6.5

1. Find the 6^{th} term of the G.P. : $-6, -3, -\frac{3}{2}, \dots$

$a_1 = -6 \qquad r = \frac{\cancel{-}3}{\cancel{-}6} = \frac{1}{2}$

$a_6 = a_1 r^5 = -6 \times \left(\frac{1}{2}\right)^5 = -6 \times \frac{1}{32}$

$a_6 = -\frac{3}{16}$

2. Find the 8^{th} term of the sequence, $3, 3^2, 3^3, \dots$

$a_1 = 3 \qquad r = \frac{3^{\cancel{2}}}{\cancel{3}} = 3$

$a_8 = a_1 r^7 = 3 \times (3)^7 = 3^8 = 6561$

3. The n^{th} terms of the sequences $1, 2, 4, 8, \dots$ and $256, 128, 64, \dots$ are equal. Find the value of n.

$1, 2, 4, 8, \dots$	$256, 128, 64, \dots$
$a_1 = 1 \qquad r = \frac{2}{1} = 2$	$a_1' = 256 \qquad r = \frac{128}{256} = \frac{1}{2}$
$a_n = a_1 r^{n-1}$	$a_n' = a_1' r^{n-1}$
$a_n = 1 \cdot (2)^{n-1}$	$a_n' = 256 \cdot \left(\frac{1}{2}\right)^{n-1}$

According to the question.

$a_n = a_n'$

$1 \times (2)^{n-1} = 256 \times \left(\frac{1}{2}\right)^{n-1}$

$$(2)^{n-1} \times (2)^{n-1} = 256$$

$$2^{n-1+n-1} = 256$$

$$2^{2n-2} = 256$$

$$2^{2(n-1)} = 2^8$$

$$2(n-1) = 8$$

$$n-1 = 4$$

$$\boxed{n = 5}$$

4. Find the first five terms of each sequence described:

(i) $a_1 = 243, r = 1/3$

$a_2 = a_1 r = 243 \times 1/3 = 81$

$a_3 = a_1 r^2 = 243 \times (1/3)^2 = 243 \times \frac{1}{9} = 27$

$a_4 = a_1 r^3 = 243 \times (1/3)^3 = 243 \times \frac{1}{27} = 9$

$a_5 = a_1 r^4 = 243 \times (\frac{1}{3})^4 = 243 \times 1/81 = 3$

$243, 81, 27, 9, 3$

(ii) $a_1 = 579, r = -1/2$

$a_2 = a_1 r = 579 \times -1/2 = -\frac{579}{2}$

$a_3 = a_1 r^2 = 579 \times (-1/2)^2 = 579 \times \frac{1}{4} = \frac{579}{4}$

$a_4 = a_1 r^3 = 579 \times (-1/2)^3 = 579 \times -1/8 = -\frac{579}{8}$

$a_5 = a_1 r^4 = 579 \times (-1/2)^4 = 579 \times \frac{1}{16} = 579/16$

$-579, -\frac{579}{2}, \frac{579}{4}, -\frac{579}{8}, \frac{579}{16}$

5. Find the 12^{th} term of $1+i, 2i, -2+2i, \ldots$

$a_1 = 1+i \quad r = \frac{2i}{1+i} \times \frac{1-i}{1-i}$

$= \frac{2i(1-i)}{(1)^2-(i)^2} = \frac{2i-2i^2}{1-i^2}$

$= \frac{2i-2(-1)}{1-(-1)} = \frac{2i+2}{1+1}$

$= \frac{2i+2}{2} = \frac{\cancel{2}(i+1)}{\cancel{2}}$

$r = i+1$

$a_{12} = a_1 r^{11} = (1+i)(1+i)^{11} = (1+i)^{12}$

$a_{12} = [(1+i)^2]^6 = (1^2+i^2+2i)^6$

$a_{12} = (\cancel{1}-\cancel{1}+2i)^6 = (2i)^6 = 64(i^2)^3$

$a_{12} = 64(-1)^3 = 64 \times -1 = -64$

6. If the 4^{th} and 9^{th} terms of a G.P. are 54 and 13122 respectively. Find the G.P. Also find its general term.

$a_1 r^3 = 54 \rightarrow (i)$

$a_1 r^8 = 13122 \rightarrow (ii)$

$\frac{\cancel{a_1} r^8}{\cancel{a_1} r^3} = \frac{13122}{54}$

$\cancel{r^2 = 243}$ $\quad r^5 = 243$

$\cancel{r = \sqrt{243}}$ $\quad r^5 = 3^5$

$\boxed{r = 3}$

$a_1(3)^3 = 54$

$a_1 = \dfrac{54}{27} = 2$

$2, 6, 18, \ldots$ is G.P.

$a_n = a_1 r^{n-1} = 2(3)^{n-1}$

7. If a, b, c, d are in G.P., prove that:

(i) $a-b,\ b-c,\ c-d$ are in G.P.

Since a, b, c, d are in G.P. so

let

$a = a_1 r^0,\ b = a_1 r^1,\ c = a_1 r^2,\ d = a_1 r^3.$

$a - b = a_1 r^0 - a_1 r^1 = a_1(1-r)$

$b - c = a_1 r^1 - a_1 r^2 = a_1 r^1(1 - r)$

$c - d = a_1 r^2 - a_1 r^3 = a_1 r^2(1-r)$

So

$a-b,\ b-c,\ c-d$

$a_1(1-r),\ a_1r(1-r),\ a_1r^2(1-r)$

Common ratio $= \dfrac{\cancel{a_1} r \cancel{(1-r)}}{\cancel{a_1}\cancel{(1-r)}} = r$

Also

$$C.r = \frac{\cancel{a_1} r^2 \cancel{(1-r)}}{\cancel{a_1} r \cancel{(1-r)}} = r$$

it forms a G.P.

(ii) $a^2-b^2, b^2-c^2, c^2-d^2$ are in G.P.

$a^2-b^2 = a_1^2-(a_1 r)^2 = a_1^2(1-r^2)$

$b^2-c^2 = (a_1 r)^2-(a_1 r^2)^2 = a_1^2 r^2 - a_1^2 r^4$

$b^2-c^2 = a_1^2 r^2(1-r^2)$

$c^2-d^2 = (a_1 r^2)^2-(a_1 r^3)^2 = a_1^2 r^4 - a_1^2 r^6$

$= a_1^2 r^4(1-r^2)$

so

$a^2-b^2, b^2-c^2, c^2-d^2$

$a_1^2(1-r^2), a_1^2 r^2(1-r^2), a_1^2 r^4(1-r^2)$

$$C.R = \frac{\cancel{a_1^2} r^2 \cancel{(1-r^2)}}{\cancel{a_1^2}\cancel{(1-r^2)}} = r^2$$

$$C.R = \frac{\cancel{a_1^2} r^4 \cancel{(1-r^2)}}{\cancel{a_1^2} r^2 \cancel{(1-r^2)}} = r^2$$

Common ratio same so

it forms an G.P.

(iii) $a^2+b^2, b^2+c^2, c^2+d^2$ are in G.P.

~~Let~~ As a, b, c, d are in G.P.

$\frac{a}{b} = \frac{c}{d}$ $\quad \frac{b}{a} = \frac{c}{b}$ $\quad \frac{c}{b} = \frac{d}{c}$

$ad = bc \qquad b^2 = ac \qquad c^2 = bd$

Consider

$(a^2+b^2)(c^2+d^2)$

$a^2c^2+a^2d^2+b^2c^2+b^2d^2$

$(ac)^2+(ad)^2+(bc)^2+(bd)^2$

$(b^2)^2+(bc)^2+(bc)^2+(c^2)^2$

$(b^2)^2+2b^2c^2+(c^2)^2$

$(b^2+c^2)^2$

Hence proved.

8. If $(p+q)^{th}$ term of a G.P. is m and $(p-q)^{th}$ term is n, then find the p^{th} term.

$a_{p+q} = a_1 r^{p+q-1} = m \rightarrow (i)$

$a_{p-q} = a_1 r^{p-q-1} = n \longrightarrow (ii)$

$(a_1 r^{p+q-1})(a_1 r^{p-q-1}) = mn$

$a_1^2 r^{p+q-1+p-q-1} = mn$

$a_1^2 r^{2p-2} = mn$

$a_1^2 r^{(p-1)2} = mn$

$\sqrt{a_1^2 r^{2(p-1)}} = \sqrt{mn}$

$a_1 r^{p-1} = \sqrt{mn}$

which is the p^{th} term.

9. Find three consecutive numbers in G.P. whose sum is 26 and their product is 216.

Let three numbers are

$\frac{a}{r}, a, ar$ in G.P.

$\frac{a}{r} + a + ar = 26 \rightarrow (i)$

$\frac{a}{\cancel{r}} \times a \times a\cancel{r} = 216$

$a^3 = 216$

$(a)^3 = (6)^3$

$\boxed{a = 6}$

Put $a=6$ in (i)

$\frac{6}{r} + 6 + 6r = 26$

$\frac{6 + 6r + 6r^2}{r} = 26$

$6r^2 + 6r + 6 = 26r$

$6r^2 - 20r + 6 = 0$

$2(3r^2 - 10r + 3) = 0$

$3r^2 - 10r + 3 = 0$

$3r^2 - 9r - r + 3 = 0$

$3r(r-3)-1(r-3)=0$

$(r-3)(3r-1)=0$

$\boxed{r=3}$ $\quad$ $3r=1$

$\boxed{r=\frac{1}{3}}$

If $r=3$

$\frac{6}{3}, 6, 6(3)$

$2, 6, 18$

If $r=\frac{1}{3}$

$\frac{6}{\frac{1}{3}}, 6, 6(\frac{1}{3})$

$18, 6, 2$

10. The 3^{rd} term of a G.P. is the square of 1^{st} term. If the 2^{nd} term is 9 then find the 6^{th} term.

let the series

$a_1, a_1r, a_1r^2, a_1r^3, \dots$

A.T.T.Q

$a_1r^2 = a_1^2 \rightarrow (i)$

$a_1r = 9 \rightarrow (ii)$

from (i)

$a_1 = r^2$

Put in eq (ii)

$r^2 \cdot r = 9$

$r^3 = 9$

$$r = \sqrt[3]{9}$$

$$a_1 = r^2$$

$$a_1 = 9^{1/3 \times 2} = 9^{2/3}$$

$$a_6 = a_1 r^5$$

$$= 9^{2/3} \cdot 9^{1/3 \times 5}$$

$$= 9^{2/3} \cdot 9^{5/3}$$

$$= 9^{7/3}$$

$$= 9^{2+1/3} = 9^2 \cdot 9^{1/3}$$

$$= 81 \cdot \sqrt[3]{9}$$

11. If $\frac{1}{a}$, $\frac{1}{b}$ and $\frac{1}{c}$ are in G.P. ~~is the~~ Show that the common ratio is $\pm\sqrt{\frac{a}{c}}$.

for G.P.

$$\left(\frac{1}{b}\right)^2 = \left(\frac{1}{a}\right)\left(\frac{1}{c}\right)$$

$$\frac{1}{b^2} = \frac{1}{ac}$$

$$b^2 = ac$$

$$b = \sqrt{ac}$$

Common ration $= \dfrac{1/b}{1/a}$

$$r = \frac{a}{b}$$

Put value of b

$$r = \frac{a}{\sqrt{ac}}$$

$$r = \frac{\sqrt{a}.\sqrt{a}}{\sqrt{a}.\sqrt{c}}$$

$$r = \pm\sqrt{\frac{a}{c}}$$

Proved

12. If the numbers 1, 4, and 3 are subtracted from three consecutive terms of an A.P., the resulting numbers are in G.P. Find the original numbers if their sum is 21.

Let three numbers

a_1-d, a_1, a_1+d are in A.P.

$$a_1-\cancel{d}+a_1+a_1+\cancel{d}=21$$

$$3a_1 = 21$$

$$\boxed{a_1 = 7}$$

$7-d-1, 7-4, 7+d-3$ are G.P.

$6-d, 3, 4+d$

As numbers are in A.P.

$$(3)^2 = (6-d)(4+d)$$
$$9 = 24 + 6d - 4d - d^2$$
$$d^2 - 4d - 15 = 0$$
$$d^2 - 5d + 3d - 15 = 0$$
$$d(d-5) + 3(d-5) = 0$$
$$(d-5)(d+3) = 0$$
$$d = 5 \qquad d = -3$$

If $d=5$	If $d=-3$
$7-5,\ 7,\ 7+5$	$7+3,\ 7,\ 7-3$
$2,\ 7,\ 12$	$10,\ 7,\ 4$

13. If three consecutive numbers in A.P. are increased by 1, 4, 15 respectively, the resulting numbers are in G.P. ~~are q and p respectively~~. Find the original numbers if their sum is 6.

Sol Let three numbers of A.P.

$$a_1 - d,\ a_1,\ a_1 + d$$

Sum

$$a_1 - \cancel{d} + a_1 + a_1 + \cancel{d} = 6$$
$$3a_1 = 6$$
$$\boxed{a_1 = 2}$$

$a-d+1,\ a+4,\ a+d+15$

$2-d+1,\ 2+4,\ 2+d+15$

$3-d,\ 6,\ 17+d$

for G.P.

$(6)^2 = (3-d)(17+d)$

$36 = 51 + 3d - 17d - d^2$

$d^2 + 14d - \cancel{51} + 36 = 0$

$d^2 + 14d - 15 = 0$

$d^2 + 15d - d - 15 = 0$

$d(d+15) - 1(d+15) = 0$

$(d+15)(d-1) = 0$

$d = -15 \qquad d = 1$

If $d = -15$	If $d = 1$
$3-(-15), 6, 17+(-15)$	$3-1, 6, 17+1$
$3+15, 6, 17-15$	$2, 6, 18$
$18, 6, 2$	

14. If p^{th}, q^{th} term of a G.P. are q and p respectively, Show that $(p+q)^{th}$ term is $(q^p \div p^q)^{\frac{1}{p-q}}$

Let the G.P.

$a, ar, ar^2, ar^3 \ldots$

$a_n = ar^{n-1}$

$$a_p = a_1 r^{p-1}$$

$$a_1 r^{p-1} = q \rightarrow (i)$$

$$a_q = a_1 r^{q-1}$$

$$a_1 r^{q-1} = p \rightarrow (ii)$$

diving eq(ii) by (i)

$$\frac{\cancel{a_1} r^{p-1}}{\cancel{a_1} r^{q-1}} = \frac{q}{p}$$

$$r^{p\cancel{1}-q+\cancel{1}} = q/p$$

$$r^{p-q} = q/p$$

$$r = \left(\frac{q}{p}\right)^{\frac{1}{p-q}} \longrightarrow (iii)$$

from eq(ii)

$$a_1 = \frac{q}{r^{p-1}} \longrightarrow (iv)$$

Now $(p+q)^{th}$ term

$$a_{p+q} = a_1 r^{p+q-1}$$

$$a_{p+q} = \frac{q}{r^{p-1}} \cdot r^{p+q-1}$$

$$= q \cdot r^{\cancel{p}+q\cancel{-1}\cancel{-p}\cancel{+1}}$$

$$= q \cdot r^{q}$$

$$= q \cdot \left(q/p\right)^{\frac{1}{p-q} \times q}$$

$$= q^{1} \cdot q^{\frac{q}{p-q}} \times \frac{1}{(p)^{\frac{q}{p-q}}}$$

$$= q^{1+\frac{q}{p-q}} \cdot \frac{1}{(p)^{\frac{q}{p-q}}}$$

$$= q^{\frac{p-\cancel{q}+\cancel{q}}{p-q}} \cdot \frac{1}{(p)^{\frac{q}{p-q}}}$$

$$= q^{\frac{p}{p-q}} \cdot \frac{1}{(p^{q})^{\frac{1}{p-q}}}$$

$$= (q^{p})^{\frac{1}{p-q}} \cdot \frac{1}{(p^{q})^{\frac{1}{p-q}}}$$

$$= \left(\frac{q^{p}}{p^{q}}\right)^{\frac{1}{p-q}}$$

15. If $a, 2a+2, 3a+3, \ldots$ are in G.P., then find the fifth term.

$$\frac{2a+2}{a} = \frac{3a+3}{2a+2}$$

$$\frac{2(a+1)}{a} = \frac{3(\cancel{a+1})}{2(\cancel{a+1})}$$

$$4(a+1) = 3a$$

$$4a+4-3a = 0$$

$$a+4 = 0$$

$$\boxed{a = -4}$$

$$r = \frac{2a+2}{a}$$

$$r = \frac{2(a+1)}{a} = \frac{2(-4+1)}{-4}$$

$$r = \frac{2(-3)}{-4} = \frac{\cancel{+}6}{\cancel{+}4} = \frac{3}{2}$$

$$a_5 = a_1 r^4$$

$$= (-4)\left(\frac{3}{2}\right)^4$$

$$= -\cancel{4} \times \frac{81}{\cancel{16}\,4}$$

$$= -\frac{81}{4}$$

HASSAN MEHBOOB
S.S.S (Math)