6.6

1. Find G.M. between:

(i) -2 and 8

$G.M. = \pm\sqrt{ab}$

$G.M. = \pm\sqrt{(-2)(8)}$

$G.M. = \pm\sqrt{-16}$

$G.M. = \pm\sqrt{16}\,i$

$G.M. = \pm 4i$

(ii) $-2i$ and $8i$

$G.M. = \pm\sqrt{ab}$

$G.M. = \pm\sqrt{(-2i)(8i)}$

$G.M. = \pm\sqrt{-16i^2}$

$G.M. = \pm\sqrt{16}$

$G.M. = \pm 4$

(iii) 6 and 9

$G.M. = \pm\sqrt{ab}$

$G.M. = \pm\sqrt{6\times 9}$

$G.M. = \pm\sqrt{54}$

$G.M. = \pm 3\sqrt{6}$

2. Insert four real geometric means between 3 and 96.

$3, G_1, G_2, G_3, G_4, 96$

$a_1 = 3$ $\qquad a_6 = 96$

$a_2 = a_1 r$

$G_1 = a_1 r = 3 \times 2 = 6$

$G_2 = a_1 r^2 = 3 \times (2)^2 = 12$

$G_3 = a_1 r^3 = 3 \times (2)^3 = 24$

$G_4 = a_1 r^4 = 3 \times (2)^4 = 48$

$a_1 r^{6-1} = 96$

$3 \times r^5 = 96$

$r^5 = 96/3$

$r^5 = 32$

$r^5 = 2^5$

$\boxed{r = 2}$

3. If both x and y are positive distinct real numbers, show that the geometric mean between x and y is less than their arithmetic mean.

Sol As x and y are positive and distinct so $\sqrt{x}$ and $\sqrt{y}$ are also positive and distinct.

$(\sqrt{x} - \sqrt{y})^2 > 0$

$(\sqrt{x})^2 + (\sqrt{y})^2 - 2\sqrt{x}\sqrt{y} > 0$

$x + y > 2\sqrt{xy}$

$\frac{x+y}{2} > \sqrt{xy}$

$A.M > G.M$

4. For what value of n, $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is

the positive geometric mean between a and b?

$$\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \sqrt{ab}$$

$$a^n + b^n = a^{1/2} \cdot b^{1/2}(a^{n-1} + b^{n-1})$$

$$a^n + b^n = a^{n-1+1/2} \cdot b^{1/2} + a^{1/2} \cdot b^{n-1+1/2}$$

$$a^n + b^n = a^{n-1/2} \cdot b^{1/2} + a^{1/2} b^{n-1/2}$$

$$a^n - a^{n-1/2} \cdot b^{1/2} = a^{1/2} b^{n-1/2} - b^n$$

$$a^{n-1/2}(a^{n-(n-1/2)} - b^{1/2}) = b^{n-1/2}(a^{1/2} - b^{n-(n-1/2)})$$

$$a^{n-1/2}(a^{1/2} - b^{1/2}) = b^{n-1/2}(a^{1/2} - b^{1/2})$$

$$\frac{a^{n-1/2}}{b^{n-1/2}} = \frac{\cancel{(a^{1/2} - b^{1/2})}}{\cancel{(a^{1/2} - b^{1/2})}}$$

$$\left(\frac{a}{b}\right)^{n-1/2} = 1.$$

$$\left(\frac{a}{b}\right)^{n-1/2} = \left(\frac{a}{b}\right)^0$$

$$n - \frac{1}{2} = 0$$

$$\boxed{n = 1/2}$$

5. The A.M. of two positive integral numbers exceeds their (positive) G.M. by 2 and their sum is

20. find the numbers.

Sol $A.M. = G.M. + 2$ Let num. are 'x', 'y'

$x+y = 20 \rightarrow (i)$

$\frac{x+y}{2} = \sqrt{xy} + 2$

$\frac{\cancel{20}^{10}}{\cancel{2}} = \sqrt{xy} + 2$

$10 - 2 = \sqrt{xy}$

$8 = \sqrt{xy}$

taking square on b/s.

$(8)^2 = (\sqrt{xy})^2$

$64 = xy \rightarrow (2)$

from (i)

$y = 20 - x$

Put in (2)

$64 = x(20-x)$

$64 = 20x - x^2$

$x^2 - 20x + 64 = 0$

$x^2 - 16x - 4x + 64 = 0$

$x(x-16) - 4(x-16) = 0$

$(x-16)(x-4) = 0$

$x - 16 = 0 \qquad x - 4 = 0$

$x = 16 \qquad x = 4$

if $x = 16$

$y = 20 - 16 = 4$

if $x = 4$

$y = 20 - 4 = 16$

So the numbers are 16, 4 either 4, 16.

6. The A.M. between two ~~positive~~ numbers is 5 and their (positive) G.M. is 4. Find the numbers.

Let the numbers are x, y.

$$A.M. = \frac{x+y}{2}$$

$$\frac{x+y}{2} = 5$$

$$x + y = 10 \rightarrow (i)$$

G.M.

$$\sqrt{xy} = 4$$

$$xy = 16 \rightarrow (ii)$$

from (i)

$$x = 10 - y$$

Put in (ii)

$(10-y)y=16$

$10y-y^2-16=0$

$y^2-10y+16=0$

$y^2-8y-2y+16=0$

$y(y-8)-2(y-8)=0$

$(y-8)(y-2)=0$

$y-8=0$	$y-2=0$
$\boxed{y=8}$	$\boxed{y=2}$
$x=10-y$	$x=10-y$
$=2$	$=8$
$2, 8$	$8, 2$

7. The arithmetic mean between two positive numbers a and b is double their geometric mean. Prove that $a:b=2+\sqrt{3}:2-\sqrt{3}$.

$$\frac{a+b}{2}=2\sqrt{ab}$$

$$a+b=4\sqrt{ab}$$

Taking square

$$(a+b)^2=16(ab)$$

$$a^2+b^2+2ab-16ab=0$$

$$a^2+b^2-14ab=0 \longrightarrow (i)$$

Let $\frac{a}{b} = x$

$a = bx$

Put in (i)

$(bx)^2 + b^2 - 14(bx)(b) = 0$

$b^2x^2 + b^2 - 14b^2x = 0$

$x^2 + 1 - 14x = 0$

$x^2 - 14x + 1 = 0$

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{14 \pm \sqrt{196-4}}{2}$$

$$x = \frac{14 \pm \sqrt{192}}{2}$$

$$= \frac{14 \pm 8\sqrt{3}}{2}$$

$x = 7 \pm 4\sqrt{3}$

To prove $\frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

$$x = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2} = \frac{2^2+(\sqrt{3})^2+2(2)(\sqrt{3})}{4-3}$$

$$= \frac{4+3+4\sqrt{3}}{1}$$

$$x = 7+4\sqrt{3}$$

From (i) and (ii)

$$x = \frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

8. If one geometric mean G and two arithmetic means p and q are inserted between two positive numbers, Show that

$$G^2 = (2p-q)(2q-p)$$

Let numbers are x, y

$$x, G, y$$

$$G^2 = xy \rightarrow (i)$$

$$x, p, q, y$$

$$a_4 = a_1 + 3d$$

$$y = x+3d$$

$$\boxed{\frac{y-x}{3} = d}$$

$$a_2 = a_1 + d$$

$$p = x + \frac{y-x}{3}$$

$$p = \frac{y+2x}{3}$$

$$3p = y + 2x \quad \rightarrow (ii)$$

$$a_3 = a_1 + 2d$$

$$q = x + 2\left(\frac{y-x}{3}\right)$$

$$q = \frac{3x+2y-2x}{3}$$

$$3q = x + 2y \quad \rightarrow (iii)$$

$$3p = 2\cancel{x} + y$$

$$\underline{\underset{-}{6q} = \underset{-}{\cancel{2}x} \underset{-}{+} 4y}$$

$$3(p-2q) = -3y$$

$$\boxed{y = 2q - p}$$

Put in (ii)

$$3p = 2p - p + 2x$$

$$4p - 2q = 2x$$

$$\cancel{2}(2p-q) = \cancel{2}x$$

$$\boxed{2p - q = x}$$

$$G^2 = (2p-q)(2q-p)$$

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