6.4

1. Sum the series:

(i) $3+6+9+\dots+a_{20}$

$a_1 = 3 \quad d = 6-3 = 3 \quad n = 20$

$$S_n = \frac{n}{2}(2a_1+(n-1)d)$$

$$= \frac{20}{2}(2(3)+(20-1)3)$$

$$= 10(6+19(3))$$

$$= 10(6+57)$$

$$= 10(63)$$

$$S_n = 630$$

(ii) $\frac{4}{\sqrt{5}} + \sqrt{5} + \frac{6}{\sqrt{5}} + \dots + a_n$

$a_1 = \frac{4}{\sqrt{5}}$ $\qquad d = \sqrt{5} - \frac{4}{\sqrt{5}}$

$$d = \frac{5-4}{\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$= \frac{n}{2}\left[2\left(\frac{4}{\sqrt{5}}\right) + (n-1)\frac{1}{\sqrt{5}}\right)$$

$$= \frac{n}{2}\left(\frac{8}{\sqrt{5}} + \frac{n-1}{\sqrt{5}}\right)$$

$$= \frac{n}{2}\left(\frac{8+n-1}{\sqrt{5}}\right)$$

$$= \frac{n(n+7)}{2\sqrt{5}}$$

2. Find S_n for each arithmetic series:

(i) $a_1 = 4$, $n = 25$, $a_n = 100$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$= \frac{25}{2}(4 + 100)$$

$$= \frac{25}{\cancel{2}}(\overset{52}{\cancel{104}})$$

$S_n = 25 \times 52 = 1300$

(ii) $a_1 = 40, \ n = 20, \ d = -3$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$= \frac{20}{2}[2(40) + (n-1)(-3)]$$

$= 10[80 + (20-1)(-3)]$

$= 10[80 + 19(-3)]$

$= 10(80 - 57)$

$= 10(23)$

$= 230$

(iii) $a_n = 52$, $n = 21$, $d = -4$

$a_n = a_1 + (n-1)d$

$52 = a_1 + (21-1)(-4)$

$52 = a_1 + (19)(-4)$

$52 = a_1 - 80$

$52 + 80 = a_1$

$a_1 = 132$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$= \frac{21}{2}(2(132) + (21-1)(-4)]$$

$$= \frac{21}{2}(264 + (20)(-4)]$$

$$= \frac{21}{2}(264 - 80)$$

$$= \frac{21}{\cancel{2}}(\overset{92}{\cancel{184}})$$

$$S_n = 21 \times 92 = 1932$$

3. Find a_1 for the series: $d = 8$, $n = 19$, $S_n = 1786$.

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$1786 = \frac{19}{\cancel{2}}[\cancel{2}(a_1) + (19-1)(\overset{4}{\cancel{8}})]$$

$$1786 = 19[a_1 + (18)(4)]$$

$$\frac{1786}{19} = a_1 + 72$$

$$94 - 72 = a_1$$

$$\boxed{a_1 = 22}$$

4. How many terms of the series: $96+93+90+\cdots$ amount to 1071.

$a_1 = 96 \qquad d = 93-96 = -3$

$S_n = 1071 \qquad n = ?$

$$S_n = \frac{n}{2}\left(2a_1+(n-1)d\right)$$

$$1071 = \frac{n}{2}\left[2(96)+(n-1)(-3)\right]$$

$$1071 \times 2 = n\left[192-3n+3\right]$$

$$2142 = n(195-3n)$$

$$2142 = 195n-3n^2$$

$$3n^2-195n+2142=0$$

$$n = \frac{-(-195) \pm \sqrt{(-195)^2-4(3)(2142)}}{2(3)}$$

$$= \frac{195 \pm \sqrt{38025-25704}}{6}$$

$$= \frac{195 \pm \sqrt{12321}}{6}$$

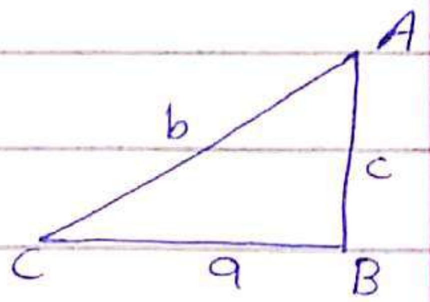
$$= \frac{195 \pm 111}{6}$$

$n = \frac{195+111}{6}$ $\qquad$ $n = \frac{195-111}{6}$

$= \frac{306}{6}$ $\qquad$ $= \frac{84}{6}$

$n = 51$ $\qquad$ $n = 14$

5. If the three sides of a right-angled triangle having perimeter 36 cm are in A.P., find them.



$a+b+c = 36$

If the side of right-angle triangle are in A.P. then they have ratio 3:4:5

Now,

$a = 3x$ $\qquad$ $b = 4x$ $\qquad$ $c = 5x$

$3x + 4x + 5x = 36$

$12x = 36$

$\boxed{x = 3}$

$a = 9$ $\qquad$ $b = 12$ $\qquad$ $c = 15$

9, 12, 15

6. Sum the series:

(i) $3+5-7+9+11-13+15+17-19+\dots$ to $3n$ term.

$-2+7+16+\dots\; n$ term

$a_1=-2 \qquad d=7-(-2)=9 \qquad n=n$

$$S_n=\frac{n}{2}(2a_1+(n-1)d)$$

$$=\frac{n}{2}(2(-2)+(n-1)9)$$

$$=\frac{n}{2}(-4+9n-9)$$

$$=\frac{n}{2}(9n-13)$$

$$S_n=\frac{n(9n-13)}{2}$$

(ii) $1+4-7+10+13-16+19+22-25+\dots$ to $3n$ term.

(i) $3+5-7+9+11-13+15+17-19+\dots$ to $3n$ term.

$1+7+13+\dots\; n$ term

$a_1=1 \qquad d=7-1=6$

$$S_n=\frac{n}{2}(2a_1+(n-1)d)$$

$$=\frac{n}{2}(2(1)+(n-1)6)$$

$$=\frac{n}{2}(2+6n-6)$$

$= \frac{n}{2}(6n-4)$

$= \frac{n}{\cancel{2}}\cancel{2}(3n-2)$

$= n(3n-2)$

~~7. Find a_1 for the arithmetic series: $d=8$, $n=19$, $S_n=1786$.~~

7. Find the sum of 20 terms of the series whose r^{th} term is $3r+1$.

$T_r = 3r+1$

If $r=1$

$T_1 = 3(1)+1 = 3+1 = 4$

$T_2 = 3(2)+1 = 6+1 = 7$

$T_3 = 3(3)+1 = 9+1 = 10$

So the series is

$4+7+10+\ldots$ 20 terms

$a_1 = 4$, $d = 7-4 = 3$, $n = 20$

$S_n = \frac{n}{2}(2a_1+(n-1)d)$

$= \frac{20}{2}(2(4)+(20-1)3)$

$= 10(8+(19)(3))$

$= 10(8+57)$

$S_n = 10(65) = 650$

8. The 5^{th} and 9^{th} term of an A.P. are 11 and 17 respectively. Find the sum of 20 terms.

$$a_1 + 4d = 11 \rightarrow (i)$$

$$a_1 + 8d = 17 \rightarrow (ii)$$

$$-4d = -6$$

$$d = \frac{\cancel{+6}^{3}}{\cancel{+4}_{2}}$$

$$\boxed{d = 3/2}$$

Put $d = 3/2$ in eq (i)

$$a_1 + \cancel{4}^{2}(3/\cancel{2}) = 11$$

$$a_1 + 6 = 11$$

$$a_1 = 11 - 6 = 5$$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$= \frac{20}{2}\left[2(5) + (20-1)\left(\frac{3}{2}\right)\right]$$

$$= 10\left[10 + (19)\left(\frac{3}{2}\right)\right]$$

$$= 10\left[10 + \frac{57}{2}\right]$$

$$= \cancel{10}^{5}\left(\frac{20+57}{\cancel{2}}\right)$$

$$= 5(77)$$

$$S_n = 385$$

9. Obtain the sum of all integers in the first 1000 positive integers which are neither divisible by 5 nor by 2.

Sol: $(1+3+7+9)+(11+13+17+19)+(21+23+27+29)+ \dots (991+993+997+999)$

$20+60+100+\dots 3980$

$a_1=20 \quad d=60-20=40 \quad a_n=3980$

$a_n=a_1+(n-1)d$

$3980=20+(n-1)40$

$3980-20=(n-1)40$

$\frac{3960}{40}=(n-1)$

$99=n-1$

$99+1=n$

$100=n$

$\boxed{n=100}$

$S_{100}=\frac{100}{2}(2(20)+(100-1)40)$

$=50(40+(99)(40))$

$=50(40+3960)$

$S_{100}=50(4000)=200000$

10. The sum of 9 terms of an A.P. is 171 and its eighth term is 31. Find the series.

$S_9 = 171 \qquad a_8 = 31$

$\frac{9}{2}(2a_1 + (9-1)d) = 171$

$9/2(2a_1 + 8d) = 171$

$\frac{9}{\cancel{2}} \times \cancel{2}(a_1 + 4d) = 171$

$a_1 + 4d = 171/9$

$a_1 + 4d = 19 \rightarrow (i)$

$a_1 + 7d = 31 \rightarrow (ii)$

$-3d = -12$

$d = \cancel{-}12/\cancel{-}3$

$\boxed{d = 4}$

Put in (i)

$a_1 + 4(4) = 19$

$a_1 + 16 = 19$

$a_1 = 19 - 16 = 3$

$3 + 7 + 11 + \dots$

11. The 5^{th} term of an A.P. is 21 and the sum of first six

terms is 90. Find the 18^{th} term.

$$a_5 = 21$$

$$a_1 + 4d = 21 \rightarrow (i)$$

$$S_n = 90$$

$$n = 6$$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$90 = \frac{6}{2}(2a_1 + (6-1)d)$$

$$90 = 3(2a_1 + 5d)$$

$$\frac{90}{3} = 2a_1 + 5d$$

$$2a_1 + 5d = 30 \rightarrow (ii)$$

Multiply eq(i) by 2.

$$\cancel{2a_1} + 8d = 42$$

$$\underline{\cancel{2a_1} + 5d = 30}$$

$$3d = 12$$

$$\boxed{d = 4}$$

$$a_1 + 4(4) = 21$$

$$a_1 + 16 = 21$$

$$a_1 = 21 - 16 = 5$$

$$a_{18} = a_1 + 17d$$

$$a_{18} = 5 + 17(4) = 5 + 68 = 73$$

12. The Sum of three numbers in an A.P. is 24 and their product is 440. Find the numbers.

Let three numbers are

$a-d, a, a+d$

Sum of the numbers $= 24$

$a-\cancel{d}+a+a+\cancel{d} = 24$

$3a = 24$

$a = 24/3$

$\boxed{a = 8}$

$8-d, 8, 8+d$

$(8-d)(8)(8+d) = 440$

$64 - d^2 = \frac{440}{8}$

$64 - d^2 = 55$

$64 - 55 = d^2$

$\sqrt{9} = \sqrt{d^{\cancel{2}}}$

$\pm 3 = d$

$\boxed{d = \pm 3}$

If $d = 3$ or If $d = -3$

$8-3, 8, 8+3$ $\quad$ $8+3, 8, 8-3$

$5, 8, 11$ $\quad$ $11, 8, 5$

13. The first four terms of an A.P. are 2,6,10 and 14. Find the least number of terms needed so that the sum of the terms is greater than 2000.

$a_1 = 2 \quad d = 6-2 = 4 \quad n = ?$

$S_n > 2000$

$\frac{n}{2}[2a_1 + (n-1)d] > 2000$

$n[2(2) + (n-1)4] > 4000$

$n(\cancel{4} + 4n - \cancel{4}] > 4000$

$4n^2 > 4000$

$n^2 > \frac{\overset{1000}{\cancel{4000}}}{\cancel{4}}$

$\sqrt{n^{\cancel{2}}} > \sqrt{1000}$

$n > 31.62$

$n = 32$

14. Find fours numbers in A.P. whose sum is 32 and the sum of whose squares is 276.

Let

$a-3d, a-d, a+d, a+3d$ in A.P.

then

$$a - 3\cancel{d} + a - \cancel{d} + a + \cancel{d} + a + 3\cancel{d} = 32$$

$$4a = 32$$

$$\boxed{a = 8}$$

$$(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 276$$

$$(8-3d)^2 + (8-d)^2 + (8+d)^2 + (8+3d)^2 = 276$$

$$64 + 9d^2 - \cancel{48d} + 64 + d^2 - \cancel{16d} + 64 + d^2 + \cancel{16d} + 64 + 9d^2 + \cancel{48d} = 276$$

$$256 + 20d^2 = 276$$

$$20d^2 = 276 - 256$$

$$20d^2 = 20$$

$$\sqrt{d^2} = \sqrt{1}$$

$$d = \pm 1$$

$8 - 3d,\ 8 - d,\ 8 + d,\ 8 + 3d$

Put $d = 1$

$8 - 3(1),\ 8 - 1,\ 8 + 1,\ 8 + 3(1)$

$8 - 3,\ 7,\ 9,\ 8 + 3$

$5, 7, 9, 11$ or $11, 9, 7, 5$

15. Find the five numbers in A.P. whose sum is 25 and the sum of whose squares is 135.

Let the numbers be

$a - 2d,\ a - d,\ a,\ a + d,\ a + 2d$

$a-\cancel{2d}+a-\cancel{d}+a+a+\cancel{d}+a+\cancel{2d}=25$

$5a=25$

$\boxed{a=5}$

$(a-2d)^2+(a-d)^2+a^2+(a+d)^2+(a+2d)^2=135$

$(5-2d)^2+(5-d)^2+(5)^2+(5+d)^2+(5+2d)^2=135$

$25+4d^2-\cancel{20d}+25+d^2-\cancel{10d}+25+25+d^2+\cancel{10d}+25+4d^2+\cancel{20d}=135$

$125+10d^2=135$

$10d^2=10$

$\sqrt{d^2}=\sqrt{1}$

$d=\pm 1$

$a-2d,\ a-d,\ a,\ a+d,\ a+2d$

$5-2(1),\ 5-1,\ 5,\ 5+1,\ 5+2(1)$

$5-2,\ 4,\ 5,\ 6,\ 5+2$

$3, 4, 5, 6, 7$ or $7, 6, 5, 4, 3$

16. If $\frac{1}{a+b}, \frac{1}{c+a}, \frac{1}{b+c}$ are in A.P. then

Show that a^2, b^2, c^2 are in A.P.

$$\frac{1}{c+a}-\frac{1}{a+b}=\frac{1}{b+c}-\frac{1}{c+a}$$

$$\frac{(a+b)-(c+a)}{(a+b)(c+a)}=\frac{(c+a)-(b+c)}{(b+c)(c+a)}$$

$$\frac{\cancel{a}+b-c-\cancel{a}}{(a+b)(c+a)} = \frac{\cancel{c}+a-b-\cancel{c}}{(b+c)(c+a)}$$

$$\frac{(b-c)(b+c)}{(\cancel{a}+b)(\cancel{c}+a)(\cancel{b}+c)} = \frac{(a-b)(a+b)}{(\cancel{a}+b)(\cancel{b}+c)(\cancel{c}+a)}$$

$$b^2-c^2 = a^2-b^2$$

a^2, b^2, c^2 are in A.P.

$$a^2-b^2 = b^2-c^2$$

Hence proved difference between a^2-b^2 is same as b^2-c^2 So a^2, b^2, c^2 are in A.P.

17. The sum of the first four terms of an A.P. is 56. The sum of the last four terms is 112. If its first term is 11, then find number of terms.

$$S_4 = a_1+a_2+a_3+a_4$$

$$S_4' = a_{n-3}+a_{n-2}+a_{n-1}+a_n$$

$a_1 = 11$, $S_4 = 56$, $S_4' = 112$

$$S_4 = \frac{4}{2}\left(2(11)+(4-1)d\right)$$

$$56 = 2(22+3d)$$

$$28 = 22+3d$$

$6 = 3d$

$\boxed{d = 2}$

$a_{n-3} = 11 + (n-3-1)2$

$a_{n-3} = 11 + 2n - 8 = 2n + 3$

$a_{n-2} = 3n + 5$

$a_{n-1} = 2n + 7$

$a_n = 2n + 9$

$a_{n-3} + a_{n-2} + a_{n-1} + a_n = 8n + 24$

$112 = 8n + 24$

$112 - 24 = 8n$

$88 = 8n$

$\boxed{n = 11}$

18. The first, second and last terms of an A.P. are a, b and c respectively. Show that the sum of A.P. is $\dfrac{(b+c-2a)(c+a)}{2(b-a)}$.

Sol $a, b, \dots, c$

$d = b - a$

$a_n = c$

$a_1 + (n-1)d = c$

$a + (n-1)(b-a) = c$

$$n-1 = \frac{c-a}{b-a}$$

$$n = \frac{c-a}{b-a} + 1$$

$$n = \frac{b+c-2a}{b-a}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$= \frac{(b+c-2a)(a+c)}{2(b-a)}$$

19. Show that the sum of n A.Ms. between a and b is n times the single A.M. between them.

$a, A_1, A_2, \ldots, A_n, b$

$A_1 + A_2 + A_3 + \ldots A_n = n\left(\frac{a+b}{2}\right)$

$$a_{n+2} = b$$

$$a_1 + (n+2-1)d = b$$

$$(n+1)d = b-a$$

$$d = \frac{b-a}{n+1}$$

$$A_1 = a_1 + d = a + \left(\frac{b-a}{n+1}\right)$$

$$A_2 = a_1 + 2d = a + 2\left(\frac{b-a}{n+1}\right)$$

$$A_3 = a_1 + 3d = a + 3\left(\frac{b-a}{n+1}\right)$$

$$\vdots$$

$$A_n = a_1 + nd = a + n\left(\frac{b-a}{n+1}\right)$$

$$+$$

$$A_1 + A_2 + \dots A_n = na + (1+2+3+\dots n)\,\frac{b-a}{n+1}$$

$$= na + \frac{n}{2}\left[2(1) + (n-1)\right]\frac{b-a}{n+1}$$

$$= na + \frac{n}{2}(2+n-1)\,\frac{b-a}{n+1}$$

$$= na + \frac{n}{2}\cancel{(n+1)}\,\frac{b-a}{\cancel{n+1}}$$

$$\therefore = n\left(a + \frac{b-a}{2}\right)$$

$$= n\left(\frac{2a+b-a}{2}\right)$$

$$= n\left(\frac{a+b}{2}\right)$$

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