Ex = 6.10

1- Sum the following series upto n terms.

i) $1\times3+2\times5+3\times7+\ldots\ldots$

$1, 2, 3, \ldots\ldots$

$a_1 = 1 \quad , d = 2-1 = 1$

$a_n = a_1 + (n-1)d$

$a_n = 1 + (n-1)1$

$a_n = \cancel{1} + n - \cancel{1}$

$a_n = n$

2nd factor:

$3, 5, 7, \ldots\ldots$

$a_1 = 3 \quad , d = 5-3 = 2$

$a_n = a_1 + (n-1)d$

$a_n = 3 + (n-1)2$

$a_n = 3 + 2n - 2$

$a_n = 2n+1$

General term T_K is

$T_K = (K)(2K+1)$

$T_K = 2K^2 + K$

sum of n term is

$S = \sum_{K=1}^{n} (2K^2 + K)$

$S = 2\sum_{K=1}^{n} K^2 + \sum_{K=1}^{n} K$

$$= 2\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{2(2n+1)}{3} + 1\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{2(2n+1)+3}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{4n+2+3}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{4n+5}{3}\right]$$

$$= \frac{n(n+1)(4n+5)}{6}$$ Ans

ii) $1\times 5 + 2\times 8 + 3\times 11 + \ldots\ldots$

First fector:

$1, 2, 3, \ldots$

$a_1 = 1$, $d = 2-1 = 1$

$a_n = a_1 + (n-1)d$

$a_n = 1 + (n-1)1$

$a_n = \cancel{1} + n - \cancel{1}$

$a_n = n$

2nd factor

$5, 8, 11, \ldots$

$a_1 = 5$, $d = 8-5 = 3$

$a_n = a_1 + (n-1)d$

$a_n = 5 + (n-1)3$

$a_n = 5 + 3n - 3$

$a_n = 3n + 2$

Genaral term T_k is,

$T_k = (k)(3k+2)$

$T_k = 3k^2 + 2k$

Sum of the first n terms is

$$S = \sum_{k=1}^{n} (3k^2 + 2k)$$

$$= 3\sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k$$

$$= 3\left[\frac{n(n+1)(2n+1)}{6} + \frac{2(n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3(2n+1)}{3} + 2\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3(2n+1)+6}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{6n+3+6}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{6n+9}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{\cancel{3}(\overset{2}{\cancel{3}}n+3)}{\cancel{3}}\right]$$

$$= \frac{n(n+1)}{2}\left[2n+3\right]$$

$$= \frac{n(n+1)(2n+3)}{2} \quad \underline{Ans}$$

iii) $1 \times 2 + 2 \times 5 + 3 \times 8 + \ldots\ldots$

First factor	2nd term
$1, 2, 3, \ldots$	$2, 5, 8, \ldots$
$a_1 = 1 \quad d = 2-1 = 1$	$a_1 = 2, \ d = 5-2 = 3$
$a_n = a_1 + (n-1)d$	$a_n = a_1 + (n-1)d$
$a_n = 1 + (n-1)1$	$a_n = 2 + (n-1)3$
$a_n = \cancel{1} + n - \cancel{1}$	$a_n = 2 + 3n - 3$
$a_n = n$	$a_n = 3n - 1$

General term of T_k is

$$T_k = (k)(3k-1)$$

$$T_k = 3k^2 - k$$

Sum of n terms is

$$S = \sum_{k=1}^{n} (3k^2 - k)$$

$$= 3\sum_{k=}^{n} k^2 - \sum_{k=1}^{n} k$$

$$= 3\left[\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3(2n+1)}{3} - 1\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3(2n+1) - 3}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{6n+3-3}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{\cancel{6}^{2}n}{\cancel{3}_{1}}\right]$$

$$\times = \frac{n(n+1)}{2}\left[\frac{\cancel{6}(n+1)}{\cancel{3}_{1}}\right]$$

$$\times = \frac{n(n+1)}{2}\left[2n+1\right]$$

$$= \frac{n(n+1).\cancel{2}n}{\cancel{2}}$$

$$= n^2(n+1) \quad \text{Ans}$$

iv) $1\times3\times5+2\times4\times6+3\times5\times7+\ldots\ldots$

1st factor

$1, 2, 3, \ldots$

$a_1 = 1$, $d = 2-1 = 1$

$a_n = a_1 + (n-1)d$

$a_n = 1 + (n-1)1$

$a_n = \cancel{1} + n - \cancel{1}$

$a_n = n$

2nd factor

$3, 4, 5, \ldots$

$a_1 = 3$, $d = 4-3 = 1$

$a_n = a_1 + (n-1)d$

$a_n = 3 + (n-1)1$

$a_n = 3 + n - 1$

$a_n = n + 2$

3rd term

$5, 6, 7, \ldots$

$a_1 = 5$, $d = 6-5 = 1$

$a_n = a_1 + (n-1)d$

$a_n = 5 + (n-1)1$

$a_n = 5 + n - 1$

$a_n = n + 4$

General term Tk is

$Tk = (k)(k+2)(k+4)$

$Tk = k(k^2+4k+2k+8)$

$Tk = k[k^2+6k+8]$

$Tk = k^3+6k^2+8k$

Sum of n terms is

$$Sn = \sum_{k=1}^{n}(k^3+6k^2+8k)$$

$$= \sum_{k=1}^{n}k^3+6\sum_{k=1}^{n}k^2+8\sum_{k=1}^{n}k$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \cancel{6}\left[\frac{n(n+1)(2n+1)}{\cancel{6}}\right] + \cancel{8}^{4}\left[\frac{n(n+1)}{\cancel{2}}\right]$$

$$= \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{1} + 4[n(n+1)]$$

$$= n(n+1)\left[\frac{n(n+1)}{4} + \frac{2n+1}{1} + 4\right]$$

$$= n(n+1)\left[\frac{n(n+1)+4(2n+1)+16}{4}\right]$$

$$= n(n+1)\left[\frac{n^2+n+8n+4+16}{4}\right]$$

$$= n(n+1)\left[\frac{n^2+9n+20}{4}\right]$$

$$= \frac{n(n+1)}{4}\left[n^2+9n+20\right]$$

$$= \frac{n(n+1)}{4}\left[n^2+4n+5n+20\right]$$

$$= \frac{n(n+1)}{4}\left[n(n+4)+5(n+4)\right]$$

$$= \frac{n(n+1)(n+4)(n+5)}{4} \quad \text{Ans}$$

v) $1\times2\times4+2\times3\times7+3\times4\times10+\ldots\ldots$

$1, 2, 3, \ldots$

$a_1=1, \quad d=2-1=1$

$a_n=a_1+(n-1)d$

$a_n=1+(n-1)1$

$a_n=\cancel{1}+n-\cancel{1}$

$a_n=n$

2nd term

$2, 3, 4, \ldots$

$a_1=2, \quad d=3-2=1$

$a_n=a_1+(n-1)d$

$a_n=2+(n-1)1$

$a_n=2+n-1$

$a_n=n+1$

3rd term

$4, 7, 10, \ldots$

$a_1=4, \quad d=7-4=3$

$a_n=a_1+(n-1)d$

$a_n=4+(n-1)3$

$a_n=4+3n-3$

$a_n=3n+1$

General term of T_K is

$T_K = (K)(K+1)(3K+1)$

$T_K = K(3K^2+K+3K+1)$

$T_K = (K)(3K^2+4K+1)$

$T_K = 3K^3+4K^2+K$

Sum of n term is

$S_n = \sum_{K=1}^{n} (3K^3+4K^2+K)$

$= 3\sum_{K=1}^{n} K^3 + 4\sum_{K=1}^{n} K^2 + \sum_{K=1}^{n} K$

$= 3\left[\frac{n(n+1)}{2}\right]^2 + 4\left[\frac{n(n+1)(2n+1)}{6}\right] + \left[\frac{n(n+1)}{2}\right]$

$= \frac{n(n+1)}{2}\left[\frac{3.n(n+1)}{2} + \frac{4.(2n+1)}{3} + 1\right]$

$= \frac{n(n+1)}{2}\left[\frac{9n(n+1)+8(2n+1)+6}{6}\right]$

$= \frac{n(n+1)}{2}\left[\frac{9n^2+9^n+16n+8+6}{6}\right]$

$\cancel{= \frac{n(n+1)}{2}\left[\frac{25n^2+\overset{17}{8}n+14\,6}{6}\right]}$

$= \frac{n(n+1)}{2}\left[\frac{9n^2+25n+14}{6}\right]$

$$= \frac{n(n+1)}{2}\left[\frac{9n^2+18n+7n+14}{6}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{9n(n+2)+7(n+2)}{6}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{(n+2)(9n+7)}{6}\right]$$

$$= \frac{n(n+1)(n+2)(9n+7)}{12}$$ Ans

vi) $2^2+4^2+6^2+\ldots\ldots$

$2, 4, 6, \ldots$

$a_1 = 2$, $d = 4-2 = 2$

$a_n = a_1 + (n-1)d$

$a_n = 2 + (n-1)2$

$a_n = \cancel{2} + 2n - \cancel{2}$

$a_n = 2n$

$a_n = (2n)^2$

$a_n = 4n^2$

Genaral term TK is

$T_K = 4K^2$

Sum of n term

$S_n = \sum_{K=1}^{n} (4K)^2$

$$S_n = 4\sum_{k=1}^{n} k^2$$

$$= \cancel{4}_2\left[\frac{n(n+1)(2n+1)}{\cancel{6}\,3}\right]$$

$$S_n = \frac{2n(n+1)(2n+1)}{3} \quad \text{Ans}$$

vii) $3^2 + 6^2 + 9^2 + \ldots\ldots$

$3, 6, 9, —$

$a_1 = 3 \quad , d = 6 - 3 = 3$

$a_n = a_1 + (n-1)d.$

$a_n = 3 + (n-1)3$

$a_n = 3 + 3n - 3$

$a_n = 3n$

$a_n = (3n)^2$

$a_n = 9n^2$

General term T_k is

$T_k = 9k^2$

Sum of n term is

$$S_n = \sum_{k=1}^{n} (9k^2)$$

$$S_n = 9\sum_{k=1}^{n} k^2$$

$$= \cancel{9}_3\left[\frac{n(n+1)(2n+1)}{\cancel{6}\,2}\right]$$

$$S_n = \frac{3n(n+1)(2n+1)}{2}$$ Ans

viii) $4\times1^2+7\times2^2+10\times3^2+\ldots$

$4, 7, 10, \ldots$

$a_1=4 \quad d=7-4=3$

$a_n=a_1+(n-1)d$

$a_n=4+(n-1)3$

$a_n=\cancel{4+4n-4}\ 4+3n-3$

$a_n=\cancel{4n}\ 3n+1$

2nd term

$1, 2, 3, \ldots$

$a_n=a_1+(n-1)d$

$a_n=1+(n-1)1$

$a_n=1+n-1$

$a_n=n$

$a_n=n^2$

Genral term

$T_k=(4k)(k^2)$

$T_k=4k^3$

Sum of nth term

$S_n=4\sum_{k=1}^{n}k^3$

$=4\left[\frac{n(n+1)}{2}\right]^2$

$=\frac{4(n(n+1))^2}{4}$

$=n(n+1)$

$T_k = k(k+1)$

$T_k =$

$T_k=(3k+1)k^2$

$T_k=3k^3+k^2$

sum of n term

$S_n=\sum_{k=1}^{n}(3k^3+k^2)$

$S_n=3\sum_{k=1}^{n}k^3+\sum_{k=1}^{n}k^2$

$=3\left[\frac{n(n+1)}{2}\right]^2+\left[\frac{n(n+1)(2n+1)}{6}\right]$

$=\frac{n(n+1)}{2}\left[\frac{3n(n+1)}{2}+\frac{2n+1}{3}\right]$

$=\frac{n(n+1)}{2}\left[\frac{9n(n+1)+4n+2}{6}\right]$

$$= \frac{n(n+1)(9n^2+9n+4n+2)}{12}$$

$$= \frac{n(n+1)(9n^2+13n+2)}{12}$$ Ans

ix) $3+(3+7)+(3+7+11)+ \cdots\cdots$

$3+7+11 \cdots\cdots$

$a_1 = 3$, $d = 7-3 = 4$

$a_n = a_1 + (n-1)d$

$a_n = 3 + (n-1)4$

$a_n = 3 + 4n - 4$

$a_n = 4n - 1$.

General Term T_k is

$T_k = 4k - 1$

$$S_n = \sum_{k=1}^{n} (4k-1)$$

$$= \cancel{4}2\left[\frac{n(n+1)}{\cancel{2}1}\right] - n$$

$$= 2n(n+1) - n$$

$$= 2n^2 + 2n - n$$

$$= 2n^2 + n$$

$T_k = 2k^2 + k$.

Sum of n term

$$S_n = 2\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{2(2n+1)}{3} + 1\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{2(2n+1)+3}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{4n+2+3}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{4n+5}{3}\right]$$

$$= \frac{n(n+1)(4n+5)}{6}$$ Ans

x) 8 $1^2 + (1^2+2^2) + (1^2+2^2+3^2) + \ldots$

$1+2+3, \ldots$

$a_1 = 1$, $d = 2-1 = 1$

$a_n = a_1 + (n-1)d$

$a_n = 1 + (n-1)1$

$a_n = 1 + n - 1$

$a_n = n$

$a_n = n^2$

$T_k = k^2$

$\cancel{T_k = k}$

$$S_n = \sum_{k=1}^{n} k^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$= \cancel{n^2+n}\ \frac{n(2n^2+n+2n+1)}{6}$$

$$= \frac{n(2n^2+3n+1)}{6}$$

$$= \frac{2n^3+3n^2+n}{6}$$

$$T_k = \frac{1}{6}\left[2k^3+3k^2+k\right]$$

sum of n terms

$$S_n = \frac{1}{6}\left[2\sum_{k=1}^{n} k^3 + 3\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k\right]$$

$$= \frac{1}{6}\left[\frac{2n(n+1)}{2}\right]^2 + \left[\frac{3n(n+1)(2n+1)}{6}\right] + \left[\frac{n(n+1)}{2}\right]$$

$$= \frac{1}{6}\cdot\frac{n(n+1)}{2}\left[\frac{\cancel{2}n(n+1)}{\cancel{2}} + \frac{\cancel{3}(2n+1)}{\cancel{6}} + 1\right]$$

$$= \frac{n(n+1)}{12}\left[n^2+n+2n+1+1\right]$$

$$= \frac{n(n+1)}{12}\left[n^2+n+2n+2\right]$$

$$= \frac{n(n+1)}{12}\left[n(n+1)+2(n+1)\right]$$

$$= \frac{n(n+1)(n+1)(n+2)}{12}$$

$$= \frac{n(n+1)^2(n+2)}{12} \quad \text{Ans}$$

Q:2 Sum the Series.

i) $1^2-2^2+3^2-4^2+\dots\dots+(2n-1)^2-(2n)^2$

$$T_n = (2n-1)^2-(2n)^2$$

$$= \cancel{4n^2}+1-4n-\cancel{4n^2}$$

$$T_n = -4n+1$$

$$T_k = -4k+1$$

Sum of n term

$$S_n = \sum_{k=1}^{n}\left[-4k+1\right]$$

$$= -4\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$\cancel{= -2(n^2+3n)} \times$$

$$= -4\left[\frac{n(n+1)}{2} + n\right]$$

$$= \cancel{4}\left[\frac{-4n(n+1)+2n}{2}\right]$$

$$= \cancel{4}\left[\frac{-4n^2-4n+2n}{2}\right]$$

$$= \cancel{4}\left[\frac{n^2+3n}{2}\right] = -\frac{4n^2-2n}{2}$$

$$= \frac{-\cancel{2}n(2n+1)}{\cancel{2}}$$

$= -n(2n+1)$ Ans

ii) $\frac{1^2}{1} + \frac{1^2+2^2}{2} + \frac{1^2+2^2+3^2}{3} + \cdots$ n term

$1+2+3, \ldots$

$a_n = a_1 + (n-1)d$

$a_n = 1 + (n-1)1$

$a_n = \cancel{1} + n - \cancel{1}$

$a_n = n$

$a_n = n^2$

$T_K = \sum_{K=1}^{n} K^2$

$$= \frac{\frac{\cancel{n}(n+1)(2n+1)}{6}}{\cancel{n}}$$

$$= \frac{(n+1)(2n+1)}{6}$$

$$= \frac{2n^2+n+2n+1}{6}$$

$$= \frac{2n^2+3n+1}{6}$$

$$T_K = \frac{1}{6}\left[2\sum_{K=1}^{n} K^2 + 3\sum_{K=1}^{n} K + \sum_{K=1}^{n} 1\right]$$

$$S_n = \frac{1}{6}\left[\frac{2n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n\right]$$

$$= \frac{1}{6}\cdot n\left[\frac{\cancel{2}(n+1)(2n+1)}{\cancel{6}\,3} + \frac{3(n+1)}{2} + \frac{1}{1}\right]$$

$$= \frac{n}{6}\left[\frac{2(n+1)(2n+1) + 9(n+1) + 6}{6}\right]$$

$$= \frac{n}{6}\left[\frac{2(2n^2+n+2n+1) + 9n+9+6}{6}\right]$$

$$= \frac{n}{6}\left[\frac{2(2n^2+3n+1) + 9n+15}{6}\right]$$

$$= \frac{n}{6}\left[\frac{4n^2+6n+2+9n+15}{6}\right]$$

$$= \frac{n(4n^2+15n+17)}{36} \quad \text{Ans}$$

Q:-3 Find the sum to n terms of the series whose n^{th} terms are given.

i) $5n^2+2n+3$

$T_n = 5n^2+2n+3$

$T_k = 5k^2+2k+3$

$T_k = \sum_{k=1}^{n}(5k^2+2k+3)$

$= 5\sum_{k=1}^{n}k^2 + 2\sum_{k=1}^{n}k + 3$

$$= \left[\frac{5n(n+1)(2n+1)}{6}\right] + \left[\frac{\cancel{2}n(n+1)}{\cancel{2}}\right] + 3n$$

$$= \left[\frac{5n(2n^2+n+2n+1)}{6}\right] + \left[\frac{2n^2+2n}{2}\right] + n$$

$$= \cancel{5n(2n^2+3n}$$

$$= n\left[\frac{5(2n^2+3n+1)}{6} + n+1+3\right]$$

$$= n\left[\frac{5(2n^2+3n+1)}{6} + \frac{n+4}{1}\right]$$

$$= n\left[\frac{5(2n^2+3n+1)+6n+24}{6}\right]$$

$$= n\left[\frac{10n^2+15n+5+6n+24}{6}\right]$$

$$= n\left[\frac{10n^2+21n+29}{6}\right]$$

$$= \frac{n}{6}\left[10n^2+21n+29\right]$$ Ans

ii) n^2+2n-3

$T_n = n^2+2n-3$

$T_k = k^2+2k-3$

$$S_n = \sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k - \sum_{k=1}^{n} 3$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2} - 3n$$

$$= n\left[\frac{(n+1)(2n+1)}{6} + \frac{\cancel{2}(n+1)}{\cancel{2}} - 3\right]$$

$$= n\left[\frac{(n+1)(2n+1)}{6} + n+1-3\right]$$

$$= n\left[\frac{2n^2+n+2n+1}{6} + n-2\right]$$

$$= n\left[\frac{2n^2+3n+1}{6} + \frac{n-2}{1}\right]$$

$$= n\left[\frac{2n^2+3n+1+6n-12}{6}\right]$$

$$= n\left[\frac{2n^2+9n-11}{6}\right]$$ Ans

4. Given n^{th} terms of the series, find the sum to $2n$ terms.

(i) $3n^2+5n+2$

$T_n = 3n^2+5n+2$

$T_k = 3k^2+5k+2$

$S_n = \sum_{k=1}^{n}(3k^2+5k+2)$

$S_n = 3\sum_{k=1}^{n}k^2 + 5\sum_{k=1}^{n}k + \sum_{k=1}^{n}2$

$$= \frac{3n(n+1)(2n+1)}{6} + \frac{5n(n+1)}{2} + 2n$$

$$= n\left[\frac{\cancel{3}(n+1)(2n+1)}{\cancel{6}\,2} + \frac{5(n+1)}{2} + 2\right]$$

$$= n\left[\frac{2n^2+3n+1}{2} + \frac{5n+1}{2} + 2\right]$$

$$= n\left[\frac{2n^2+3n+1+5n+1+4}{2}\right]$$

$$= \frac{n}{\cancel{2}}\,\frac{(2n^2+8n+6)}{2}$$

$$= n\left[\frac{\cancel{2}(n^2+4n+3)}{\cancel{2}}\right]$$

$$= n(n^2+4n+3)$$

replace $n = 2n$

$$S_n = 2n\left[(2n)^2 + (4)(2n) + 3\right]$$

$$S_{2n} = 2n\left[4n^2+8n+3\right]$$

Ans

ii) n^2+n-2

$T_n = n^2+n-2$

$T_k = k^2+k-2$

$$S_n = \sum_{k=1}^{n} (k^2+k-2)$$

$$= \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k - \sum_{k=1}^{n} 2$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} - 2n$$

$$= n\left[\frac{2n^2+3n+1}{6} + \frac{n+1}{2} - \frac{2}{1}\right]$$

$$= n\left[\frac{2n^2+3n+1+3n+3-12}{6}\right]$$

$$= n\left[\frac{2n^2+6n-8}{6}\right]$$

$$= n\left[\frac{\cancel{2}^{1}(n^2+3n-4)}{\cancel{6}\,3}\right]$$

$$= \frac{n}{3}(n^2+3n-4]$$

replace $n = 2n$

$$= \frac{2n}{3}\left[(2n)^2+3(2n)-4\right]$$

$$= \frac{2n}{3}\left[4n^2+6n-4\right]$$

$$S_{2n} = \frac{2n}{3}\left(4n^2+6n-4\right) \quad \text{Ans}$$

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