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CHAPTER 7

Factorial Notation:

Let n be a positive integer, then the product $n(n-1)(n-2)\ldots 3.2.1$ denoted by $n!$ or $\underline{|n}$ and read as n factorial.

i.e $n! = n(n-1)(n-2)\ldots\ldots 3.2.1$

and $n! = n(n-1)!$ where $0! = 1$

De-rangement Formula:

Let D_n be the number of de-rangements for n different objects, then

$$D_n = n!\left\{\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots + (-1)^n\frac{1}{n!}\right\}$$

where n is the number of items to be de-arranged.

EXERCISE 7.1

Q: 1 Evaluate each of the following:

i) $\dfrac{10!}{0!\,8!}$

Solution: $\dfrac{10!}{0!\,8!} = \dfrac{10.9.8!}{1.8!} = 90$

ii) $\dfrac{12!}{3!(12-3)!}$

Solution: $\dfrac{12!}{3!(12-3)!} = \dfrac{12.11.10.9!}{3.2.1.9!} = 220$

iii) $\dfrac{1440}{3!\,4!} + \dfrac{2400}{5!\,2!}$

Solution: $\dfrac{1440}{3!\,4!} + \dfrac{2400}{5!\,2!} = \dfrac{1440}{3.2.1.4.3.2.1} + \dfrac{2400}{5.4.3.2.1.2.1}$

$= \dfrac{1440}{144} + \dfrac{2400}{240}$

$= 10 + 10 = 20$

iv) $\dfrac{(n+2)!}{(n+1)!}$

Solution: $\dfrac{(n+2)!}{(n+1)!} = \dfrac{(n+2)(n+1)!}{(n+1)!}$

$= n+2$

Q: 2 Write each of the following in factorial form

i) $n^3 - n$

Solution: $n^3 - n = n(n^2-1)$

$= n(n+1)(n-1)$

$= (n+1)n(n-1)$

$= \dfrac{(n+1)n(n-1)(n-2)!}{(n-2)!}$

$= \dfrac{(n+1)!}{(n-2)!}$

ii) $n(n-1)(n-2)\ldots(n-r+1)$

Solution: $n(n-1)(n-2)\ldots(n-r+1)$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)(n-r)!}{(n-r)!}$$

$$= \frac{n!}{(n-r)!}$$

Q:3 Find n, if $(n+4)! = 3024 \cdot n!$

Solution: $(n+4)! = 3024 \cdot n!$

$$\Rightarrow \frac{(n+4)!}{n!} = 3024$$

$$\Rightarrow \frac{(n+4)(n+3)(n+2)(n+1)n!}{n!} = 3024$$

$$\Rightarrow (n+4)(n+3)(n+2)(n+1) = 9.8.7.6$$

$$\Rightarrow n+4 = 9$$

$$\Rightarrow n = 5$$

Q: 4 If $\frac{1}{7!} + \frac{1}{8!} = \frac{x}{9!}$, find x.

Solution: $\frac{1}{7!} + \frac{1}{8!} = \frac{x}{9!}$

$$\Rightarrow \frac{1}{7!} + \frac{1}{8.7!} = \frac{x}{9.8.7!}$$

$$\Rightarrow \left(1 + \frac{1}{8}\right)\frac{1}{7!} = \frac{x}{72.7!}$$

$$\Rightarrow \frac{9}{8} = \frac{x}{72}$$

$$\Rightarrow x = \frac{9.72}{8}$$

$$\therefore x = 81$$

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Q:5 Prove that $\dfrac{(2n+1)!}{n!} = [1.3.5 \ldots (2n-1)(2n+1)]\, 2^n$

Solution: $\dfrac{(2n+1)!}{n!} = \dfrac{(2n+1)2n(2n-1)(2n-2)\ldots 6.5.4.3.2.1}{n!}$

$$= \frac{[(2n+1)(2n-1)\ldots 5.3.1][2n(2n-2)\ldots 6.4.2]}{n!}$$

$$= \frac{[1.3.5\ldots(2n-1)(2n+1)][2.4.6\ldots(2n-2)2n \quad n\text{-factors}]}{n!}$$

$$= \frac{[1.3.5\ldots(2n-1)(2n+1)][1.2.3\ldots(n-1)(n)]2^n}{n!}$$

$$= \frac{[1.3.5\ldots(2n-1)(2n+1)]\, n!\, 2^n}{n!}$$

$$= [1.3.5\ldots(2n-1)(2n+1)]\, 2^n$$

Q.6: Express as a single fraction $\dfrac{(n+2)!}{(r+2)!} + \dfrac{(n+1)!}{(r+1)!}$

Solution: $\dfrac{(n+2)!}{(r+2)!} + \dfrac{(n+1)!}{(r+1)!} = \dfrac{(n+2)(n+1)!}{(r+2)(r+1)!} + \dfrac{(n+1)!}{(r+1)!}$

$$= \left[\frac{n+2}{r+2} + 1\right]\frac{(n+1)!}{(r+1)!}$$

$$= \left[\frac{n+2+r+2}{r+2}\right]\frac{(n+1)!}{(r+1)!}$$

$$= \frac{(n+r+4)(n+1)!}{(r+2)(r+1)!}$$

$$= \frac{(n+1)!\,(n+r+4)}{(r+2)!}$$

Q: 7 There are Four distinct colord balls and four Boxes of same color as those of the balls. Determine the possible ways the balls, one in each box, can be placed such that a ball doesnot go to a box of its own colour?

Solution:

we label 4 boxes as Blue, Green, Yellow, Red
we label 4 balls as Blue, Green, Yellow, Red

Boxes

$\boxed{B}$ $\boxed{G}$ $\boxed{Y}$ $\boxed{R}$

Balls

B G Y R

No. of Possible ways	Boxes $\boxed{B}$	$\boxed{G}$	$\boxed{Y}$	$\boxed{R}$
1.	G	Y	R	B
2.	G	R	B	Y
3.	G	B	Y	R
4.	Y	R	B	G
5.	Y	B	G	R
6.	Y	G	R	B
7.	R	B	G	Y
8.	R	G	Y	B
9.	R	Y	B	G

Total No. of Possible ways = 9.

Alternate Method

There are Total 4 objects which we want to be De-arranged.

$$n = 4$$

$$D_n = \text{De-rangment Formula} = n!\left[\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!}\right]$$

$$D_4 = 4!\left[\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = 24\left[1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right] = 9$$