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Chapter # 09:

Division of Polynomials:

Used:

Data Science
Digital communication
Engineering

Polynomial:

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_3 x^3 + a_2 x^2 + a_1 x^1 + a_0$$

n = positive integers:

$a_0, a_1, a_2, \ldots, a_n$ = Real number

a_n = leading Co-efficient

n = degree of polynomial

e.g:

$P(x) = ax^3 + bx^2 + c$ ✓

$P(x) = 3x^{-2} + 2x^1 + 3$ ×

$P(x) = 5x^{1/2} + 3x^1 + 4$ ×

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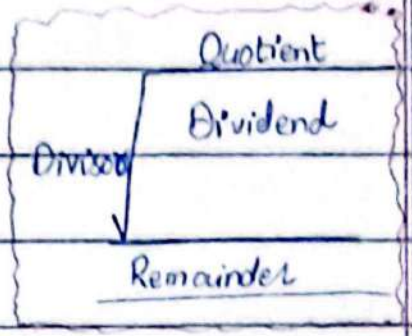
Simple Divison:



Exercise # 9.1

Question-01

(ii) $(3x^2 - x + 2) \div (x - 1)$

$$
\begin{array}{r|l}
 & 3x + 2 \\
\hline
x - 1 & 3x^2 - x + 2 \\
 & -3x^2 + 3x \\
\hline
 & +2x + 2 \\
 & -2x + 2 \\
\hline
 & 4
\end{array}
$$

Quotient = $3x + 2$

Remainder = 4

(iii)

$(x^3 + 12x^2 - 3x + 4) \div (x - 2)$

$$
\begin{array}{r|l}
 & x^2 + 14x + 25 \\
\hline
x - 2 & x^3 + 12x^2 - 3x + 4 \\
 & -x^3 + 2x^2 \\
\hline
 & 14x^2 - 3x + 4 \\
 & -14x^2 + 28x
\end{array}
$$

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$$25x + 4$$
$$-25x \mp 250$$
$$54$$

Qoutient = $x^2 + 14x + 25$

Remainded = 54

(V)

$$(3x^4 - 5x^3 + 4x - 6) \div (x^2 - 3x + 5)$$

$$3x^2 + 4x - 3$$
$$x^2 - 3x + 5 \;\big|\; 3x^4 - 5x^3 + 4x - 6$$
$$-3x^4 \mp 9x^3 \quad + 15x^2$$
$$4x^3 - 15x^2 + 4x - 6$$
$$-4x^3 \mp 12x^2 + 20x$$
$$-3x^2 - 16x - 6$$
$$\mp 3x^2 \mp 9x \mp 15$$
$$-25x + 9$$

Quotient = $3x^2 + 4x - 3$

Remainderd = $-25x + 9$

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(iii) $x^4-5x^3-8x^2+13x+12 \div x-6$

Ques $x^4+x^3+x^2+x+1 \div x-1$

$$x-6 \;\overline{)\; x^4-5x^3-8x^2+13x+12 \;\big|\; x^3+x^2-2x+1}$$

$$\mp x^4 \pm 6x^3$$

$$x^3-8x^2+13x+12$$

$$\mp x^3 \pm 6x^2$$

$$-2x^2+13x+12$$

$$\mp 2x^2 \pm 12x$$

$$x+12$$

$$\mp x \pm 6$$

$$18$$

Quotient $= x^3+x^2-2x+1$

Remainder $= 18$

(iv)

$5x^4-3x^3+2x^2-1 \div x^2+4$

$$x^2+4 \;\overline{)\; 5x^4-3x^3+2x^2-1 \;\big|\; 5x^2-3x-18}$$

$$\mp 5x^4 \quad \div +20x^2$$

$$-3x^3-18x^2-1$$

$$\mp 3x^3 \quad +12x$$

$$-18x^2+12x-1$$

$$\mp 18x^2 \quad \mp 72$$

$$12x-71$$

Quotient $= 5x^2-3x-18$

Remainder $= 12x-71$

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Question - 02

Remainder Theorem:

If a plynomial $f(x)$ of degree $f(x)$ of degree $n \geq 1$ is divided by $x-a$ till no x-term exists in the remainder, then $f(a)$ is the remainder

Proof: let $f(x)$ be a polynomial divided by $(x-a)$ and quoient is $Q(x)$ and remainder R then

$$x-a \;\overline{)\; f(x)}\;\; Q(x)$$

$$R$$

$$f(x) = Q(x)(x-a) + R$$

Put $x - a = 0$

$x = a$

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$f(a) = [Q(a)](a-a) + R$

$= [Q(a)](0) + R$

(ii)

$x^2 + 5x + 6, \quad x - 2$

$x - 2 = 0$

$x = 2$

$f(x) = x^2 + 5x + 6$

$f(2) = (2)^2 + 5(2) + 6$

$f(2) = 4 + 10 + 6$

$= 20$

(iii)

$x^3 + 5x^2 + 6, \quad x + 1$

$x + 1 = 0$

$x = -1$

$f(x) = x^3 + 5(x^2) + 6$

$f(-1) = (-1)^3 + 5(-1)^2 + 6$

$= -1 + 5(1) + 6$

$= -1 + 5 + 6$

$= 10$

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(iii)

$x^4+x^3+x^2+x+1$, $x+1$

$x+1=0$

$x=+1$

$f(+1)=(+1)^4+(+1)^3+(+1)^2+(+1)+1$

$=1+1+1+1+1$

$=5$

(iv)

x^4+x^2+1 , $x+3$

$x+3=0$

$x=-3$

$f(x)=x^4+x^2+1$

$f(-3)=(-3)^4+(-3)^2+1$

$=81+9+1$

$=91$

(v)

x^4+x^3+2, $x+2$

$x+2=0$

$x=-2$

$f(x)=x^4+x^2+2$

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$f(-2) = (-2)^4 + (-2)^3 + 2$

$= 16 - 8 + 2$

$= 18 - 8$

$= 10$

Question – 03:

(ii)

$x+1, x^2-1$

let: $f(x) = x^2 - 1$

Put: $x + 1 = 0$

$x = -1$

$f(-1) = (-1)^2 - 1$

$= 1 - 1$

$= 0$

So, $x+1$ is a factor of $f(x)$

(iii)

$x-2, x^2-5x+6$

Let: $f(x) = x^2 - 5x + 6$

Put $x - 2 = 0$

$x = 2$

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$f(2) = (2)^2 - 5(2) + 6$

$= 4 - 10 + 6$

$= 10 - 10$

$= 0$

So, $x - 2$ is a factor of $f(x)$.

(iii)

$x + 1, x^3 + x^2 + x - 3$

Let $f(x) = x^3 + x^2 + x - 3$

Put $x + 1 = 0$

$x = -1$

$f(-1) = (-1)^3 + (-1)^2 + (-1) - 3$

$= -1 + 1 - 1 - 3$

$= -2 - 2$

$= -4$

So, $x + 1$ is not a factor of $f(x)$

(iv)

$x - 2, x^3 + x^2 - 7x + 2$

Let $f(x) = x^3 + x^2 - 7(x) + 2$

Put $x - 2 = 0$

$x = 2$

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$f(2) = (2)^3 + (2)^2 - 7(2) + 2$

$= 8 + 4 - 14 + 2$

$= 14 - 14$

$= 0$

So, $f(u)$ is a factor of $(u-2)$

(v)

$u-3,\ u^4 - 3u^3 + u^2 - u + 1$

Put $u-3=0$

$u = 3$

$f(u) = u^4 - 3u^3 + u^2 - u + 1$

$f(3) = (3)^4 - 3(3)^3 + (3)^2 - (3) + 1$

$= 81 - 3(27) + 9 - 3 + 1$

$= 81 - 81 + 9 - 3 + 1$

$= 7$

So, $u-3$ is not a factor of $(u-3)$

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Question-04

(i)

$x^3 - 7x + 6, \quad x = 2$

By Synthetic division

	1	0	-7	6
2	x	2	4	-6
	1	2	-3	0

Since $R = 0$ So, $x = 2$ is zero given plynomial, then

$x^2 + 2x - 3 = 0$

$x^2 + 3x - x - 3 = 0$

$x(x+3) - 1(x+3) = 0$

$(x+3)(x-1) = 0$

$x + 3 = 0 \quad | \quad x - 1 = 0$

$x = -3 \quad | \quad x = 1$

$S.S = \{-3, 1, 2\}$

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(iii)

$x^2 - 28x - 48$, $x = -4$

By synthetic division.

	1	0	-28	-48
-4		-4	16	48
	1	-4	-12	0

Since $R = 0$, So $x = -4$ is zero

give polynomial, then

$x^2 - 4x - 12 = 0$

$x^2 - 6x + 2x - 12 = 0$

$x(x-6) + 2(x-6) = 0$

$(x+2)(x-6) = 0$

$x + 2 = 0$	$x - 6 = 0$
$x = -2$	$x = 6$

$S.S = \{6, -4, -2\}$

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$2x^4 + 7x^3 - 4x^2 - 27x - 18$, $x = 2$, $x = -3$

using Synthetic divison.

	2	7	-14	-27	-18
2		4	22	36	18
	2	11	18	9	0

Since R=0 So $x = 2$ and $x = -3$ are zeros of given polynomial, then

	2	11	18	9
-3		-6	-15	-9
	2	5	3	0

$2x^2 + 5x + 3 = 0$

$2x^2 + 2x + 3x + 3 = 0$

$2x(x+1) + 3(x+1) = 0$

$(2x+3)(x+1) = 0$

$2x + 3 = 0$ | $x + 1 = 0$

$2x/2 = -3/2$ | $x = -1$

$x = -3/2$

$S.S = \{-1, -3/2, 2, -3\}$

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Question – 05.

$x^4 - 10x^2 - 2x + 4 \div (x+3)$

let $x+3=0$

$x=-3$

By synthetic division

	1	0	-10	-2	4
-3	×	-3	9	3	-3
	1	-3	-1	1	1

So, Quotient: $x^3 - 3x^2 - x + 1$

Remainder: 1

Question – 06

$(x^3 - px^2 + qx + 2) \div (x+1)(x-2)$

let

$x+1=0$ | $x-2=0$

$x=-1$ | $x=2$

By synthitic divison.

	1	$-p$	q	2
-1	×	-1	$1+p$	$-1-p-q$
	1	$-1-p$	$1+q+p$	$1-p-q$

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2	x	2	2-2P
	1	1-P	3-P+q

By given condision.

$$1 - P - q = 0 \quad \text{---} \ (i)$$

$$3 - P + q = 0 \quad \text{---} \ (ii)$$

Adding (i) & (ii)

$$1 - P - q = 0$$

$$3 - P + q = 0$$

$$4 - 2P = 0$$

$$4 = 2P$$

$$\boxed{P = 2}$$

Put the value of P is equation (i)

$$1 - 2 - q = 0$$

$$\boxed{q = -1}$$

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Question–07

$4x^4 + 2x^3 + kx^2 + 13 \quad , \quad x+1$

let: $x+1=0$

$x=-1$

By synthetic division

	4	2	k	0	13
-1	x	-4	2	-2-k	2+k
	4	-2	2+k	-2-k	15+k

By given Condision

$15+k=16$

$k=16-15$

$\boxed{k=1}$

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Question – 09

x^3+Px^2+qx+3 , $x+1$, $x-2$

let $f(x)=x^3+Px^2+qx+3$

$x+1=0$, $x=-1$

$x-2=0$, $x=2$

Put $x=-1$ in $f(x)$

$f(-1)=(-1)^3+P(-1)^2+q(-1)+3$

$=-1+P-q+3$

$=2+P-q$

By factor thraim $f(1)=0$

$2+P-q=0$ ---- (i)

Put $x=2$ in $f(x)$

$f(2)=(+2)^3+P(2)^2+q(2)+3$

$=8+P(4)+2q+3$

$=11+4P+2q$

By factor throum

$f(2)=0$

$11+4P+2q=0$ ---- (ii)

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Multiplay (2) by (1) and then adding in (2)

$$11+4P+2q=0$$
$$4+2P-2q=0$$
$$15+6P=0$$
$$6P=-15$$
$$P=-15/6$$

Put in (i) $P=-5/6$

$$2-5/2-q=0$$
$$\frac{4-5}{2}=q$$
$$\boxed{q=-1/2}$$

Question – 10

$2x^3+4x^2+ax+b$

let $f(x)=2x^3+4x^2+ax+b$

$x=-2$

$x=2$

Put $x=-2$ in $f(x)$

$$f(-2)=2(-2)^3+4(-2)^2+a(-2)+b$$
$$=2(-8)+4(4)+a(-2)+b$$
$$=-16+16-2a+b$$
$$f(-2)=-2a+b$$

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By factor theroum $f(-2)=0$

$$-2a+b=0 \quad \text{---- (1)}$$

Put $x=2$ in $f(x)$

$$f(2)=2(2)^3+4(2)^2+a(2)+b$$

$$=2(8)+4(4)+2a+b$$

$$=16+16+2a+b$$

$$f(2)=32+2a+b$$

$$32+2a+b=0 \quad \text{---- (2)}$$

Multiply (2) by (1) and then adding in (2)

$$32+2a+b$$

$$-2a+b$$

$$32+2b=0$$

$$2b=-32$$

$$b=-32/2$$

Put in (1) $\boxed{b=-16}$

$$-2a-16=0$$

$$-16=+2a$$

$$a=-16/2$$

$$\boxed{a=-8}$$

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