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PERMUTATION

Permutation is an arrangement of objects in a specific order.

A permutation of n different objects taken $r\ (\leq n)$ at a time is written as ${}^nP_r$ or $P(n,r)$ and is defined as

$${}^nP_r = \frac{n!}{(n-r)!} \quad \text{where } r \leq n.$$

THEOREM:

Prove that ${}^nP_r = n(n-1)(n-2)\ldots(n-r+1) = \dfrac{n!}{(n-r)!}$

Proof: As there are n different objects to fill up r places. So, the first place can be filled in n ways. Since repetitions are not allowed, the second place can be filled in $(n-1)$ ways, the third place is filled in $(n-2)$ ways and so on. The r^{th} place has $n-(r-1) = n-r+1$ choices to be filled in. Therefore, by the fundamantal principle of counting, r places can be filled by n different objects in $n(n-1)(n-2)\ldots(n-r+1)$ ways

$$\therefore \quad {}^nP_r = n(n-1)(n-2)\ldots\ldots(n-r+1)$$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)(n-r)(n-r-1)\ldots 3.2.1}{(n-r)(n-r-1)\ldots\ldots 3.2.1}$$

$$\Rightarrow \quad {}^nP_r = \frac{n!}{(n-r)!}$$

which completes the proof.

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EXERCISE 7.2

Q: 1 Evaluate the following:

i) $^{10}P_5$

Solution: $^{10}P_5 = \frac{10!}{(10-5)!} = \frac{10!}{5!} =$

$= \frac{10.9.8.7.6.5!}{5!}$

$= 30240$

ii) $^{5}P_2$

Solution: $^{5}P_2 = \frac{5!}{(5-2)!}$

$= \frac{5!}{3!}$

$= \frac{5.4.3!}{3!}$

$= 20$

iii) $^{7}P_7$

Solution: $^{7}P_7 = \frac{7!}{(7-7)!}$

$= \frac{7!}{0!}$

$= \frac{7!}{1} = 5040$

iv) $^{10}P_3 =$

Solution: $^{10}P_3 = \frac{10!}{(10-3)!}$

$= \frac{10.9.8.7!}{7!}$

$= 720$

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Q: 2

Find the value of n, when:

i) $^nP_3 = 504$

Solution: $^nP_3 = 504$

$$\Rightarrow \frac{n!}{(n-3)!} = 504$$

$$\Rightarrow \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 9.8.7$$

$$\Rightarrow n(n-1)(n-2 = 9.8.7$$

$$\Rightarrow n = 9.$$

ii) $^{15}P_n = 15.14.13.12.11$

Solution: $^{15}P_n = 15.14.13.12.11$

$$\frac{15!}{(15-n)!} = 15.14.13.12.11$$

$$\Rightarrow \frac{15!}{(15-n)!} = \frac{15.14.13.12.11.10!}{10!}$$

$$\text{or} \quad \frac{15!}{(15-n)!} = \frac{15!}{(15-5)!}$$

$$\Rightarrow n = 5.$$

iii) $^nP_5 : {}^{n-2}P_2 = 540 : 1$

Solution:

$$^nP_5 / {}^{n-2}P_2 = 540/1$$

$$\Rightarrow \frac{n!/(n-5)!}{(n-2)!/(n-2-2)!} = \frac{540}{1}$$

$$\Rightarrow \frac{n!\,(n-4)!}{(n-2)!\,(n-5)!} = \frac{540}{1}$$

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$$\Rightarrow \quad \frac{n(n-1)(n-2)!\,(n-4)(n-5)!}{(n-2)!\,(n-5)!} = \frac{540}{1}$$

$$\Rightarrow \quad n(n-1)(n-4) = 540$$

$$\Rightarrow \quad n(n-1)(n-4) = 10.9.6$$

$$\Rightarrow \quad n = 10$$

Q: 3 Prove from the first Principle that:

i) $^{n}P_{r} = n.\,^{n-1}P_{r-1}$

Solution:

$$n.\,^{n-1}P_{r-1} = n.\frac{(n-1)!}{[n-1-(r-1)]!}$$

$$= \frac{n(n-1)!}{(n-1-r+1)!}$$

$$= \frac{n!}{(n-r)!}$$

$$= {}^{n}P_{r}$$

So, $^{n}P_{r} = n.\,^{n-1}P_{r-1}$

ii) $^{n}P_{r} = {}^{n-1}P_{r} + r.\,^{n-1}P_{r-1}$

Solution: Take $^{n-1}P_{r} + r.\,^{n-1}P_{r-1}$.

$$^{n-1}P_{r} + r.\,^{n-1}P_{r-1} = \frac{(n-1)!}{(n-1-r)!} + r\frac{(n-1)!}{[n-1-(r-1)]!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + r\frac{(n-1)!}{(n-r)!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + r\frac{(n-1)!}{(n-r)(n-r-1)!}$$

$$= \left[1 + \frac{r}{n-r}\right]\frac{(n-1)!}{(n-r-1)!}$$

$$= \left[\frac{n-r+r}{n-r}\right]\frac{(n-1)!}{(n-r-1)!}$$

$$= \frac{n(n-1)!}{(n-r)(n-r-1)!}$$

$$= \frac{n!}{(n-r)!}$$

$$= {}^{n}P_{r}$$

So, $${}^{n}P_{r} = {}^{n-1}P_{r} + r.\ {}^{n-1}P_{r-1}$$

Q:4 How many words can be formed from the letters of following words using all letters when no letter is to be repeated.

i) PYTHON

Solution: $n = 6,\ r = 6$

$$6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 720$$

ii) NETWORK

Solution: $n = 7,\ r = 7$

$$^{7}P_7 = \frac{7!}{(7-7)!} = \frac{7!}{0!} = 5040$$

iii) GOMPUTER

Solution: $n = 8,\ r = 8$

$$8P_8 = \frac{8!}{(8-8)!} = \frac{8!}{0!} = 40320$$

Q: 5 How many signals can be given by 6 flags of different colours, using 2 colours flags at a time.

Solution: Total No. of flags $= n = 6$

No. of flags at a time $= r = 2$

Total No. of Signals $= {}^{6}P_{2} = 6.5 = 30$

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Q: 6 How many signals can be given by 5 flags of different colours when any No. of flags are used at a time.

Solution:

Total No. of flags $= n = 5$

No. of Signals using 1 flag $= {}^5P_1 = 5$

No. of Signals using 2 flag $= {}^5P_2 = 5.4 = 20$

No. of Signals using 3 flag $= {}^5P_3 = 5.4.3 = 60$

No. of Signal, Using 4 flag $= {}^5P_4 = 5.4.3.2 = 120$

No of signals using 5 flag $= {}^5P_5 = 5.4.3.2.1 = 120$

Total No. of Signals $= 5+20+60+120+120 = 325$

Q: 7 How many 4 digit numbers can be formed with distinct digit with each digit odd.

Solution: The odd digits are 1, 3, 5, 7, 9

Total No. of odd digits $= n = 5$

No. of digits taken 4 at a time $= r = 4$

Total No. of 4 digit numbers $= {}^5P_4 = 5.4.3.2$

$= 120$

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Q: 8 How many numbers between 100 & 1000 Can be formed by using the digits 0, 1, 2, 3, 4, 5 without repetition. How many of them are divisible by 5.

Solution: Total available digits 0, 1, 2, 3, 4, 5

number between 100 and 1000 are 3-digit numbers from 100 to 999.

Total No. of digits $= n = 6$

Taken 3-digits at a time $= r = 3$

Total No. of 3-digit numbers $= {}^6P_3 = 6.5.4 = 120$

These 3-digit numbers also include number like 012, 035, 041 etc, So, we exclude these numbers. 0 is fixed $\boxed{0}\square\square$ So, remaing digit is 2 digit number. As 0 is fixed we have only 1, 2, 3, 4, 5 digits remaining. ~~No~~ Now

Total No. of available digits $= n = 5$

Taken 2-digits at a time $= r = 2$

Total No. of 2-digit no. $= {}^5P_2 = 5.4 = 20$

Total No. between 100 and 1000 are $= 120 - 20$
$= 100$

Q.9 Find the number greater than 35000 that can be formed from the digits 1,2,3,4,5,6 without repeating any digit.

Solution:

Total available digits are 1, 2, 3, 4, 5, 6
all 6-digit numbers are greater than 35000, so
Total Numbers greater than 35000 are of the form

i) □ □ □ □ □ □ $= {}^6P_6 = 720$

ii) $\boxed{3}\,\boxed{5}$ □ □ □ $= {}^4P_3 = 24$

iii) $\boxed{3}\,\boxed{6}$ □ □ □ $= {}^4P_3 = 24$

iv) $\boxed{4}$ □ □ □ □ $= {}^5P_4 = 120$

v) $\boxed{5}$ □ □ □ □ $= {}^5P_4 = 120$

vi) $\boxed{6}$ □ □ □ □ $= {}^5P_4 = 120$

Total no. of digits greater than 35000

$= 720 + 24 + 24 + 120 + 120 + 120$

$= 1128$

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Q:10 Find the number of 5-digit numbers that can be formed from the digits 1,2,4,6,8 (when no digit is repeated), but
i) The digits 2 and 8 are next to each other.
ii) The digits 2 and 8 are not next to each other.

Solution: Given digits are 1,2,4,6,8.
Consider 2,8 as one element.
Then we have to find permutations of 4 digits taken 4 at a time.
So, no. of permutations containing $28 = {}^4P_4$
$= 4!$
$= 24$
and
no. of permutations containing $82 = {}^4P_4$
$= 4!$
$= 24$

So, required no. of 5-digits no's is $= 24 + 24$
$= 48.$

ii) Total no. of 5-digit no's taken 5 at a time is
$= {}^5P_5$
$= 5!$
$= 5.4.3.2.1$
$= 120$

and total no. of 5-digit no's with 2 and 8 next to each other
$= 48$

So the total no. of 5-digit no's with 2 and 8 are not next to each other $= 120 - 48 = 72.$

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Q:11 How many 6-digit no's. can be formed, without repeating any digit from the digits 0, 1, 2, 3, 4, 5? In how many of them will 0 be at the ten's place?

Solution: Given digits are 0, 1, 2, 3, 4, 5.

For a 6-digit number, 0 can not be placed at the extreme left position, because then the number will be 5-digit.

Total no. of arrangements of 6 digits taken 6 at a time is $= 5 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 600$

Now total number of arrangements of 6 digits with 0 at the extrem left position

$= {}^5P_5$

$= 5!$

$= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 120$

So total no. of 6-digit no's $= 720 - 120$

$= 600$

Now no. of 6-digit no's with 0 at ten's place $= {}^5P_5$

$= 5!$

$= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 120$

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Q:12 How many 5-digit multiples of 5 can be formed from the digits 2, 3, 5, 7, 9 when no digit is repeated.

Solution: Given digits are 2, 3, 5, 7, 9.
For a 5-digit number to be a multiple of 5, 5 will be at the extreme right position.
So the total no. of 5-digit multiples of 5 is $= {}^4P_4$
$= 4!$
$= 4\cdot3\cdot2\cdot1$
$= 24.$

Q:13 In how many ways can 8 books including 2 on English be arranged on a shelf in such a way that the English books are never together?
Solution:
Let the English books be E_1 and E_2.
Consider E_1, E_2 as one book,
then we have to find the permutation of 7 books taken 7 at a time
Now no. of permutation containing $E_1E_2 = {}^7P_7$
$= 7!$
$= 7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$
$= 5040$
Now no. of permutation containing $E_2E_1 = {}^7P_7$
$= 7!$
$= 7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$
$= 5040$
So the number of permutations containing

Two English books E_1, E_2 together $= 5040 + 5040$
$= 10080.$

Now no. of permutation of 8 books taken all at a time
$= {}^8P_8$
$= 8!$
$= 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$= 40320$

Hence the no. of permutations of 8 books with two English books are never together
$= 40320 - 10080$
$= 30240.$

Q:14 Find the number of arrangements of 3 books on English and 5 books on urdu for placing them on a Shelf such that the book on the same Subjects are together.

Solution:

Let we denote the English book by E and urdu book by U, then no. of permutations of the form EEEEEUUU is $= {}^5P_5 \cdot {}^3P_3 = 5! \cdot 3! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1$
$= 720$

Now no. of permutations of the form UUU EEEEE is $= {}^3P_3 \cdot {}^5P_5 = 3! \cdot 5!$
$= 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

So, the number of permutations of 8 books with books on the same Subject are together $= 720 + 720 = 1440.$

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Q15: In how many ways can 5 boys and 4 girls be seated on a bench so that girls and the boys occupy alternate seats?

Sol:
Let we denote the boys by B and girl by G.
So the seating plan so that the girls and the boys occupy alternate seats is of the form BGBGBGBGB.
So the required no. of ways $= {}^5P_5 \cdot {}^4P_4$

$= 5! \cdot 4!$

$= 5\cdot4\cdot3\cdot2\cdot1 \cdot 4\cdot3\cdot2\cdot1$

$= 120 \times 24$

$= 2880.$

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