# 6.2

1. Find the common difference and write the next two terms of each arithmetic sequence.

(i) $9, 16, 23, \ldots$

Common difference is 7.

1st term $= 23 + 7 = 30$

2nd term $= 30 + 7 = 37$

(ii) $5, 5+\sqrt{2}, 5+2\sqrt{2}, \ldots$

Common difference is $\sqrt{2}$.

1st term $= 5 + 3\sqrt{2}$

2nd term $= 5 + 4\sqrt{2}$

2. Write the first three terms of each arithmetic sequence, with given information.

(i) $a_1 = 2, d = 13$

$a_2 = a_1 + (n-1)d$

$a_2 = 2 + (1)(13)$

$a_2 = 2 + 13 = 15$

$a_3 = a_1 + (n-1)d$

$a_3 = 2+(3-1)\,13$

$a_3 = 2+(2)(13)$

$a_3 = 2+26 = 28$

$2, 15, 28$

(ii) $a_1 = 12,\ d = -13$

$a_2 = a_1+(n-1)d$

$a_2 = 12+(2-1)\times(-13)$

$a_2 = 12+(1)(-13)$

$a_2 = 12-13 = -1$

$a_3 = a_1+(n-1)d$

$a_3 = 12+(3-1)\times -13$

$a_3 = 12+(2)(-13)$

$a_3 = 12-26 = -14$

$12, -1, -14$

3. Find $a_{n+1}$ and $a_{2n}$ if $a_n = 4+3n$.

$a_n = 4+3n$

$a_{n+1} = 4+3(n+1)$

$a_{n+1} = 4+3n+1 = 3n+7$

$a_{2n} = 4+3(2n)$

$a_{2n} = 4+6n$

4. Find the indicated term of each of the following arithmetic sequence:

(i) $a_1 = 3,\ d = 7,\ a_{14}$

$a_{14} = a_1 + (n-1)d$

$a_{14} = 3 + (14-1)(7)$

$a_{14} = 3 + (13)(7)$



$a_{14} = 3 + 91$

$a_{14} = 94$

(ii) $8, 3, -2, \ldots, a_{12}$

$a_1 = 8$ , $d = 3 - 8 = -5$

$a_{12} = a_1 + (n-1)d$

$a_{12} = 8 + (12-1)(-5)$

$a_{12} = 8 + (11)(-5)$

$a_{12} = 8 - 55$

$a_{12} = -47$

5. The $18^{th}$ and $30^{th}$ terms of an arithmetic sequence are 367 and 499 respectively. How many terms of this sequence are less than 1000?

$a_1 + 17d = 367 \rightarrow (i)$

$a_1 + 29d = 499 \rightarrow (ii)$

$-12d = -132$

$d = \frac{-132}{-12}$

$\boxed{d = 11}$

Puting value of $d$ in eq(i)

$a_1 + 17(11) = 367$

$a_1 + 187 = 367$

$a_1 = 367 - 187$

$\boxed{a_1 = 180}$

$a_n < 1000$

$a_1 + (n-1)d < 1000$

$180 + (n-1)11 < 1000$

$(n-1)11 < 1000 - 180$

$(n-1)11 < 820$

$n - 1 < \frac{820}{11}$

$n - 1 < 74.55$

$n < 74.55 + 1$

$\boxed{n < 75}$

75 terms of this sequence is less

than 1000.

6. Is 301 a term of the A.P. 5, 11, 17, ... ?

$a_1 = 5 \qquad d = 11 - 5 = 6$

let $n = 301$

$a_1 + (n-1)d = 301$

$5 + (n-1)6 = 301$

$(n-1)6 = 301 - 5$

$(n-1) = 296/6$

$(n-1) \cancel{296+1 = 297} = 49.33$

$n = 49.33 + 1 = 50.33$

No, 301 is not a term of this sequence.

7. If $2x, x+8, 3x+1$ are in A.P., then find the value of $x$.

$a_1 = 2x$

$d = x + 8 - 2x = 8 - x$

$d = 3x + 1 - (x+8) = 3x + 1 - x - 8 = 2x - 7$

$8 - x = 2x - 7$

$8 + 7 = 2x + x$

$15 = 3x$

$\boxed{x = 5}$

8. Which term of the A.P. 3, 8, 13, ... is 123?

$a_1 = 3 \quad d = 8-3 = 5$

$n = 123$

$a_1 + (n-1)d = 123$

$3 + (n-1)5 = 123$

$(n-1)5 = 123 - 3$

$(n-1) = \frac{120}{5}$

$n - 1 = 24$

$n = 24 + 1 = 25$

9. Which term of the A.P., 30, 29.5, 29, ... is the first negative term?

$a_1 = 30 \quad d = 29.5 - 30 = -0.5$

let $a_n = -0.5$

$-0.5 = a_1 + (n-1)(d)$

$-0.5 = 30 + (n-1)(-0.5)$

$-0.5 - 30 = (n-1)(-0.5)$

$$-30.5 = (n-1)(-0.5)$$

$$\frac{-30.5}{-0.5} = n-1$$

$$61 = n-1$$

$$61+1 = n$$

$$n = 62$$

So $62^{th}$ term is the first negative.

10. The $7^{th}$ and $21^{th}$ terms of an A.P. are 37 and 107 respectively. Find the A.P. and its $100^{th}$ term.

$$a_1 + 6d = 37 \rightarrow (i)$$

$$\underline{\cancel{-a_1} \pm 20d = -107} \rightarrow (ii)$$

$$\cancel{-}14d = \cancel{-}70$$

$$d = \frac{70}{14}$$

$$\boxed{d = 5}$$

Puting value of $d$ in eq (i)

$$a_1 + 6(5) = 37$$

$$a_1 + 30 = 37$$

$$a_1 = 37 - 30$$

$$a_1 = 7$$

$a_{100} = a_1 + (n-1)d$

$a_{100} = a_1 + (100-1)d$

$a_{100} = 7 + (99)(5)$

$a_{100} = 7 + 495$

$a_{100} = 502$

A.P. $= 7, 12, 17, \ldots 502$.

11. If $\frac{1}{a-c}, \frac{1}{b-c}, \frac{1}{b-a}$ are in A.P., the

Show that $\frac{a-b}{a-c} = \frac{a-c}{b-a}$.

$a_1 = \frac{1}{a-c}$

$d = \frac{1}{b-c} - \frac{1}{a-c}$

$d = \frac{(a-c)-(b-c)}{(b-c)(a-c)}$

$d = \frac{a-\cancel{c}-b+\cancel{c}}{(b-c)(a-c)}$

$d = \frac{a-b}{(b-c)(a-c)}$

$d = \frac{1}{b-a} - \frac{1}{b-c}$

$$= \frac{(b-c)-(b-a)}{(b-a)(b-c)}$$

$$= \frac{\cancel{b}-c-\cancel{b}+a}{(b-a)(b-c)}$$

$$d = \frac{a-c}{(b-a)(b-c)}$$

$$\frac{a-b}{(\cancel{b-c})(a-c)} = \frac{a-c}{(b-a)(\cancel{b-c})}$$

$$\frac{a-b}{a-c} = \frac{a-c}{b-a}$$

Proved.

12.

12. How many numbers of three digits are divisible by 7?

$105, 112, 119, \ldots 994$

$a_1 = 105 \qquad d = 112-105 = 7$

let $a_n = 994$

$994 = a_1 + (n-1)d$

$994 = 105 + (n-1)7$

$994 - 105 = (n-1)7$

$$\frac{889}{7} = n-1$$

$127 = n-1$

$127+1 = n$

$\boxed{n = 128}$

13. Find the $8^{th}$ term from the end of the A.P. 8, 11, 14, ..., 185.

$a_1 = 8 \quad d = 11-8 = 3$

$a_n = 185$

$185 = a_1 + (n-1)d$

$185 = 8 + (n-1)3$

$185 - 8 = (n-1)3$

$\frac{177}{3} = n-1$

$59 = n-1$

$59+1 = n$

$\boxed{n = 60}$

The $8^{th}$ term from end $(60-7) = 53^{th}$

$a_{53} = a_1 + (n-1)d$

$a_{53} = a_1 + (53-1)d$

$a_{53} = 8 + (52)(3)$

$a_{53} = 8 + 156$

$a_{53} = 164$

14. Find the $n^{th}$ term of the progression $\left(\frac{3}{7}\right)^{10}, \left(\frac{10}{7}\right)^{10}, \left(\frac{17}{7}\right)^{10}, \ldots$. Is the progression an A.P.?

Let

$$\frac{3}{7}, \frac{10}{7}, \frac{17}{7}, \ldots$$

$$a_1 = \frac{3}{7} \qquad d = \frac{10}{7} - \frac{3}{7} = \frac{10-3}{7} = \frac{\cancel{7}}{\cancel{7}} = 1$$

$$a_n = a_1 + (n-1)d$$

$$a_n = \frac{3}{7} + (n-1)(1)$$

$$a_n = \frac{3+7(n-1)}{7}$$

$$a_n = \frac{3+7n-7}{7}$$

$$a_n = \frac{7n-4}{7}$$

$n^{th}$ term of given sequence is

$$a_n = \left(\frac{7n-4}{7}\right)^{10}$$

As we know that common difference of A.P. is same, so

$\therefore d = d$

$a_2 - a_1 = a_3 - a_2$

$\left(\frac{10}{7}\right)^{10} - \left(\frac{3}{7}\right)^{10} = \left(\frac{17}{7}\right)^{10} - \left(\frac{10}{7}\right)^{10}$

$35.401 \neq 7101.4856$

So, given progression not in A.P.

15. If the arithmetic progressions 3, 10, 17, ... and 63, 65, 67, ... are such that their $n^{th}$ terms are equal, then find the value of n.

| 3, 10, 17, ... | 63, 65, 67, ... |
| --- | --- |
| $a_1 = 3 \quad d = 10 - 3 = 7$ | $a_1 = 63 \quad d = 65 - 63 = 2$ |
| $a_n = a_1 + (n-1)d$ | $a_n' = a_1 + (n-1)d$ |
| $a_n = 3 + (n-1)7$ | $a_n' = 63 + (n-1)2$ |
| $a_n = 3 + 7n - 7$ | $a_n' = 63 + 2n - 2$ |
| $a_n = 7n - 4$ | $a_n' = 61 + 2n$ |

$a_n = a_n'$

$7n - 4 = 61 + 2n$

$7n - 2n = 61 + 4$

$5n = 65$

$n = 65/5$

$\boxed{n = 13}$

16. If the $p^{th}$ term of an A.P. is $q$ and the $q^{th}$ term is $p$, prove that its $n^{th}$ term is $(p+q-n)$.

$a_p = q$

$a_q = p$

$\cancel{a_1} + (p-1)d = q \quad \rightarrow (i)$

$\cancel{a_1} + (q-1)d = p \quad \rightarrow (ii)$

$(p-q)d = q-p$

$d = \dfrac{q-p}{p-q}$

$d = \dfrac{-\cancel{(p-q)}}{\cancel{p-q}}$

$\boxed{d = -1}$

Putting value of $d$ in eq (i)

$a_1 + (p-1)(-1) = q$

$a_1 - p + 1 = q$

$a_1 = q + p - 1$

$a_n = a_1 + (n-1)d$

$a_n = q + p - 1 + (n-1)(-1)$

$a_n = q + p \cancel{-1} - n + \cancel{1}$

$a_n = q + p - n$

Proved

17. If $\frac{1}{a}$, $\frac{1}{b}$ and $\frac{1}{c}$ are in A.P., Show that $b = \frac{2ac}{a+c}$

As $\frac{1}{b}$ is middle term, so in A.P.

$$\frac{1}{b} = \left(\frac{1}{a} + \frac{1}{c}\right) \div 2$$

$$\frac{1}{b} = \left(\frac{c+a}{ac}\right) \times \frac{1}{2}$$

$$\frac{1}{b} = \frac{a+c}{2ac}$$

$$b = \frac{2ac}{a+c}$$

Proved.

18. If $\frac{1}{a}$, $\frac{1}{b}$ and $\frac{1}{c}$ are in A.P., Show that the common difference is $\frac{a-c}{2ac}$.

$d = a_2 - a_1$

$d = \frac{1}{b} - \frac{1}{a}$

$\frac{1}{b} = d + \frac{1}{a}$

$d = a_3 - a_2$

$d = \frac{1}{c} - \frac{1}{b}$

$d = \frac{1}{c} - \left(d + \frac{1}{a}\right)$

$$d = \frac{1}{c} - d - \frac{1}{a}$$

$$d + d = \frac{1}{c} - \frac{1}{a}$$

$$2d = \frac{a - c}{ac}$$

$$d = \frac{a - c}{2ac}$$

Proved.

19. If $a_k$ and $a_n$ denotes two different terms of an A.P., Show that its $n^{th}$ term is $a_k + (n-k)\left(\frac{a_k - a_m}{k - m}\right)$.

$$a_k = a_1 + (k-1)d$$
$$a_m = a_1 + (m-1)d$$
$$a_k - a_m = (k-m)d$$
$$\frac{a_k - a_m}{k-m} = d$$

$$a_n = a_1 + (n-1)d$$
$$= a_1 + (n-k+k-1)d$$
$$= a_1 + (k-1)d + (n-k)d$$
$$a_n = a_k + (n-k)\left(\frac{a_k - a_m}{k-m}\right)$$

2a. If $a_1, a_2, a_3, \ldots, a_n$ are positive and in A.P., prove that

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \cdots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

$$a_2 - a_1 = d$$
$$(\sqrt{a_2})^2 - (\sqrt{a_1})^2 = d$$
$$(\sqrt{a_2} - \sqrt{a_1})(\sqrt{a_2} + \sqrt{a_1}) = d$$
$$\sqrt{a_2} - \sqrt{a_1} = \frac{d}{\sqrt{a_2} + \sqrt{a_1}}$$

Similarly

$$\sqrt{a_3}-\sqrt{a_2} = \frac{d}{\sqrt{a_3}+\sqrt{a_2}}$$

Similarl

$\vdots$

$$\sqrt{a_n}-\sqrt{a_{n-1}} = \frac{d}{\sqrt{a_n}+\sqrt{a_{n-1}}}$$

$$\frac{1}{d}\left[\frac{d}{\sqrt{a_1}+\sqrt{a_2}}+\frac{d}{\sqrt{a_2}+\sqrt{a_3}}+\cdots+\frac{d}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right]$$

$$= \frac{1}{d}\left[\cancel{\sqrt{a_2}}-\sqrt{a_1}+\cancel{\sqrt{a_3}}-\cancel{\sqrt{a_2}}+\cdots+\sqrt{a_n}-\cancel{\sqrt{a_{n-1}}}\right]$$

$$= \frac{1}{d}\left[\sqrt{a_n}-\sqrt{a_1}\right]\times\frac{\sqrt{a_n}+\sqrt{a_1}}{\sqrt{a_n}+\sqrt{a_1}}$$

$$= \frac{1}{d}\left(\frac{a_n-a_1}{\sqrt{a_n}+\sqrt{a_1}}\right)$$

$$= \frac{1}{\cancel{d}}\left(\frac{\cancel{a_1}+(n-1)\cancel{d}-\cancel{a_1}}{\sqrt{a_n}+\sqrt{a_1}}\right)$$

$$= \frac{n-1}{\sqrt{a_n}+\sqrt{a_1}}$$

21. If the roots of the equation $(b-c)x^2+(c-a)x+(a-b)=0$ are equal.

For equal roots

$b^2 - 4ac = 0$

$(c-a)^2 - 4(b-c)(a-b) = 0$

$c^2 + a^2 - 2ac = 4(ab - b^2 - ac + bc)$

$c^2 + a^2 - 2ac = 4ab - 4b^2 - 4ac + 4bc$

$c^2 + a^2 + (4b^2) + 2ac - 4ab - 4bc = 0$

$c^2 + a^2 + (-2b)^2 + 2(a)(c) + 2(a)(-2b) + 2(c)(-2b) = 0$

$(c + a - 2b)^2 = 0$

$c + a - 2b = 0$

$c + a - b - b = 0$

$c - b = b - a$

So, a, b, c are in A.P.

22. If the sides of a right-angled triangle are in A.P., Find the ratio of its sides.

Let a-d, a, a+d are the sides of triangle.

$(H)^2 = (B)^2 + (P)^2$

$(a+d)^2 = (a)^2 + (a-d)^2$

$\cancel{a^2} + \cancel{d^2} + 2ad = \cancel{a^2} + a^2 + \cancel{d^2} - 2ad$

$0 = a^2 - 2ad - 2ad$

$0 = a^2 - 4ad$

$0 = a(a - 4d)$

$a = 0$ (not posible)

$a - 4d = 0$

$a = 4d$

$\frac{a}{4} = d$

$a-d, a, a+d$

$a - \frac{a}{4}, a, a + \frac{a}{4}$

$\frac{4a-a}{4}, a, \frac{4a+a}{4}$

$\frac{3a}{4}, a, \frac{5a}{4}$

Multiply by $\frac{4}{a}$

$\frac{\cancel{4}}{\cancel{a}} \times \frac{3\cancel{a}}{\cancel{4}} : \cancel{a} \times \frac{4}{\cancel{a}} : \frac{5\cancel{a}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{a}}$

$3 : 4 : 5$

23. If the $n^{th}$ term of a progression is a linear expression in $n$, then prove that this progression is an A.P.

$T_n = Pn + q$

$T_1 = P + q$

$T_2 = 2p+q$

$T_3 = 3p+q$

$T_4 = 4p+q$

$T_2 - T_1 = 2p+q-(p+q)$

$= 2p+\cancel{q}-p-\cancel{q}$

$= p$

$T_3 - T_2 = 3p+q-(2p+q)$

$= 3p+\cancel{q}-2p-\cancel{q}$

$= p$

Hence progression is in A.P.

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