

Chs 03

Theory of Quadratic Functions

Quadratic Functions-

A quadratic function is a polynomial function of degree two. It is typically expressed in standard form:

$$f(x) = ax^2 + bx + c$$

where a, b and c are real numbers, and $a \neq 0$.

Ex 3.1

Q28 (i) $f(x) = x^2 + 6x + 13$

soln $f(x) = x^2 + 6x + 9 + 4$

$$f(x) = x^2 + 6x + (3)^2 + 4$$

$$f(x) = (x+3)^2 + 4$$

$$f(x) = [x - (-3)]^2 + 4$$

Vertex is at $(-3, 4)$

Here, $a = 1 > 0$

So, function $f(x)$ has minimum value, which is 4 at $x = -3$. Ans.

(ii) $f(x) = x^2 + 4x$

soln $f(x) = x^2 + 4x + (2)^2 - (2)^2$

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$$f(x) = (x+2)^2 - 4$$

$$f(x) = [x - (-2)]^2 - 4$$

$$\text{Vertex} = (-2, -4)$$

Hence, $a = 1 > 0$.

So, function $f(x)$ has minimum value of -4 at $x = -2$. Ans.

$$(iii) f(x) = -x^2 + 8x + 29$$

$$\begin{aligned} \text{sols: } f(x) &= -(x^2 - 8x - 13) \\ &= -(x^2 - 8x - 13 + (4)^2 - (4)^2) \\ &= -[(x-4)^2 - 13 - (16)] \\ &= -[(x-4)^2 - 13 - 16] \\ &= -[(x-4)^2 - 29] \end{aligned}$$

$$f(x) = -(x-4)^2 + 29$$

$$\text{Vertex} = (4, 29)$$

Hence, $a = -1 < 0$

So, function $f(x)$ has maximum value 29 at $x = 4$. Ans.

$$(iv) f(x) = -x^2 - 3x - 5$$

$$\begin{aligned} \text{sols: } f(x) &= -(x^2 + 3x + 5) \\ &= -\left[x^2 + 3x + \left(\frac{3}{2}\right)^2 + 5 - \left(\frac{3}{2}\right)^2\right] \end{aligned}$$

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$$= - \left[\frac{(x+3)^2 + 5 - 9}{4} \right]$$

$$= - \left[\frac{(x+3)^2 + 20 - 9}{4} \right]$$

$$= - \left[\frac{(x+3)^2 + 11}{4} \right]$$

$$= - \left[x - \left(\frac{-3}{2} \right)^2 - \frac{11}{4} \right]$$

vertex is at $= \left(\frac{-3}{2}, \frac{-11}{4} \right)$

$$a = -1 < 0$$

So, $f(x)$ has maximum value of

$$\frac{-11}{4} \text{ at } x = \frac{-3}{2}. \text{ Ans}$$

iv) $f(x) = 3x^2 + 6x - 13$

Sol: $f(x) = 3(x^2 + 2x) - 13$

$$= 3(x^2 + 2x + 1 - 1) - 13$$

$$= 3[(x^2 + 2x + 1) - 1] - 13$$

$$= 3(x+1)^2 - 3 - 13$$

$$= 3(x+1)^2 - 16$$

$$f(x) = 3[x - (-1)]^2 - 16$$

$$a = 3 > 0$$

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So, $f(x)$ has minimum value of -16
at $x = -1$. Ans:

$$(iv) f(x) = -2x^2 - x + 21$$

$$\text{sol}^n f(x) = -2 \left(\frac{x^2 + x}{2} \right) + 21$$

$$= -2 \left[\frac{x^2 + x + \left(\frac{x}{4}\right)^2}{2} + 21 - \left(\frac{x}{4}\right)^2 \right]$$

$$= -2 \left[\frac{(x+1)^2}{4} + 21 \right] - \frac{1}{16}$$

$$= -2 \left(\frac{x+1}{4} \right)^2 + \frac{21}{2} + 21$$

$$= -2 \left(\frac{x+1}{4} \right)^2 + \frac{1}{8} + 21$$

$$= -2 \left(\frac{x+1}{4} \right)^2 + \frac{1 + 168}{8}$$

$$= -2 \left(\frac{x+1}{4} \right)^2 + \frac{169}{8}$$

$$f(x) = -2 \left[x - \left(-\frac{1}{4} \right) \right]^2 + \frac{169}{8}$$

vertex is at $\left(-\frac{1}{4}, \frac{169}{8} \right)$

Hence, $a = -2 < 0$

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So, $f(x)$ has maximum value of 169at $x = -1$. Ans.

4

Q28- (i) $f(x) = x^2 - 4x$ sol:- let, $y = x^2 - 4x$ Now take x and y intercept. y -Interceptput $x = 0$

$$y = (0)^2 - 4(0)$$

$$y = 0 - 0$$

$$\boxed{y = 0} \quad (0, 0)$$

 x -Interceptput $y = 0$.

$$0 = x^2 - 4x$$

$$0 = x(x - 4)$$

$$\boxed{x = 0}, \quad x - 4 = 0$$

$$(4, 0) \quad \boxed{x = 4}$$

Finding vertex,

$$h = \frac{-b}{2a} = \frac{-(-4)}{2(1)}$$

$$h = \frac{4}{2} \quad \boxed{h = 2}$$

$$k = \frac{4(a)(b) - b^2}{4a}$$

$$= \frac{4(1)(-0) - (-4)^2}{4(1)}$$

$$k = \frac{0 - 16}{4}$$

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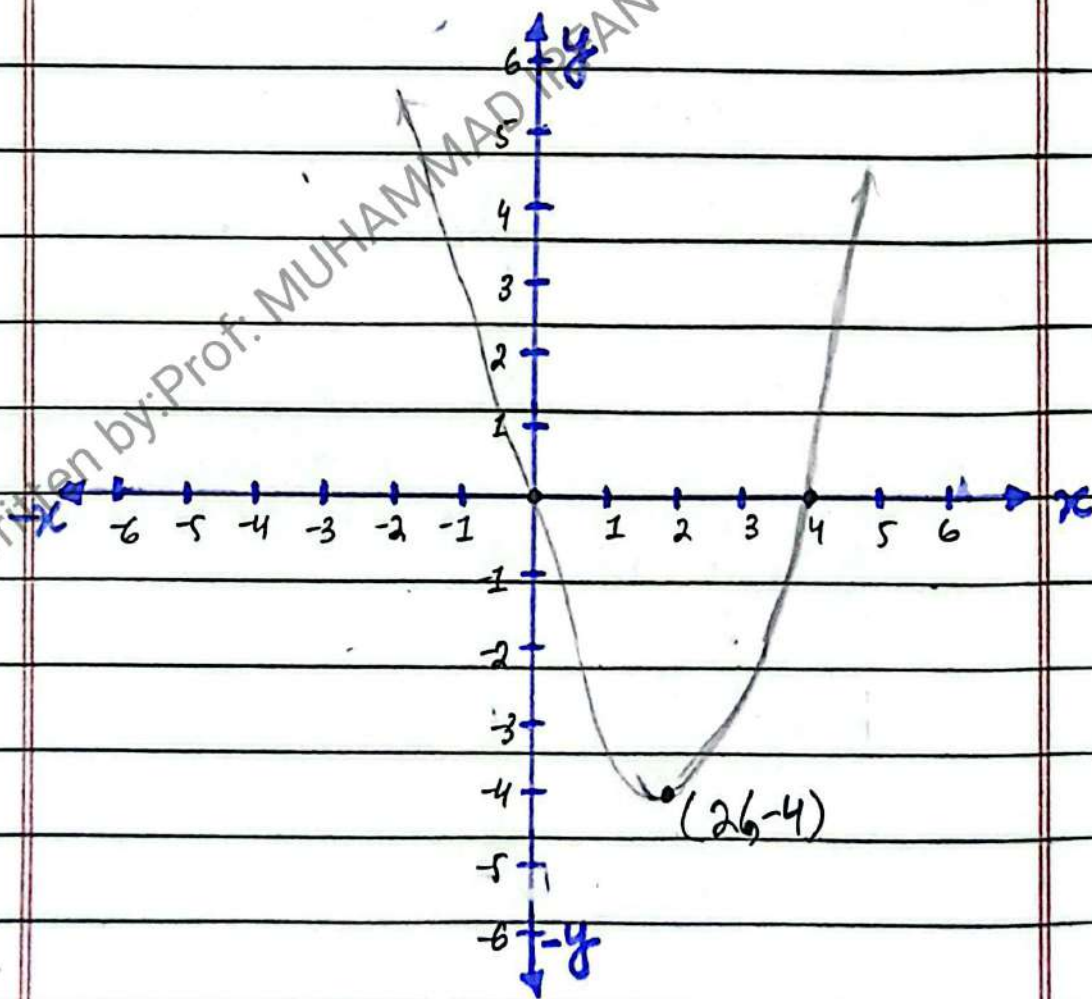
$$k = \frac{-16}{4}$$

$$k = -4$$

$$\text{Vertex } x = (2, -4)$$

Domain of $f(x)$ = Set of all real numbers = $(-\infty, +\infty)$.

Range of $f(x)$ = Minimum value is -4 . Range is all real no. above -4 .
= $[-4, \infty)$



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(ii) $f(x) = x^2 - 5x + 6$

sols: let $y = x^2 - 5x + 6$

Taking x & y intercept.

y -intercept

put $x = 0$

$$y = (0)^2 - 5(0) + 6$$

$$y = 0 - 0 + 6$$

$$\boxed{y = 6}$$

$$(0, 6)$$

x -intercept.

put $y = 0$

$$0 = x^2 - 5x + 6$$

$$0 = x^2 - 3x - 2x + 6$$

$$0 = x(x-3) - 2(x-3)$$

$$0 = (x-3)(x-2)$$

$$x-3=0, x-2=0$$

$$\boxed{x=3}, \boxed{x=2}$$

$$(3, 0), (2, 0)$$

Finding vertex:-

$$h = \frac{-b}{2a} = -\frac{(-5)}{2(1)}$$

$$h = \frac{5}{2} \quad h = 2.5$$

$$k = \frac{4(a)(c) - b^2}{4a}$$

$$= \frac{4(1)(6) - (-5)^2}{4}$$

$$= \frac{24 - 25}{4}$$

$$= \frac{-1}{4}$$

$$= -\frac{1}{4}$$

$$= -\frac{1}{4}$$

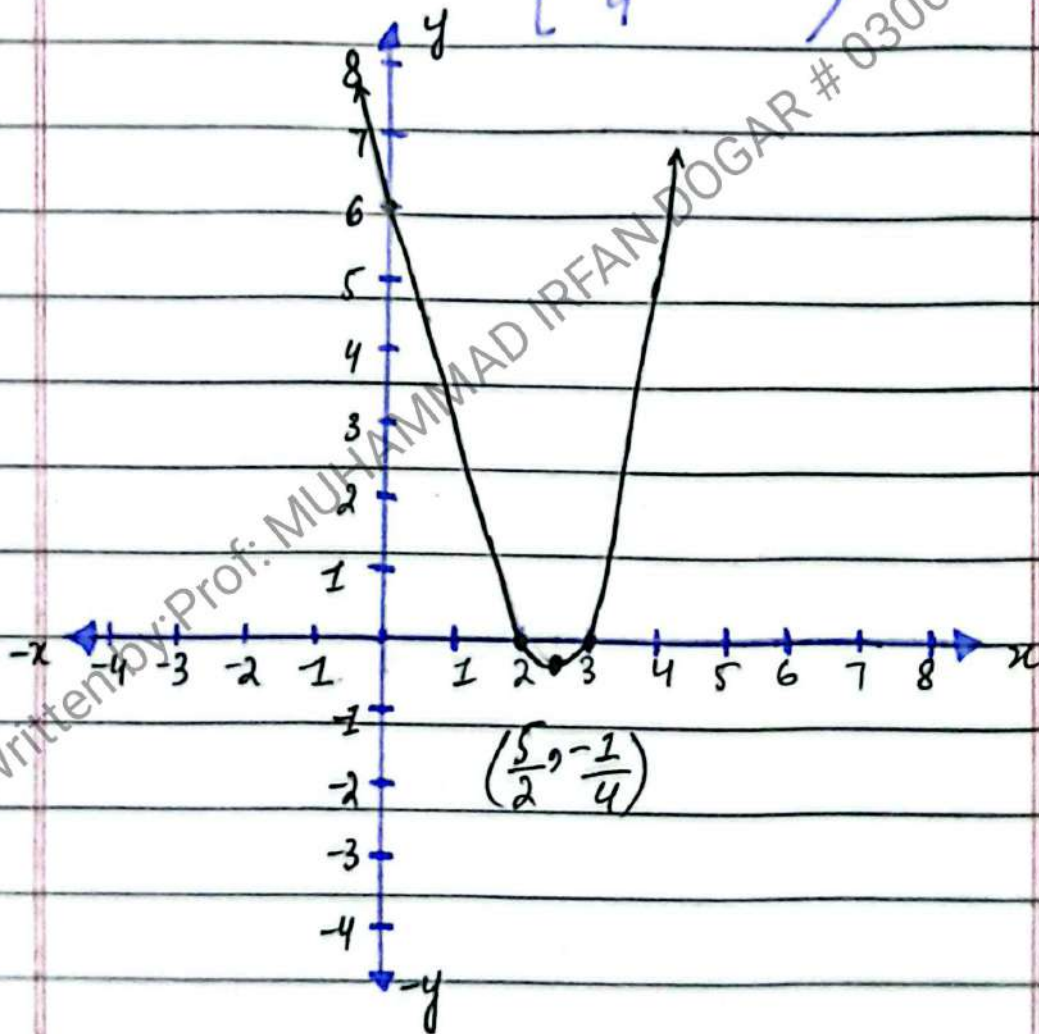
$$k = -\frac{1}{4}$$

$$k = -0.25$$

$$\text{Vertex} = (2.5, -0.25)$$

$$\text{Domain of } f(x) = (-\infty, +\infty)$$

$$\text{Range of } f(x) = \left[-\frac{1}{4}, +\infty \right)$$



$$(iii) f(x) = -x^2 + 2x - 8$$

soln let, $y = -x^2 + 2x - 8$

Taking x & y intercept.

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y-intercept

x-intercept.

put $x=0$

put $y=0$

$$y = -(0)^2 + 2(0) - 8$$
$$= -0 + 0 - 8$$

$$0 = x^2 + 2x - 8$$

Not possible

$$\boxed{y = -8}$$

$(0, -8)$

To find additional points, we
let $x=2$.

$$y = -(2)^2 + 2(2) - 8$$

$$y = -4 + 4 - 8$$

$$\boxed{y = -8} \quad (2, -8)$$

Finding vertex:-

$$h = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$$

$$k = \frac{4ac - b^2}{4a} = \frac{4(-1)(-8) - (2)^2}{4(-1)}$$

$$k = \frac{32 - 4}{-4} = \frac{28}{-4}$$

$$\boxed{k = -7}$$

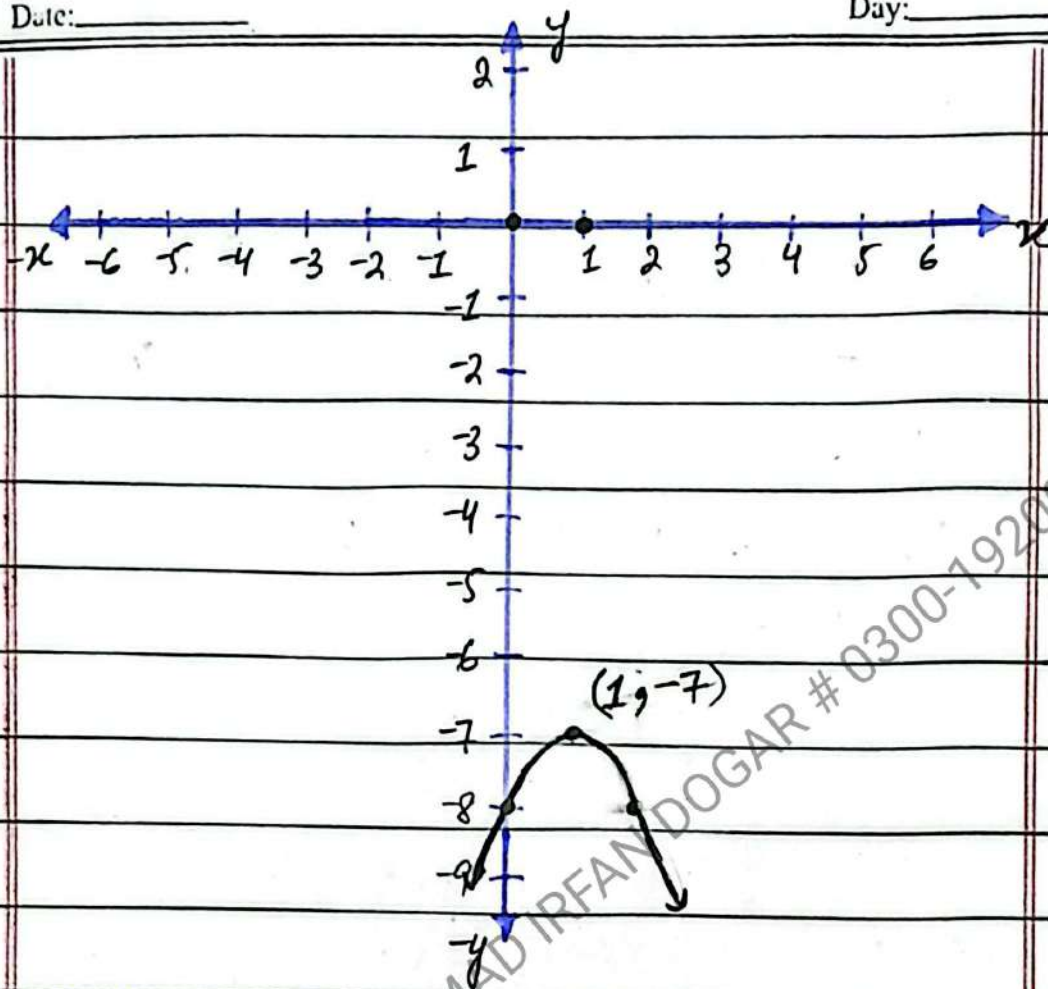
$$\text{vertex} = (1, -7)$$

$$\text{Domain of } f(x) = (-\infty, +\infty)$$

$$\text{Range of } f(x) = (-\infty, -7]$$

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(ii) $f(x) = x^2 - 4x + 4$

sol: let, $y = x^2 - 4x + 4$

y-intercept

put $x=0$

$$y = (0)^2 - 4(0) + 4$$

$$y = 0 - 0 + 4$$

$$\boxed{y = 4}$$

$$(0, 4)$$

x-intercept.

put $y=0$

$$0 = x^2 - 4x + 4$$

$$\sqrt{0} = \sqrt{(x-2)^2}$$

$$x - 2 = 0$$

$$\boxed{x = 2} (2, 0)$$

To find additional points, we let $x=3$.

$$y = (3)^2 - 4(3) + 4$$

$$y = 9 - 12 + 4 \quad \boxed{y = 1} (3, 1)$$

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$$\text{vertex} = h = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2}$$

$$[h=2]$$

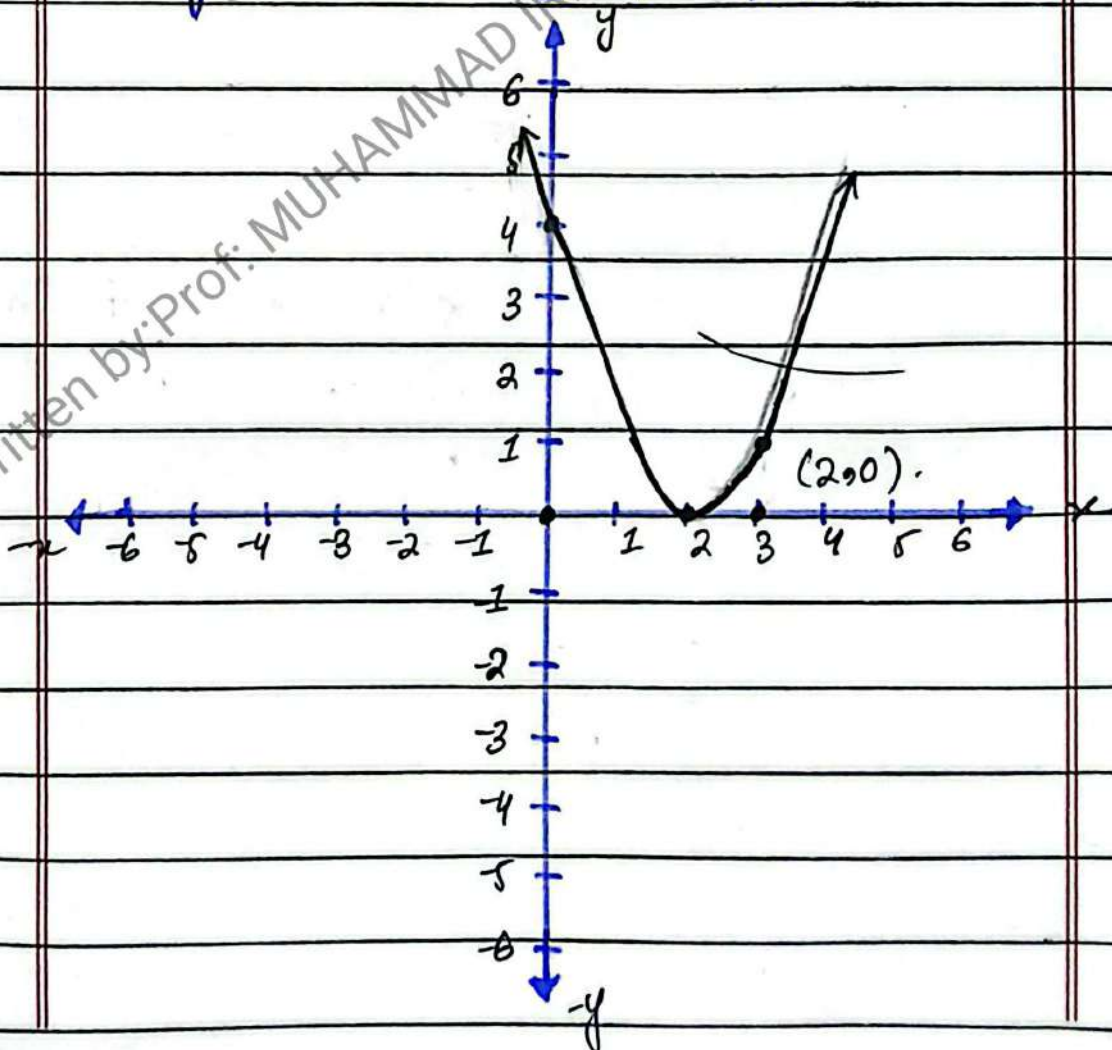
$$k = \frac{4(1)(4) - (-4)^2}{4}$$

$$= \frac{16 - 16}{4} = \frac{0}{4}$$

$$[k=0] \text{ vertex} = (2, 0).$$

Domain of $f(x) = (-\infty, +\infty)$

Range of $f(x) = [0, \infty)$.



$$(v) f(x) = x^2 + 2x - 8.3$$

Solⁿ: Let, $y = x^2 + 2x - 8.3$

y-intercept

x-intercept.

put $x=0$

put $y=0$

$$y = (0)^2 + 2(0) - 8.3$$

$$0 = x^2 + 2x - 8.3$$

$$y = 0 + 0 - 8.3$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-8.3)}}{2(1)}$$

$$\boxed{y = -8.3}$$

$2(1)$

$$(0, -8.3)$$

$$x = \frac{-2 \pm \sqrt{4 + 33.2}}{2}$$

$$x = \frac{-2 \pm \sqrt{37.2}}{2}$$

$$x = \frac{-2 + \sqrt{37.2}}{2}, \quad x = \frac{-2 - \sqrt{37.2}}{2}$$

$$x = +2.05, \quad x = -4.05$$

$$(2.05, 0), \quad (-4.05, 0)$$

vertex $h = -a$

$2(1)$

$$\boxed{h = -1}$$

$$k = \frac{4(1)(-8.3) - (2)^2}{4}$$

$$= \frac{-33.2 - 4}{4} = \boxed{-9.3 = k}$$

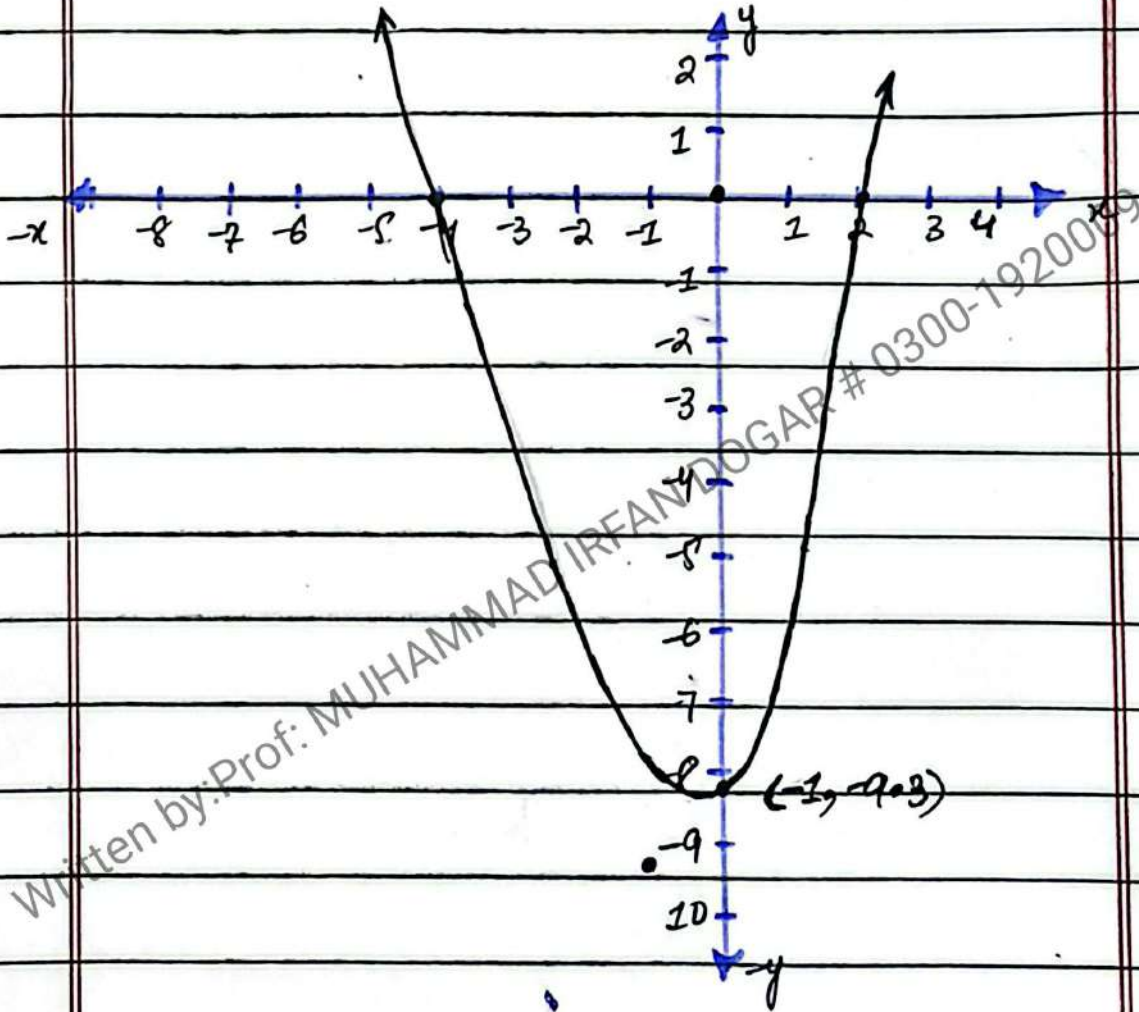
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$$\text{Vertex} = (-1, -9.3)$$

$$\text{Domain of } f(x) = (-\infty, \infty)$$

$$\text{Range of } f(x) = [-9.3, +\infty)$$



$$(28) f(x) = 6 - x - x^2$$

soln let, $y = 6 - x - x^2$

$$y = -x^2 - x - 6$$

y-intercept

put $x=0$.

x-intercept

put $y=0$.

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$$y = -(0)^2 - (0) + 6$$

$$-x^2 - x + 6 = 0$$

$$y = 6$$

$$x^2 + x - 6 = 0$$

$$(0, 6)$$

$$x^2 + 3x - 2x - 6 = 0$$

$$x(x+3) - 2(x+3) = 0$$

$$x+3 = 0, x-2 = 0$$

$$x = -3, x = 2$$

$$(-3, 0), (2, 0)$$

$$\text{Vertex } x = h = -\frac{(-1)}{2(-1)} = 1$$

$$k = 4(-1)(6) - (-1)^2$$

$$4(-6) - 1$$

$$= -24 - 1$$

$$-4$$

$$= -25 \quad [k = -6.25]$$

$$-4$$

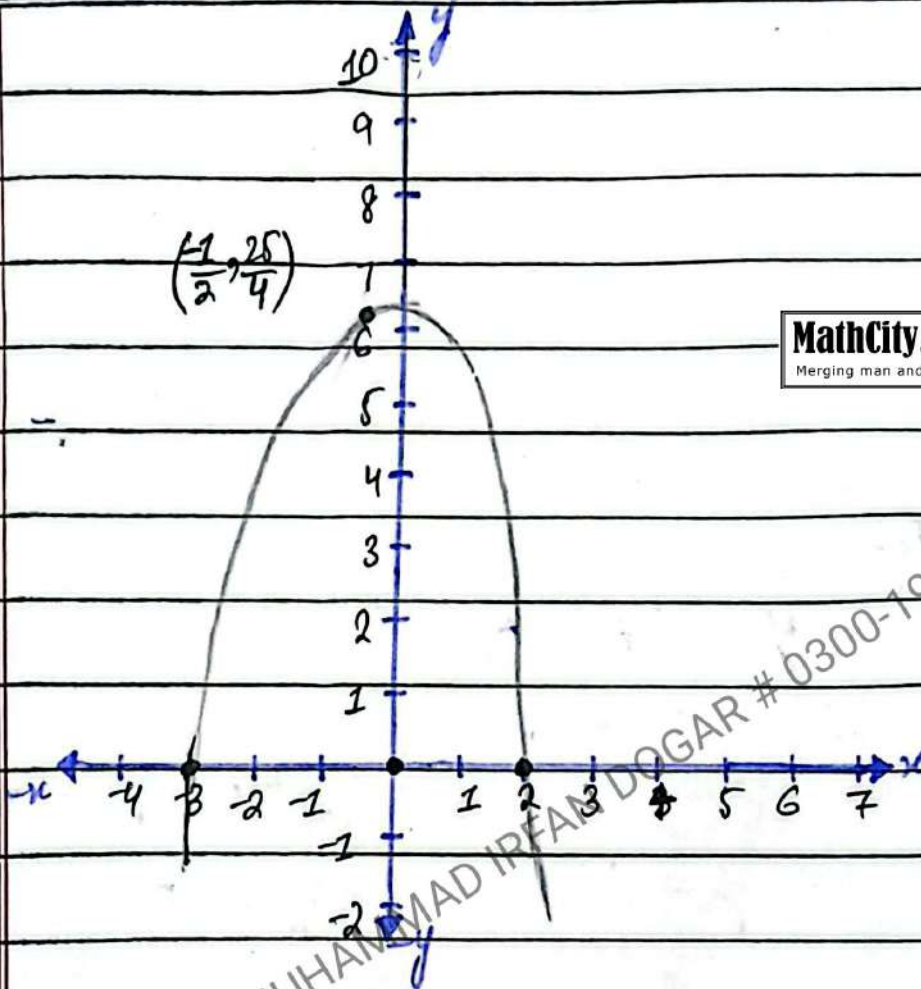
$$\text{Vertex } F(-0.5, -6.25)$$

$$\text{Domain of } f(x) = (-\infty, +\infty)$$

$$\text{Range of } f(x) = \left(-\infty, \frac{25}{4}\right]$$

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Q38 (i) $f(x) = x^2 - 3, x \leq 0$

Soln $f(x) = x^2 - 3$

$$y = x^2 - 3$$

$$y + 3 = x^2$$

$$\sqrt{x^2} = \sqrt{y + 3}$$

$$x = \pm \sqrt{y + 3}$$

Now, replace 'y' by 'x'

inverse $f^{-1}(x) = \pm \sqrt{x + 3}$

$$f^{-1}(x) = -\sqrt{x + 3}$$

Domain $f(x) = (-\infty, 0]$

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$$\text{Range of } f(x) = a = 1 > 0$$

$$f(x) \geq -3 \text{ when } x \leq 0$$

$$\text{Range } f(x) = [-3, +\infty)$$

$$\text{Domain of } f^{-1} = [-3, +\infty)$$

$$\text{Range of } f^{-1} = (-\infty, 0]. \text{ Ans.}$$

$$(ii) f(x) = x^2 + 6x + 4, x < -3$$

$$\text{Soln. } f(x) = y$$

$$y = x^2 + 6x + 4$$

Replace x and y

$$x = y^2 + 6y + 4 \quad y^2 + 6y + 4 - x = 0$$

$$y = \frac{-6 \pm \sqrt{6^2 - 4(1)(4-x)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 - 16 + 4x}}{2}$$

$$= \frac{-6 \pm \sqrt{20 + 4x}}{2}$$

$$= \frac{-6 \pm \sqrt{4(5+x)}}{2}$$

$$= \frac{-6 \pm 2\sqrt{5+x}}{2}$$

$$= \cancel{2} \left(\frac{-3 \pm \sqrt{5+x}}{\cancel{2}} \right)$$

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$$y = -3 \pm \sqrt{5+x}$$

Replace y by $f^{-2}(x)$

$$f^{-2}(x) = -3 \pm \sqrt{5+x}$$

$$f^{-2}(x) = -3 - \sqrt{5+x}$$

$$\text{Dom } f(x) = x \in \mathbb{R}$$

$$\text{Domain} = (-\infty, -3)$$

$$\text{Range of } f(x) = x^2 + 6x + 4$$

$$= x^2 + 6x + 9 - 3^2 + 4$$

$$= (x+3)^2 - 9 + 4$$

$$\text{Range} = (x+3)^2 - 5$$

Min value is -5 so

$$\text{Range is } = (-5, \infty)$$

$$\text{Domain of } f^{-2}(x) = (-5, \infty)$$

$$\text{Range of } f^{-2}(x) = (-3, \infty) \text{ Ans}$$

(iii) $f(x) = 2x^2 - 8x + 11, x \geq 2$

Sol: $y = 2x^2 - 8x + 11$

$$y = 2(x^2 - 4x) + 11$$

$$= 2(x^2 - 4x + 2^2 - 2^2) + 11$$

$$= 2[(x-2)^2 - 4] + 11$$

$$= 2(x-2)^2 - 8 + 11$$

$$= 2(x-2)^2 + 3$$

$$y - 3 = 2(x-2)^2$$

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$$\frac{y-3}{2} = (x-2)^2$$

$$\sqrt{\frac{y-3}{2}} = \sqrt{(x-2)^2}$$

$$x-2 = \pm \sqrt{\frac{y-3}{2}}$$

$$x = 2 \pm \sqrt{\frac{y-3}{2}}$$

$$f^{-1}(y) = 2 \pm \sqrt{\frac{y-3}{2}}$$

Replace 'x' and 'y'?

$$f^2(x) = 2 + \sqrt{\frac{x-3}{2}}$$

Domain of $f(x) = [2, \infty)$

Range of $f(x) = [3, \infty)$

Domain of $f^{-1}(y) = [3, \infty)$

Range of $f^{-1} = [2, \infty)$. Ans.

(iv) $f(x) = 3x^2 - 2x + 6$, $x \geq 5$

Sol. $y = 3x^2 - 2x + 6$

Replace 'x' and 'y'?

$$x = 3y^2 - 2y + 6$$

$$3y^2 - 2y + 6 - x = 0$$

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$$y = \frac{-2 \pm \sqrt{-2^2 - 4(3)(6-x)}}{2(3)}$$

$$= \frac{-2 \pm \sqrt{4 - 12(6-x)}}{6}$$

$$= \frac{-2 \pm \sqrt{4 - 72 + 12x}}{6}$$

$$= \frac{-2 \pm \sqrt{12x - 68}}{6}$$

$$= \frac{-2 \pm \sqrt{4(3x - 17)}}{6}$$

$$= \frac{-2 \left(1 \pm \sqrt{3x - 17} \right)}{6}$$

$$= \frac{-2 \left(1 \pm \sqrt{3x - 17} \right)}{6}$$

$$y = \frac{-1 \pm \sqrt{3x - 17}}{3}$$

$$f^{-1}(x) = \frac{-1 \pm \sqrt{3x - 17}}{3}$$

$$\text{Domain of } f(x) = [5, \infty)$$

$$\text{Range} = \frac{4(3)(6) - (-2)^2}{4(3)}$$

$$= \frac{72 - 4}{12}$$

$$= \frac{68}{12}$$

$$= \frac{17}{3}$$

Range = $\frac{17}{3}$ we cannot take $\frac{17}{3}$ as minimum point.

By condition $x \geq 5$, here $5 > \frac{17}{3}$

So put $x = 5$ in $f(x)$.

$$f(5) = 3(5)^2 - 2(5) + 6$$

$$= 75 - 10 + 6$$

$$f(5) = 71$$

$$\text{Range of } f(x) = [71, \infty)$$

$$\text{Domain } f^{-1} = [5, \infty)$$

$$\text{Range } f^{-1} = [5, \infty) \quad \underline{\text{Ans.}}$$

(ii) $f(x) = 2(x-3)^2 + 1$, $x \geq 3$

Solⁿ: $y = 2(x-3)^2 + 1$

$$y - 1 = 2(x-3)^2$$

$$\frac{y-1}{2} = (x-3)^2$$

$$\sqrt{\frac{y-1}{2}} = \sqrt{(x-3)^2}$$

$$x - 3 = \sqrt{\frac{y-1}{2}}$$

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$$y=3 = \sqrt{\frac{y-1}{2}} \quad \text{since, } x \geq 3$$

$$x-3 > 0,$$

$$y = 3 + \sqrt{\frac{y-1}{2}}$$

$$f^{-1}(y) = 3 + \sqrt{\frac{y-1}{2}}$$

$$f^{-1}(x) = 3 + \sqrt{\frac{y-1}{2}}$$

Domain of $f(x) = [3, \infty)$

Range of $f(x) = a = 2 > 0$

$$f(x) \geq 2, \quad x \geq 3$$

Range = $[1, \infty)$

Domain $f^{-1} = [1, \infty)$

Range $f^{-1} = [3, \infty)$. Ans.

(vi) $f(x) = -3(x+4)^2 - 5, \quad x < -4$

Sol: $y = -3(x+4)^2 - 5$

$$y + 5 = -3(x+4)^2$$

$$\frac{y+5}{-3} = (x+4)^2$$

-3

$$\frac{y+5}{-3} = (x+4)^2$$

3

$$\sqrt{x+4} = \sqrt{\frac{-(y+5)}{3}}$$

$$x+4 = \frac{\pm}{\sqrt{3}} \sqrt{-(y+5)}$$

$$x+4 = -\frac{\sqrt{-(y+5)}}{\sqrt{3}}$$

$$x = -4 - \frac{\sqrt{-(y+5)}}{\sqrt{3}}$$

$$f^{-1}(y) = -4 - \frac{\sqrt{-(y+5)}}{\sqrt{3}}$$

$$f^{-1}(x) = -4 - \frac{\sqrt{-(y+5)}}{\sqrt{3}}$$

$$\text{Domain of } f(x) = (-\infty, 4)$$

$$\text{Range of } f(x) = (-\infty, -5)$$

$$\text{Domain of } f^{-1} = (-\infty, -5)$$

$$\text{Range of } f^{-1} = (-\infty, 4) \text{ Ans.}$$

Q48 (i) $|x^2+1| = 5$

Sols $x^2+1 = \pm 5$

$$x^2+1 = 5$$

$$x^2 = 5-1$$

$$\sqrt{x^2} = \sqrt{4}$$

$$x = \pm 2$$

$$x^2+1 = -5$$

$$x^2 = -5-1$$

$$\sqrt{x^2} = \sqrt{-6}$$

$$x = \pm \sqrt{-6}$$

Imaginary.

$$S \cdot S = \{ \pm 2 \} \text{ Ans.}$$

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$$(iii) |x^2 + 5x + 4| = 0$$

$$\text{sol: } - x^2 + 5x + 4 = 0$$

$$x^2 + 4x + x + 4 = 0$$

$$x(x+4) + 1(x+4) = 0$$

$$(x+4)(x+1) = 0$$

$$x+4=0, x+1=0$$

$$\boxed{x = -4, x = -1}$$

$$\text{Sol} = \{-4, -1\} \text{ Ans.}$$

$$(iv) |x^2 - 6x + 8| = 4$$

$$\text{sol: } - x^2 - 6x + 8 = \pm 4$$

$$x^2 - 6x + 8 = 4$$

$$x^2 - 6x + 8 - 4 = 0$$

$$x^2 - 6x + 4 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2}$$

$$= \frac{6 \pm \sqrt{36 - 16}}{2}$$

$$= \frac{6 \pm \sqrt{20}}{2}$$

$$= \frac{6 \pm 2\sqrt{5}}{2}$$

$$= \frac{2(3 \pm \sqrt{5})}{2} \quad \boxed{x = 3 \pm \sqrt{5}}$$

$$x^2 - 6x + 8 = -4$$

$$x^2 - 6x + 8 + 4 = 0$$

$$x^2 - 6x + 12 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(12)}}{2}$$

$$= \frac{6 \pm \sqrt{36 - 48}}{2}$$

$$= \frac{6 \pm \sqrt{-12}}{2}$$

$$= \frac{6 \pm 2\sqrt{-3}}{2}$$

$$= \frac{2(3 \pm \sqrt{-3})}{2}$$

$$x = 3 \pm \sqrt{-3} \text{ (Imaginary)}$$

$$S.O.S = \{ 3 \pm \sqrt{5} \} \text{ Ans}$$

$$(iv) |3x^2 - 7x + 2| = x^2 - x + 1$$

$$\text{Soln } 3x^2 - 7x + 2 = \pm(x^2 - x + 1)$$

$$3x^2 - 7x + 2 = x^2 - x + 1$$

$$3x^2 - x^2 - 7x + x + 2 - 1 = 0$$

$$2x^2 - 6x + 1 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 4(2)(1)}}{4}$$

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$$= \frac{6 \pm \sqrt{36-8}}{4}$$

$$= \frac{6 \pm \sqrt{28}}{4}$$

$$= \frac{6 \pm 2\sqrt{7}}{4}$$

$$= \frac{2(3 \pm \sqrt{7})}{4 \cdot 2}$$

$$\boxed{x = \frac{3 \pm \sqrt{7}}{2}}$$

$$3x^2 - 7x + 2 = -(x^2 - x + 1)$$

$$3x^2 - 7x + 2 = -x^2 + x - 1$$

$$3x^2 + x^2 - 7x - x + 2 + 1 = 0$$

$$4x^2 - 8x + 3 = 0$$

$$4x^2 - 6x - 2x + 3 = 0$$

$$2x(2x-3) - 1(2x-3) = 0$$

$$(2x-3)(2x-1) = 0$$

$$2x-3=0, \quad 2x-1$$

$$2x=+3$$

$$\boxed{x = \frac{+3}{2}}$$

$$\boxed{x = \frac{1}{2}}$$

$$S.S = \left\{ \frac{3 \pm \sqrt{7}}{2}, \frac{3}{2}, \frac{1}{2} \right\} \text{ Ans.}$$

$$(v) |x^2 - 4| < 5$$

$$\text{Soln } -5 < x^2 - 4 < 5$$

$$-5 < x^2 - 4$$

$$x^2 - 4 < 5$$

$$-5 + 4 < x^2$$

$$x^2 < 5 + 4$$

$$-1 < x^2 \quad \text{--- (i)}$$

$$x^2 < 9 \quad \text{--- (ii)}$$

Inequality (i)

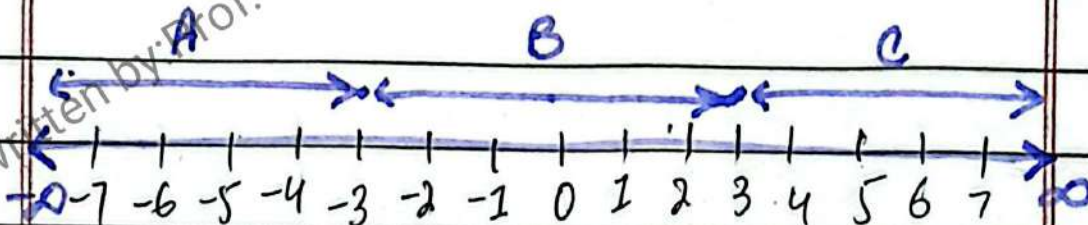
$$x^2 > 1 \quad \text{So, SoS of (i) is}$$

$$R = (-\infty, -1) \cup (1, \infty)$$

Inequality (ii)

$$x^2 < 9$$

$$x^2 = 9 \Rightarrow x = \pm 3$$



Region A:-

$$x = -4$$

$$(-4)^2 < 9$$

$$16 < 9 \quad (\text{False})$$

Region B:-

$$x = 0$$

$$(0)^2 < 9 \quad 0 < 9 \quad (\text{True})$$

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Region C₂

$$x = 4$$

$$(4)^2 < 9$$

$$16 < 9 \quad (\text{False})$$

$$\text{Sol of (ii) is } (-3, 3)$$

$$= (-\infty, \infty) \cap (-3, 3) = (-3, 3) \quad \text{Ans.}$$

(vi) $|x^2 - 3x + 2| > 4$

Sol: $-4 > x^2 - 3x + 2 > 4$

$$-4 > x^2 - 3x + 2$$

$$x^2 - 3x + 2 > 4$$

$$x^2 - 3x + 2 + 4 < 0$$

$$x^2 - 3x + 2 - 4 > 0$$

$$x^2 - 3x + 6 < 0 \quad \text{(i)}$$

$$x^2 - 3x - 2 > 0 \quad \text{(ii)}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(6)}}{2} \quad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2}$$

$$= \frac{3 \pm \sqrt{9 - 24}}{2}$$

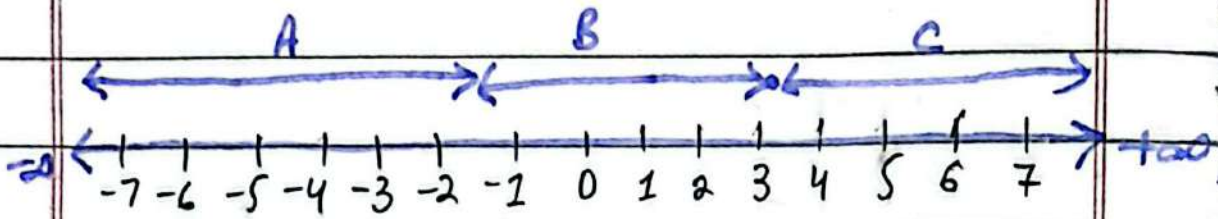
$$x = \frac{3 \pm \sqrt{9 + 8}}{2}$$

$$x = \frac{3 \pm \sqrt{-15}}{2}$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$

(Imaginary). $x = \frac{3 + \sqrt{17}}{2}, x = \frac{3 - \sqrt{17}}{2}$

$$x = 3.56, x = -0.56$$



Region A

$$x = -1 \text{ in (ii)}$$

$$(-1)^2 - 3(-1) - 2 > 0$$

$$+1 - 4 - 2 > 0$$

$$-2 > 0 \quad (\text{True})$$

Region B:-

$$x = 0 \text{ in (ii)}$$

$$(0)^2 - 3(0) - 2 > 0$$

$$0 - 0 - 2 > 0$$

$$-2 > 0 \quad (\text{False})$$

Region C:-

$$x = 4 \text{ in (i)}$$

$$(4)^2 - 3(4) - 2 > 0$$

$$16 - 12 - 2 > 0$$

$$2 > 0 \quad (\text{True})$$

∴ solⁿ is

$$\left(\frac{-2 \pm \sqrt{17}}{2} \right) \cup \left(\frac{3 \pm \sqrt{17}}{2} \right), \infty$$

Ans.

$$(vii) |x^2 - 5x + 6| \leq x + 2$$

$$\text{Sol: } -(x+2) \leq x^2 - 5x + 6 \leq x+2$$

$$-(x+2) \leq x^2 - 5x + 6, \quad x^2 - 5x + 6 \leq x+2$$

$$-x - 2 \leq x^2 - 5x + 6, \quad x^2 - 5x + 6 - x - 2 \leq 0$$

$$0 \leq x^2 - 5x + 6 + x + 2, \quad x^2 - 6x + 4 \leq 0$$

$$x^2 - 11x + 8 \geq 0$$

$$x = \frac{-(-4) \pm \sqrt{4^2 - 4(1)(8)}}{2(1)}, \quad x = \frac{6 \pm \sqrt{36 - 16}}{2}$$

$$2(1)$$

$$x = \frac{6 \pm \sqrt{20}}{2}$$

$$= \frac{4 \pm \sqrt{16 - 32}}{2}$$

$$x = \frac{6 \pm \sqrt{36 - 16}}{2}$$

$$x = \frac{4 \pm \sqrt{-16}}{2}$$

$$x = \frac{6 \pm \sqrt{20}}{2}$$

(Imaginary)

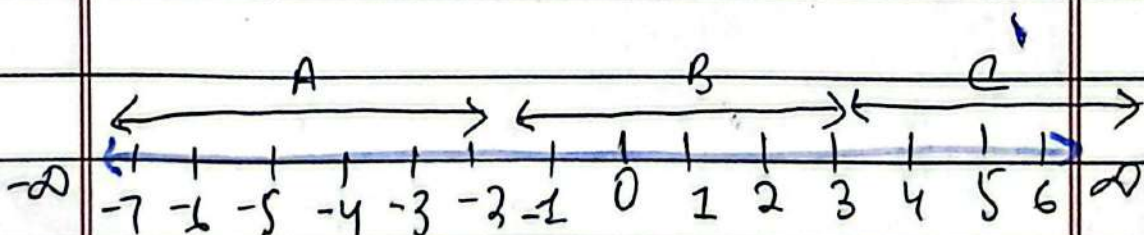
$$x = \frac{6 \pm 2\sqrt{5}}{2}$$

$$x = 3 \pm \sqrt{5}$$

$$x = 3 + \sqrt{5}$$

$$x = 3 + \sqrt{5}, \quad x = 3 - \sqrt{5}$$

$$x = 5.23, \quad x = 0.76$$



Region A:-

$$x=0 \text{ in (ii)}$$

$$(0)^2 - 6(0) + 4 \leq 0$$

$$0 - 0 + 4 \leq 0$$

$$4 \leq 0 \quad (\text{False}).$$

Region B:-

$$x=1 \text{ in (ii)}$$

$$(1)^2 - 6(1) + 4 \leq 0$$

$$1 - 6 + 4 \leq 0$$

$$-1 \leq 0 \quad (\text{True}).$$

Region C:-

$$x=6 \text{ in (ii)}$$

$$(6)^2 - 6(6) + 4 \leq 0$$

$$36 - 36 + 4 \leq 0$$

$$4 \leq 0 \quad (\text{False})$$

S.S of (ii) is $[3 - \sqrt{5}, 3 + \sqrt{5}]$

or

$$S.S = [-\sqrt{5} + 3, \sqrt{5} + 3] \text{ Ans.}$$

$$(vii) | 2x^2 - 3x - 5 | < 4 \quad \text{--- (i)}$$

$$\text{Sol: } -4 < 2x^2 - 3x - 5 < 4$$

$$-4 < 2x^2 - 3x - 5 \quad ; \quad 2x^2 - 3x - 5 < 4$$

$$(i) \text{--- } 0 < 2x^2 - 3x - 1 \quad ; \quad 2x^2 - 3x - 9 < 0 \text{--- (ii)}$$

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$$x = \frac{3 \pm \sqrt{-3^2 - 4(2x-1)}}{4} \quad 2x^2 - 6x + 3x - 9 = 0$$

4

$$2x(x-3) + 3(x-3) = 0$$

$$x = \frac{3 \pm \sqrt{9+8}}{4}$$

$$(2x+3)(x-3) = 0$$

4

$$x-3=0, 2x+3=0$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

$$\boxed{x=3}, 2x=-3$$

4

$$x = -3$$

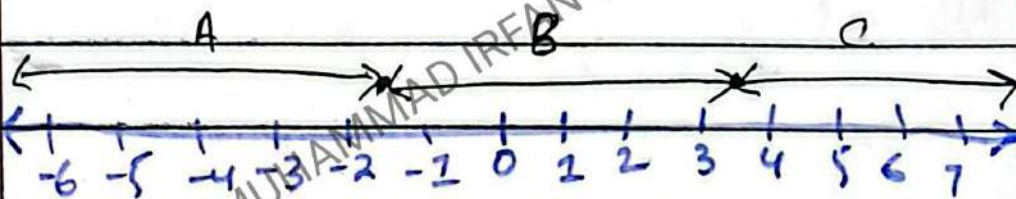
$$x = \frac{3 + \sqrt{17}}{4}, x = \frac{3 - \sqrt{17}}{4}$$

4

4

$$\boxed{x = -1.5}$$

$$\boxed{x = 1.78}, \boxed{x = -0.28}$$



Region A:

$$x = -1 \text{ in (i)}$$

$$2(-1)^2 - 3(-1) - 1 > 0$$

$$2 + 3 - 1 > 0$$

$$4 > 0 \text{ (True)}$$

Region B:

$$x = 0 \text{ in (i)}$$

$$2(0)^2 - 3(0) - 1 > 0$$

$$0 - 0 - 1 > 0$$

$$-1 > 0 \text{ (False)}$$

Region C:-

$$x=2 \text{ in (i)}$$

$$2(2)^2 - 3(2) - 2 > 0$$

$$2(4) - 6 - 2 > 0$$

$$8 - 6 - 2 > 0$$

$$2 > 0 \text{ (True).}$$

$$\text{S.O.S of (i)} = \left(-\infty, \frac{3 - \sqrt{17}}{4} \right) \cup \left(\frac{3 + \sqrt{17}}{4}, \infty \right)$$

Now inequality (ii)

Region A:-

$$x = -2 \text{ in (ii)}$$

$$2(-2)^2 - 3(-2) - 9 < 0$$

$$8 + 6 - 9 < 0$$

$$5 < 0 \text{ (False)}$$

Region B:-

$$x = 0 \text{ in (ii)}$$

$$2(0)^2 - 3(0) - 9 < 0$$

$$0 - 0 - 9 < 0$$

$$-9 < 0 \text{ (True).}$$

Region C:-

$$\text{put } x=4 \text{ in (ii)}$$

$$2(4)^2 - 3(4) - 9 < 0$$

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$$2(16) - 12 - 960$$

$$32 - 12 - 960$$

11 60 (False)

$$\text{NS of (i)} = \left(\frac{-3}{2}, 3 \right)$$

$$\text{Overall SOS} = \left(\frac{-3}{2}, \frac{3 - \sqrt{17}}{4} \right) \cup \left(\frac{3 + \sqrt{17}}{4}, 3 \right) \text{ Ans}$$

Ex 302

$$\text{Q18-(ii)} \frac{1}{3x} + \frac{4x}{6} = 1, x \neq 0$$

$$\text{Sol: } \frac{1}{3x} + \frac{4x}{6} = 1$$

$$\frac{1}{3x} + \frac{2x}{3} = 1$$

$$\frac{1 + 2x^2}{3x} = 1$$

$$1 + 2x^2 = 3x$$

$$2x^2 - 3x + 1 = 0$$

$$a = 2, b = -3, c = 1$$

Using Quadratic Formula