in metres) of a car is modeled by:

$$\rightarrow d(s) = 0.02s^2 + 0.1s, \text{ where } s \text{ is the speed of car in km/h}$$

If the maximum safe braking distance is 50 metres, find the range of speed where braking is safe.

Sol

Braking distance:  $d(s) = 0.02s^2 + 0.1s$

Safe braking means:

$$d(s) \leq 50$$

$$0.02s^2 + 0.1s \leq 50$$

$$\therefore 0.02 = \frac{1}{50}$$

Multiply by 50,

$$50(0.02s^2 + 0.1s) \leq 50 \times 50$$

$$s^2 + 5s \leq 2500$$

$$s^2 + 5s - 2500 \leq 0$$

Associated equation is:

$$s^2 + 5s - 2500 = 0$$

$$s = \frac{-5 \pm \sqrt{5^2 - 4(1)(-2500)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25 + 10000}}{2}$$

$$s = \frac{-5 \pm \sqrt{10025}}{2}$$

$$s = \frac{-5 \pm 100.12}{2}$$

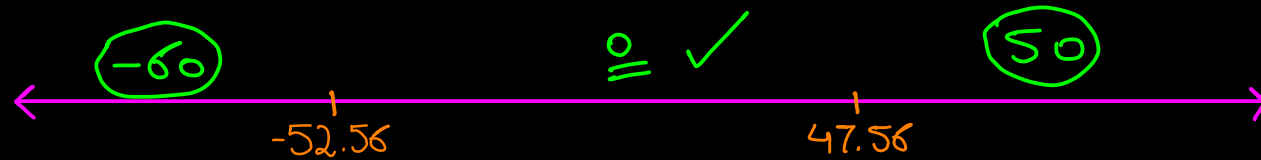
$$s = \frac{-5 + 100.12}{2}$$

$$s = 47.56$$

or

$$s = \frac{-5 - 100.12}{2}$$

$$s = -52.56$$



$$\begin{aligned} (-60)^2 + 5(-60) - 2500 &= 3600 - 300 - 2500 \\ &= 800 \neq 0 \end{aligned}$$

$$0^2 + 5(0) - 2500 = -2500 < 0$$

$$\begin{aligned} (50)^2 + 5(50) - 2500 &= 2500 + 25 - 2500 \\ &= 25 \neq 0 \end{aligned}$$

$$S.S. = [0, 47.56]$$

Hence, the range of safe speed is  $0 \leq s \leq 47.56 \text{ km/h}$

7. A rocket follows the height function  $h(t) = -5t^2 + 20t + 30$ , where  $h(t)$  is the height in metres and  $t$  is the time in seconds. Find the time interval during which the rocket is at least 40 metres above the ground.

Sol

Height function:  $h(t) = -5t^2 + 20t + 30$

Inequality:  $h(t) \geq 40$

$$-5t^2 + 20t + 30 \geq 40$$

$$-5t^2 + 20t + 30 - 40 \geq 0$$

$$-5t^2 + 20t - 10 \geq 0$$

Divide by -5,

$$t^2 - 4t + 2 \leq 0$$

Associated equation is:

$$t^2 - 4t + 2 = 0$$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 8}}{2}$$

$$t = \frac{4 \pm \sqrt{8}}{2}$$

$$t = \frac{4 \pm 2\sqrt{2}}{2}$$

$$t = 2 - \sqrt{2} \approx 0.58$$

$$t = 2 + \sqrt{2} \approx 3.41$$

