6.3

1. Find A.M. between the given numbers:

(i) $2+\sqrt{3}i,\ 2-\sqrt{3}i$

$$A.M. = \frac{a+b}{2}$$

$$A.M. = \frac{2+\cancel{\sqrt{3}i}+2-\cancel{\sqrt{3}i}}{2} = \frac{\cancel{4}\,2}{\cancel{2}} = 2$$

(ii) $(a+b)^2, (a-b)^2$

$$A.M. = \frac{a+b}{2}$$

$$A.M. = \frac{(a+b)^2+(a-b)^2}{2}$$

$$= \frac{a^2+b^2+\cancel{2ab}+a^2+b^2-\cancel{2ab}}{2}$$

$$= \frac{2a^2+2b^2}{2} = \frac{\cancel{2}(a^2+b^2)}{\cancel{2}}$$

$$A.M. = a^2+b^2$$

2. If 6, 11, 16 are three A.Ms. between a and b, find a and b.

$a, 6, 11, 16, b$

$d = 11-6 = 5$

$b = 16 + 5 = 21$

$a = 6 - 5 = 1$

3. Insert five A.Ms. between $\sqrt{2}$ and $\frac{15}{\sqrt{2}}$.

Let A_1, A_2, A_3, A_4 and A_5 are in A.P. between $\sqrt{2}$ and $\frac{15}{\sqrt{2}}$,

$\sqrt{2}, A_1, A_2, A_3, A_4, A_5, \frac{15}{\sqrt{2}}$ A.P.

$a_1 = \sqrt{2}$

$a_7 = a_1 + 6d$

$a_7 = \sqrt{2} + 6d$

$$\frac{15}{\sqrt{2}} - \sqrt{2} = 6d$$

$$\frac{15 - 2}{\sqrt{2}} = 6d$$

$$\frac{13}{\sqrt{2}} = 6d$$

$$d = \frac{13}{6\sqrt{2}}$$

$A_1 = a_1 + d$

$$A_1 = \sqrt{2} + \frac{13}{6\sqrt{2}}$$

$$A_1 = \frac{6(2)+13}{6\sqrt{2}} = \frac{25}{6\sqrt{2}}$$

$$A_2 = a_1 + 2d$$

$$= \sqrt{2} + \frac{2(13)}{6\sqrt{2}}$$

$$= \frac{6(2)+26}{6\sqrt{2}} = \frac{38}{6\sqrt{2}} = \frac{19}{3\sqrt{2}}$$

$$A_3 = a_1 + 3d$$

$$A_3 = \sqrt{2} + \frac{3(13)}{6\sqrt{2}}$$

$$= \frac{6(2)+39}{6\sqrt{2}} = \frac{51}{6\sqrt{2}} = \frac{17}{2\sqrt{2}}$$

$$A_4 = a_1 + 4d$$

$$= \sqrt{2} + \frac{4(13)}{6\sqrt{2}}$$

$$= \frac{\sqrt{2}+65}{6\sqrt{2}} = \frac{77}{6\sqrt{2}}$$

Five A.Ms are

$$\frac{25}{6\sqrt{2}}, \frac{19}{3\sqrt{2}}, \frac{17}{2\sqrt{2}}, \frac{32}{3\sqrt{2}}, \frac{77}{6\sqrt{2}}$$

4. The A.M. of two numbers is 7 and their product is 45. Find the numbers.

$$\frac{a+b}{2} = 7 \rightarrow (i)$$

$$ab = 45 \rightarrow (ii)$$

from (i)

$$a+b = 14$$

$$a = 14-b$$

Put in (ii)

$$(14-b)b = 45$$

$$14b - b^2 = 45$$

$$0 = b^2 - 14b + 45$$

$$a = 1, \quad b = -14, \quad c = 45$$

$$b = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(45)}}{2(1)}$$

$$b = \frac{14 \pm \sqrt{196-180}}{2}$$

$$= \frac{14 \pm \sqrt{16}}{2}$$

$$= \frac{14 \pm 4}{2}$$

$$b = \frac{18}{2}, \frac{10}{2} = 9, 5$$

$a = 14 - 9 = 5$

$a = 14 - 5 = 9$

So two numbers are 9 and 5.

5. If n arithmetic means are inserted between a and b, prove that $d = \frac{b-a}{n+1}$, where d is the common difference.

n arithmetic means as $A_1, A_2, A_3, \dots A_n$ then with a and b

$a, A_1, A_2, A_3, \dots A_n, b$ in A.P.

$a_{n+2} = b$

$a_1 + (n+2-1)d = b$

$a + (n+1)d = b$

$(n+1)d = b - a$

$$d = \frac{b-a}{n+1}$$

Hence, proved.

6. If A is the A.M. between a and b, prove that $(a-A)^2+(A-b)^2 = \frac{1}{2}(a-b)^2$.

$$A = \frac{a+b}{2}$$

$$= \left(a-\left(\frac{a+b}{2}\right)\right)^2 + \left(\frac{a+b}{2} - b\right)^2$$

$$= \left(\frac{2a-a-b}{2}\right)^2 + \left(\frac{a+b-2b}{2}\right)^2$$

$$= \frac{(a-b)^2}{4} + \frac{(a-b)^2}{4}$$

$$= \frac{\cancel{2}(a-b)^2}{\cancel{4}\,2}$$

$$= \frac{1}{2}(a-b)^2$$

Hence, proved

7. For what value of n, $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M. between a and b, where $a \neq b$.

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{a+b}{2}$$

cross multiplication

$$2(a^{n+1}+b^{n+1}) = (a+b)(a^n+b^n)$$

$$2a^{n+1}+2b^{n+1} = a^{n+1}+ab^n+a^nb+b^{n+1}$$

$$2a^{n+1}+2b^{n+1}-a^{n+1} = ab^n+a^nb+b^{n+1}$$

$$a^{n+1}+2b^{n+1}-b^{n+1} = ab^n+a^nb$$

$$a^{n+1}+b^{n+1} = ab^n+a^nb$$

$$a^{n+1}-a^nb = ab^n-b^{n+1}$$

$$a^n(a^1-b) = b^n(a-b)$$

$$\frac{a^n}{b^n} = \frac{\cancel{a-b}}{\cancel{a-b}}$$

$$\left(\frac{a}{b}\right)^n = 1$$

$$\left(\frac{a}{b}\right)^n = 1^0$$

$$\boxed{n = 0}$$

HASSAN MEHBOOB
S.S.S (Maths)