DATE: 01 Exercise 2.1 DAY:

Q1 Given That (a) $f(x)=x^2-1$ b) $f(x)=\sqrt{2x+3}$

(i) $f(-3)$ As $f(x)=x^2-1$

$f(-3)=(-3)^2-1=9-1=8.$

(ii) $f(0)$ As $f(x)=x^2-1$

$f(0)=0^2-1=-1$

(iii) $f(x-2)$ As $f(x)=x^2-1$

$f(x-2)=(x-2)^2-1$

$=x^2+4-4x-1$

$=x^2-4x+3$

(iv) $f(x^2+3)$ As $f(x)=x^2-1$

$f(x^2+3)=(x^2+3)^2-1$

$=x^4+9+6x^2-1$

$=x^4+6x^2+8$

b) $f(x)=\sqrt{2x+3}$

(i) $f(-3)=\sqrt{2(-3)+3}=\sqrt{-6+3}=\sqrt{-3}$

(ii) $f(0)=\sqrt{2(0)+3}=\sqrt{3}$

(iii) $f(x-2)=\sqrt{2(x-2)+3}=\sqrt{2x-1}$

(iv) $f(x^2+3)=\sqrt{2(x^2+3)+3}=\sqrt{2x^2+9}$

Q2 Find $\dfrac{f(a+h)-f(a)}{h}$ and Simplify where

(i) $f(x)=4x+7$

$f(a+h)=4(a+h)+7$

$f(a)=4a+7.$

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Now $\frac{f(a+h) - f(a)}{h}$

$\frac{\cancel{4a} + 4h + \cancel{7} - \cancel{4a} - \cancel{7}}{h}$

$\frac{4\cancel{h}}{\cancel{h}} = 4$

(iii) $f(x) = x^3 + x^2 - 1.$

$f(a+h) = (a+h)^3 + (a+h)^2 - 1$

$= a^3 + h^3 + 3a^2h + 3ah^2 + a^2 + h^2 + 2ah - 1$

$f(a) = a^3 + a^2 - 1$

Now $\frac{f(a+h) - f(a)}{h}$

$\frac{\cancel{a^3} + h^3 + 3a^2h + 3ah^2 + \cancel{a^2} + h^2 + 2ah - \cancel{1} - \cancel{a^3} - \cancel{a^2} + \cancel{1}}{h}$

$\frac{\cancel{h}(\cancel{h^2 + 3a^2h + 3ah + h})}{\cancel{h}}$

$\frac{h^3 + 3a^2h + 3ah^2 + h^2 + 2ah}{h}$

$\frac{\cancel{h}(h^2 + 3a^2 + 3ah + h + 2a)}{\cancel{h}}$

$h^2 + 3a^2 + 3ah + h + 2a.$

(ii) $f(x) = \sin x$

$f(a+h) = \sin(a+h)$

$f(a) = \sin(a)$

$\frac{f(a+h) - f(a)}{h} = \frac{\sin(a+h) - \sin a}{h}$

$= \frac{2\cos\left(\frac{a+h+a}{2}\right)\sin\left(\frac{a+h-a}{2}\right)}{h}$

$= \frac{2\cos(a+h/2)\sin h/2}{h}$

(iv) $f(x) = \tan x$

$f(a+h) = \tan(a+h)$

$f(a) = \tan a.$

$$\frac{f(a+h) - f(a)}{h}$$

$$\frac{1}{h}\left(\tan(a+h) - \tan a\right)$$

$$\frac{1}{h}\left(\frac{\sin(a+h)}{\cos(a+h)} - \frac{\sin a}{\cos a}\right)$$

$$\frac{1}{h}\left(\frac{\sin(a+h)\cos a - \cos(a+h)\sin a}{\cos(a+h)\cos a}\right)$$

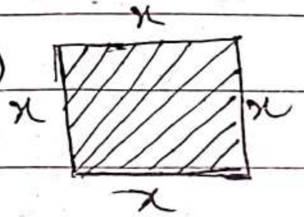
$$\frac{1}{h}\,\frac{\sin(a+h-a)}{\cos(a+h)\cos a}$$

$$\boxed{\frac{\sin h}{h\cos(a+h)\cos a}}$$

Q3) Express the following.

a) The area A of a square as a function of its perimeter P.

Soln) A square with side (x) has $P = 4x$ — ①



$A = x^2$ — ②

From ① $x = P/4$

put in ② $A = (P/4)^2$

$$\boxed{A = \frac{P^2}{16}}$$

b) The circumference C of a circle as a function of its area A.

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Soln Circumference $C = 2\pi r$ — (1)

Area $= A = \pi r^2$ — (2)

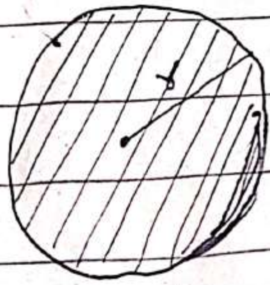
From (2) $r^2 = A/\pi$

$r = \sqrt{A/\pi}$

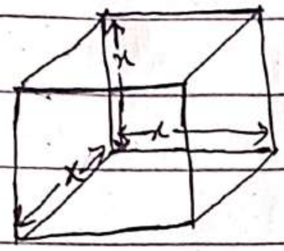
put in (1)

$C = 2\pi\sqrt{\frac{A}{\pi}}$

$C = 2\sqrt{\pi A}$



c) The surface area, S of a cube as a function of its volume V.



A cube have length, width and height are equation in length.

Let $l = x$ $w = x$ $H = x$

So $V = x^3$ — (1)

$S = 6x^2$ — (2)

From (1) $x = V^{1/3}$

put in (2)

$\boxed{S = 6V^{2/3}}$

Domain of a function: The point where function is define and have an out is called domain of function.

we often exclude those point from $\mathbb{R}$, where output become complex OR denominator become zero.

The set of output point of $f(x)$ is called Range or co-domain of $f(x)$.

(QNo 4) Find the domain and the range of the function g defined below.

(i) $g(x) = 5 - x$

Domain $= \mathbb{R}$. Not (As No square root No denominator)

Range $= \mathbb{R}$

(ii) $g(x) = \sqrt{x+2}$

g become undefine as $x + 2 < 0$

$x < -2$

So Domain $g = \mathbb{R} - (-\infty, -2)$

OR

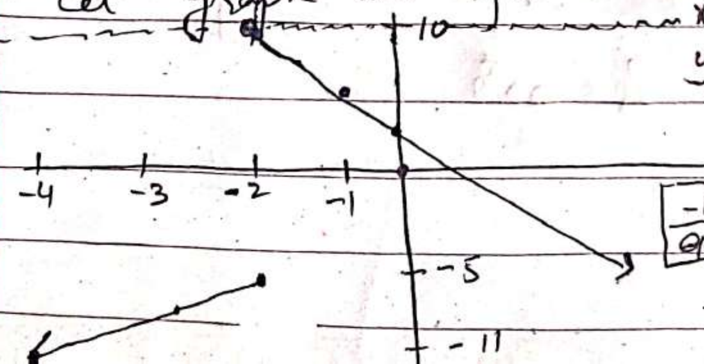
$[-2, \infty)$.

Range$(g) = [0, \infty)$ or $\mathbb{R}^+ \cup \{0\}$

(iii) $g(x) = \begin{cases} 6x + 7 & x \le -2 \\ 4 - 3x & x > -2 \end{cases}$

Let graph the function.

x	-4	-3	-2	
y	-17	-11	-5	



-1.99	-1	0	
9.9	7	4	

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By Graph

Domain = R

Range(g) = $(-\infty, 10)$

(As No value of graph is above

iv). $g(x) = |x-5|$

Domain of g = Dom(g) = $(-\infty, \infty)$

OR $\mathbb{R}$ (Real No.)

Range(g) = $(0, \infty)$.

v) $g(x) = \frac{x+2}{3-x}$

Domain(g) = $R - \{3\}$ OR

$(-\infty, 3) \cup (3, \infty)$

Range(g) = R OR $(-\infty, \infty)$

QNo5 Given $f(x) = x^3 - ax^2 + bx + 1$ if $f(2) = -3$ and $f(-1) = 0$. Find the value of a and b.

Soln $f(x) = x^3 - ax^2 + bx + 1$

$f(2) = (2)^3 - a(2)^2 + 2b + 1$

$f(2) = 8 - 4a + 2b + 1$

$f(2) = 9 - 4a + 2b$

As $f(2) = -3$

$-3 = 9 - 4a + 2b$

$4a - 2b = 12$

$2(2a - b) = 12$

$2a - b = 6$

$f(x) = x^3 - ax^2 + bx + 1.$

$f(-1) = (-1)^3 - a(-1)^2 + b(-1) + 1$

$f(-1) = \cancel{-1} - a - b + \cancel{1}$

$f(-1) = -a - b$

$0 = -a - b$ As $f(-1) = 0$

$a + b = 0$ —— (1)

$+\ 2a - b = 6$ —— (2)

$3a = 6 \Rightarrow \boxed{a = 2}$

$b + 2 = 0$ $\boxed{b = -2}$

06) A stone fall from a height of 60m on the ground, the height h after x seconds is approximately $h(x) = 40 - 10x^2$.

i) What is height of stone when.

a) $x = 1$ sec.

$h(1) = 40 - 10(1)^2 = 30m.$

b) $x = 1.5$ sec.

$h(1.5) = 40 - 10(1.5)^2 = 17.5m.$

c) $x = 1.7$ sec.

$h(1.7) = 40 - 10(1.7)^2 = 11.1m.$

ii) When does the stone strike the ground.

Put $h(x) = 0$

$40 - 10x^2 = 0$

$40 = 10x^2$

$4 = x^2$

$\boxed{x = 2 \text{ sec}}$

After two second it strike the ground

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07) Con~~sider~~ The function $f(x) = 3x - 5$

(i) Determine the domain and range of $f(x)$.

Domain $f = \mathbb{R}$ OR $(-\infty, \infty)$

Range $f = \mathbb{R}$ OR $(-\infty, \infty)$

(ii) Is the function f one-to-one. Justify your answer.

Let $f(x_1) = f(x_2)$

$3x_1 - 5 = 3x_2 - 5$

$3x_1 = 3x_2$

$x_1 = x_2$

So distinct element as distinct image hence f is one-to-one

(iii) Is the function onto if the codomain is all real number.

Soln. Let $y \in \mathbb{R}$

Such $y = 3x - 5$

$\frac{y+5}{3} = x$

for every $y \in \mathbb{R}$ we can find x such that $f(x) = y$

So f is onto.

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8) If $f: R \to R$ be defined by $f(x) = \frac{2x-3}{x+1}$

i) Find the domain and range of $f(x)$.

$Dom(f) = R - \{-1\}$ OR $(-\infty,-1) \cup (-1,\infty)$

As f is not define on -1.

$Rang(f) = R - \{2\}$ OR $\cancel{(-\infty,\infty)}$ $(-\infty,2) \cup (2,\infty)$

(ii) Determine whether $f(x)$ is onto.

Let $y \in R$ Let $y = f(x)$

$$y = \frac{2x-3}{x-1}$$

$$xy - y = 2x - 3$$

$$xy - 2x = y - 3$$

$$x(y-2) = y - 3$$

$$x = \frac{y-3}{y-2}$$

So For $y \in R$ we can find $x \in R$ such $f(x) = y$ except for 2 so f is not onto

(iii) Prove that $f(x)$ is one-to-one.

Let $f(x_1) = f(x_2)$

$$\frac{2x_1-3}{x_1-1} = \frac{2x_2-3}{x_2-1}$$

$$(2x_1-3)(x_2-1) = (2x_2-3)(x_1-1)$$

$$2x_1x_2 - 2x_1 - 3x_2 + 3 = 2x_1x_2 - 2x_2 - 3x_1 + 3$$

$$-2x_1 - 3x_2 = -2x_2 - 3x_1$$

$$x_1 = x_2$$

So distinct element has distinct images hence f is one-to-one.

Q9) Consider the function $f: R^+ \to R^+$ define by $f(x) = e^{-x}$ show that $f(x)$ is bijective.

Soln
$f: R^+ \longrightarrow R^+$

$f(x) = e^{-x}$

obviously f is a function.
(As define for all positive real No)
(Every $x \in R$ has a unique image)

1) f is one-to-one.

Let $f(x_1) = f(x_2)$

$e^{-x_1} = e^{-x_2}$

$\ln e^{-x_1} = \ln e^{-x_2}$

$-x_1 \ln e = -x_2 \ln e$

$-x_1 = -x_2$

$x_1 = x_2$

Distinct element has distinct image.
so f is one-to-one.

ii. f is onto.

Let $y \in R^+$ such

$y = f(x)$

$y = e^{-x}$

$\ln y = \ln e^{-x}$

$\ln y = -x$

$-\ln y = x$

$\ln y^{-1} = x$

$\ln(1/y) = x$

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For every $y \in R^+$ we can find $x = \ln(1/y)$ for which $f(x) = y$
so f is onto.
Since f is one-to-one and onto
hence f is bijective function.

10) Let $g : R \to R$ be given by $g(x) = x^3 - 3x$
Determine if $g(x)$ is inject and or Surjective.
Let $y = d$ $d \in R$.
Can we find $x \in R$ such ~~$f(d) = x$~~

~~$d^3 - 3d = x$ ← x is Rea~~

$$f(x) = d$$

$$x^3 - 3x - d = 0$$

which is cubic equation in x
and has discriminant.

$\Delta = -4(1)(-3)^2 - 27d^2$

$\Delta = 4 \times 27 - 27d^2$

$\Delta = 27(4 - d^2)$

For $\Delta < 0$ cubic equation has complex root.
For every number $|d| > 2$.
cubic equation complex root.
~~So g is surjective~~
But complex root exist in conjugate form.

So we have guarantee one real value of x.
So g is onto (hence surjective).