Date: Day:

Exersice # 7.4

Combination: $({}^nC_r)$

$${}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$= \frac{1}{r!}\,\frac{n!}{(n-r)!}$$

$$= \boxed{\frac{1}{r!}\,{}^nP_r}$$

$$\boxed{r!\,{}^nC_r = {}^nP_r}$$

$$\boxed{\begin{matrix}{}^nC_n = 1 \\ {}^nC_0 = 1\end{matrix}}$$

Question-01

(i)

$${}^{50}C_{50}$$

$$= \frac{50!}{50!\,(50-50)!}$$

$$= \frac{1}{0!} = \frac{1}{1} = 1 \quad \text{Ans:-}$$

Written By:
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Date: Day:

(ii)

$^{1000}C_0$

$$\frac{n!}{r!\,(n-r)!}$$

$$= \frac{1000!}{0!\,(1000-0)!}$$

$$= \frac{1000!}{0!\,(1000)!}$$

$$= \frac{1}{0!} = \frac{1}{1} = 1$$ Ans.

(iii)

$^{10}C_7$

$$\frac{10!}{7!\,(10-7)!}$$

$$= \frac{10!}{7!\,(3)!}$$

$$= \frac{10\cdot \overset{3}{\cancel{9}}\cdot 8\cdot \cancel{7!}}{\cancel{7!}\,(\cancel{3})!}$$

$$= \frac{10\cdot 3\cdot 8}{1}$$

$$= 240$$

Date: Day:

(iv)

$^{20}C_n$

$$\frac{20!}{17!\,(20-17)!}$$

$$= \frac{20!}{17!\,(3)!}$$

$$= \frac{20\cdot19\cdot18\cdot\cancel{17!}}{\cancel{17!}\,(3)!}$$

$$= \frac{20\cdot19\cdot\cancel{18}^{3}}{\cancel{6}}$$

$$= 20\cdot19\cdot3$$

$$= 1140$$

Date: Day:

(iii)

$^nP_r = 120$ $\quad r = P$

$^nC_r = 20$

We Know

$r!\ ^nC_r = {}^nP_r$

$r!\ (20) = 120$

$r! = \frac{120}{20}$

$r! = 6$

$r! = 3!$

$\boxed{r = 3}$

Question - 03

(i)

$^nC_r = 56$, $^nP_r = 336$

We know

$r!\ ^nC_r = {}^nP_r$

$r!\ (56) = 336$

$r! = \frac{366}{56}$

$r! = 6$

Date:____________ Day:____________

$$r! = 3!$$

$$\boxed{r = 3}$$

AND

$$^nP_r = 336$$

$$\frac{n!}{(n-r)!} = 336 \qquad r = 3$$

$$\frac{n!}{(n-3)!} = 336$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 336$$

$$n(n-1)(n-2) = 336$$

$$n(n-1)(n-2) = 8 \cdot 7 \cdot 6$$

$$n(n-1)(n-2) = 8(8-1)(8-2)$$

On Comparing

$$\boxed{n = 8}$$

(ii)

$$^{n-1}C_{r-1} : {}^nC_r : {}^nC_r = 1 : 3 : 7$$

As:

$$^{n-1}C_{r-1} : {}^nC_r = 1 : 3$$

We Know

$$P.O.E = P.O.M$$

Date:____________ Day:____________

$$\left({}^{n-1}C_{r-1}\right)(3) = \left({}^{n}C_{r}\right)(1)$$

$$3\left[\frac{(n-1)!}{(r-1)!\,[n-1-(r-1)]!}\right] = \frac{n!}{r!\,(n-1)!}$$

$$\frac{3\,(n-1)!}{(r-1)!\,(n-r)!} = \frac{n\,(n-1)!}{r\,(r-1)!\,(n-r)!}$$

$$3 = \frac{n}{r}$$

$$3r = n \quad \text{—— (i)}$$

and.

$$^{n}C_{r} : {}^{n+1}C_{r+1} = 3 : 7$$

$$P.O.E = P.O.M$$

$$7\left({}^{n}C_{r}\right) = 3\left({}^{n+1}C_{r+1}\right)$$

$$7\left(\frac{n!}{r!\,(n-r)!}\right) = 3\left(\frac{(n+1)!}{(r+1)!\,[n+1-(r+1)]!}\right)$$

$$\frac{7\,\cancel{n!}}{\cancel{r!}\,\cancel{(n-r)!}} = \frac{3(n+1)\,(\cancel{n})!}{(r+1)\,r!\,[n+\cancel{1}-r-\cancel{1}]!}$$

$$\frac{7\,\cancel{n!}}{\cancel{r!}\,(n\cancel{-r})!} = \frac{3\,(n+1)\,(\cancel{n})!}{(r+1)\,\cancel{r!}\,[\cancel{n-r}]!}$$

$$7 = \frac{3\,(n+1)}{(r+1)}$$

$$7(r+1) = 3\,(n+1)$$

$$7r+7 = 3n+3$$

Date: Day:

$$7r+7 = 3(3r)+3$$

$$7r+7 = 9r+3$$

$$7-3 = 9r-7r$$

$$4 = 2r$$

$$\cancel{4}/\cancel{2} = r$$

$$\boxed{r = 2}$$

$n = 3(2)$ Put in eq (i)

$$\boxed{n = 6}$$

Question-04

Prove that

(ii)

$${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$$

L.H.S ${}^{n}C_r + {}^{n}C_{r-1}$

$$\frac{n!}{r!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-(r-1))!}$$

$$= \frac{n!}{r(r-1)!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)!}$$

$$= \frac{n!}{r(r-1)!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)(n-r)!}$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

Date: Day:

$$\frac{n!}{(r-1)!\,(n-r)!}\left[\frac{n-r+1+r}{(r)\,(n-r+1)}\right]$$

$$\frac{n!}{(r-1)!(n-r)!}\left[\frac{n+1}{r\,(n-r+1)}\right]$$

$$=\frac{(n+1)\,n!}{r(r-1)!\,(n-r+1)(n-r)!}$$

$$=\frac{(n+1)!}{r!\,(n-r+1)!}$$

$$=\frac{(n+1)!}{r!\,(n+1-r)!}$$

$$= {}^{n+1}C_r$$

L.H.S = R.H.S

(ii)

$$r.\,{}^{n}C_r = (n-r+1)\,{}^{n}C_{r-1}$$

$$R.H.S = (n-r+1)\,{}^{n}C_{r+1}$$

$$=(n-r+1)\left[\frac{n!}{(r-1)!\,(n-(r-1))!}\right]$$

$$=\frac{(n-r+1)\,n!}{(r-1)!\,(n-r+1)!}$$

$$=\frac{\cancel{(n-r+1)}\,(n)!}{(r-1)!\,\cancel{(n-r+1)}(n-r)!}$$

Date: Day:

$$= \frac{n!}{(r-1)!\,(n-r)!}$$

$$= r \cdot \frac{n!}{r(r-1)!\,(n-r)!}$$

$$= r \cdot \frac{n!}{r!\,(n-r)!}$$

$$= r \cdot {}^{n}C_{r}$$

Hence prove that

L.H.S = R.H.S

Question-05

Let

$n(n+1)(n+2) \cdots (n+r-2)(n+r-1)$

be the r consection number let P denotes the product of these numbeber

then,

$$P = n(n+1)(n+2) \cdots (n+r-2)(n+r-1)$$

Consider,

$${}^{n+r-1}C_{r} = \frac{(n+r-1)!}{r!\,(n+r-1-r)!}$$

$${}^{n+r-1}C_{r} = \frac{(n+r-1)(n+r-2) \cdots (n+2)(n+1)(n)\cancel{(n-1)!}}{r!\,\cancel{(n-1)!}}$$

Date: Day:

$$^{n+r-1}C_r = \frac{(n+r-1)(n+r-2)\cdots(n+2)(n-1)(n)}{r!}$$

$$^{n+r-1}C_r = \frac{P}{r!}$$

Hence proved

Question-06

Total no of courses $= 8 = n$

Courses to be sealected $= r = 5$

So

Total no of select 5 courses out to 8 $= {}^{8}C_5$

$$= \frac{8!}{5!(8-5)!}$$

$$= \frac{8\cdot 7\cdot 6\cdot \cancel{5!}}{\cancel{5!}(3)!}$$

$$= \frac{8\cdot 7\cdot \cancel{6}}{\cancel{6}}$$

$$= 56$$ Ans.

Date:________ Day:________

Question-07

Total number of letters of english alphabets = 26

Total no of vowal letter = 5

So

Total no of arrangement of vowel letter = $^5P_5$

$$= \frac{5!}{(5-5)!}$$

$$= \frac{5!}{0!} = \frac{120}{1}$$

$$= 120$$

Ans.

Question-08

Total no of Desi dishes = 6

Total no of chinese dishes = 8

Desi dishes to be selected = 3

chinese dishes to be selected = 2

So

Total no of ways to selected 3 deshes out of 6 and 2 chinese dishes out of 8 = $^8C_2 \times {}^6C_2$

$= (28)(20)$

$= 560$ Ans:-

Question-09

Total no of playing cards = 52

Total no of black cards = 26

Total no of red cards = 26

Black cards to be selected = 3

Red cards to be Selected = 5

So

Total no of ways to selected 8 card (3 black + 5 red) are

$= {}^{26}C_3 \times {}^{26}C_5$

$= (2600)(65780)$

$= 171028000$

Question-10

Total no of red ball = 8

Total no of green ball = 7

~~Red balls sel~~ (i)

Red ball to be selected = 3

Green ball to be selected = 2

Date:________ Day:________

So.

Total no of Possible ways to selected

5 balls (3Red + 2Green) are

$= {}^{8}C_3 \times {}^{7}C_2$

$= 56 \times 21$

$= 1176$

(iii)

Red balls to be selected = 1

Green balls to be selected = 4

So,

Total no of Possible ways to selected

5 balls (1red + 4Green) are

${}^{8}C_1 \times {}^{7}C_4$

$= 8 \times 35$

$= 280$

(iiii)

Red ball to be selected = 4

Green ball to be selected = 1

So,

Total no of Possible ways to selected

5 balls (4red + 1Green)

Date:________ Day:________

$$= {}^{8}C_4 \times {}^{7}C_1$$

$$= 70 \times 7$$

$$= 490$$

(iv)

Number of diagonals formula

$\boxed{{}^{n}C_2 - n}$

Number of triangle

$\boxed{= {}^{n}C_3}$

Question-011

Solve:

Total no of sides (vertices) $= n = 15$

To form diagonals we use vertices $= r = 2$

So,

Total no of diagonals $= {}^{15}C_2 - 15$

Date:______ Day:______

$= 105 - 15$

$= 90$

And:

To from triangle we use vertices

$= r = 3$

Total no of triangle formed $= {}^{15}C_3$

$= 499$

Question-012

No of diagonals = 104

No of sides = n = ?

We Know

No of diagonals $= {}^nC_2 - n$

$104 = {}^nC_2 - n$

$104 = \frac{n!}{2!(n-2)!} - n$

$104 = \frac{n(n-1)(n-2)!}{2!(n-2)!} - n$

$104 = \frac{n(n-1)}{2} - n$

$104 = \frac{n^2 - n - 2n}{2}$

$208 = n^2 - n - 2n$

Date:______ Day:______

$$0 = n^2 - 3n - 208$$

$$n^2 - 16n + 13n - 208 = 0$$

$$n(n-16) + 13(n-16) = 0$$

$$(n-16)(n+13) = 0$$

$n - 16 = 0$ | $n + 13 = 0$

$\boxed{n = 16}$ | $\boxed{n = -13}$

Question-013

Total no of vertices $= n = 15$

To form triangle we use vertices $= r = 3$

As,

Total no of triangles formed $= {}^{15}C_3$

$\boxed{= 455}$

Total no of linnear points $= 6$

No of invalid triangle formed by 6 linear points $= {}^{6}C_3 = 20$

So,

Total no of valid traiangle $= 455 - 20$

$\boxed{= 435}$

Date: Day:

Question-014.

Total no of boys = 10

Total no of girls = 8

Boys in Commites = 6

Girls in Commites = 3

So,

Total no of ways to from Commites of 6 boys and 3 girls are = $^{10}C_6 \times {^3C_3}$

$= 210 \times 56$

$\boxed{= 11760}$

Question-015

Total no of member = n = 10

member chosen for commite = r = 7

when two particuler person include

So,

Total no of commitee formed when two particuler person include = $^{10-2}C_{7-2}$

$= {^8C_5}$

$\boxed{= 56}$

Date: Day:

Question-17

Totel no of men = 6

Totel no of woman = 8

membes in a Committee = 7

(i)

with 3 - women

No of men = 4

So totel no of ways to from a Committee of 7-members with 3-woman =

$$ {}^{8}C_{3} \times {}^{6}C_{4} $$

$$ = (56)(15) $$

$$ = 840 $$

(ii)

with at most 3 - women

Totel no of ways to from a Committe of 7 - members with 3 - women

$$ = {}^{8}C_{1} \times {}^{6}C_{6} + {}^{8}C_{2} \times {}^{6}C_{5} + {}^{8}C_{3} \times {}^{6}C_{4} $$

$$ = (8) \times 1 + 28 \times 6 + 56 \times 15 $$

$$ = 1016 $$

ate:____________ Day:____

with at last 5 woman

Totel no of ways to from a Committe of 7 members with at least 3 women.

$${}^8C_5 \times {}^6C_2 + {}^8C_6 \times {}^6C_1 + {}^8C_7 \times {}^6C_0$$

$$= 56 \times 15 + 28 \times 6 + 8 \times 1$$

$$= 1016$$