Exercise # 14.2

1. Find the cosines of the angle θ between $\underline{u}$ and $\underline{v}$:

(i) $\underline{u} = 2\underline{i} + 3\underline{j} + \underline{k}$, $\underline{v} = -\underline{i} + 2\underline{j} + 2\underline{k}$

$\underline{u}.\underline{v} = 2\times -1 + 3\times 2 + 1\times 2$

$= -2 + 6 + 2$

$= 6$

$|\underline{u}| = \sqrt{(2)^2 + (3)^2 + (1)^2}$

$= \sqrt{4+9+1}$

$= \sqrt{14}$

$|\underline{v}| = \sqrt{(-1)^2 + (2)^2 + (2)^2}$

$= \sqrt{1+4+4}$

$= \sqrt{9} = 3$

$$\cos\theta = \frac{\underline{u}.\underline{v}}{|\underline{u}||\underline{v}|}$$

$$= \frac{6}{3\sqrt{14}}$$

$$\cos\theta = \frac{2}{\sqrt{14}}$$

(ii) $\underline{u} = [-3, 2, 5]$, $\underline{v} = [1, 6, -2]$

$\underline{u}.\underline{v} = -3\times 1 + 2\times 6 + 5\times -2$

$= -3 + 12 - 10$

$= -1$

$$|\underline{u}| = \sqrt{(-3)^2 + (2)^2 + (5)^2}$$
$$= \sqrt{9 + 4 + 25}$$
$$= \sqrt{38}$$
$$|\underline{v}| = \sqrt{(1)^2 + (6)^2 + (-2)^2}$$
$$= \sqrt{1 + 36 + 4}$$
$$= \sqrt{41}$$

$$\cos\theta = \frac{\underline{u} \cdot \underline{v}}{|\underline{u}||\underline{v}|}$$

$$= \frac{-1}{\sqrt{38} \times \sqrt{41}}$$
$$= \frac{-1}{\sqrt{1558}}$$

2. If $\underline{a} + \underline{b} + \underline{c} = \underline{0}$ and $|\underline{a}| = 3$, $|\underline{b}| = 5$ and $|\underline{c}| = 7$. Find the angle between a and b.

$$\underline{a} + \underline{b} + \underline{c} = 0$$
$$\underline{a} + \underline{b} = -\underline{c}$$
$$(\underline{a}+\underline{b}) \cdot (\underline{a}+\underline{b}) = -c \cdot (\underline{a} + \underline{b})$$
$$\underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b} + \underline{b} \cdot \underline{a} + \underline{b} \cdot \underline{b} = -c \cdot (-c)$$
$$|\underline{a}||\underline{a}|\cos 0 + 2\,\underline{a} \cdot \underline{b} + |\underline{b}||\underline{b}|\cos 0 = |\underline{c}||\underline{c}|\cos 0$$
$$|\underline{a}|^2 + 2\,\underline{a} \cdot \underline{b} + |\underline{b}|^2 = |\underline{c}|^2$$
$$|3|^2 + 2\,\underline{a} \cdot \underline{b} + |5|^2 = |7|^2$$

$$9 + 2\underline{a}.\underline{b} + 25 = 49$$

$$2\underline{a}.\underline{b} = 49 - 34$$

$$\underline{a}.\underline{b} = \frac{15}{2}$$

$$\underline{a}.\underline{b} = 7.5$$

$$\cos\theta = \frac{\underline{a}.\underline{b}}{|a||b|}$$

$$\cos\theta = \frac{7.5}{3 \times 5} = 1.5$$

$$\theta = \cos^{-1}(0.5)$$

$$\theta = 60^\circ$$

3. If $|a| = 3$, $|b| = 4$ and $|\underline{a} + \underline{b}| = 5$. Find the angle between $\underline{a}$ and b.

$$|\underline{a} + \underline{b}| = 5$$

taking square on both sides.

$$|a + \underline{b}|^2 = (5)^2$$

$$(\underline{a} + \underline{b}).(\underline{a} + \underline{b}) = 25$$

$$\underline{a}.\underline{a} + \underline{a}.\underline{b} + \underline{b}.\underline{a} + \underline{b}.\underline{b} = 25$$

$$|\underline{a}|^2 + 2\underline{a}.\underline{b} + |\underline{b}|^2 = 25$$

$$|3|^2 + 2\underline{a}.\underline{b} + |4|^2 = 25$$

$$9 + 2\underline{a}.\underline{b} + 16 = 25$$

$2\,\underline{a}.\underline{b} = 25 - 25$

$2\,a.\underline{b} = 0$

$\underline{a}.\underline{b} = 0$

$$\cos\theta = \frac{\underline{a}.\underline{b}}{|a||b|}$$

$$= \frac{0}{3 \times 4} = 0$$

$\theta = \cos^{-1}(0)$

$\theta = 90^\circ$

4. Find the projection of $\underline{a}$ along $\underline{b}$ and projection of $\underline{b}$ along $\underline{a}$, when,

(i) $\underline{a} = 2\underline{i} + 3\underline{j} - \underline{k}$, $\underline{b} = \underline{i} - 2\underline{j} + 4\underline{k}$

$\underline{a}.\underline{b} = |a||b|\cos\theta$

$\underline{a}.\underline{b} = (2\underline{i} + 3\underline{j} - \underline{k}).(\underline{i} - 2\underline{j} + 4\underline{k})$

$= 2 \times 1 + 3 \times -2 + (-1) \times 4$

$= 2 - 6 - 4$

$\underline{a}.\underline{b} = -8$

$|\underline{a}| = \sqrt{(2)^2 + (3)^2 + (1)^2}$

$= \sqrt{4 + 9 + 1}$

$|\underline{a}| = \sqrt{14}$

$|\underline{b}| = \sqrt{(1)^2 + (-2)^2 + (4)^2}$

$|\underline{b}| = \sqrt{1+4+16}$

$= \sqrt{21}$

Projection of $\underline{a}$ along $\underline{b}$:

$a \cos\theta = \frac{\underline{a}.\underline{b}}{|\underline{b}|^2}\vec{b}$

$= \frac{-8}{(\sqrt{21})^2}\vec{b} = \frac{-8}{21}\vec{b}$

Projection of $\underline{b}$ along $\underline{a}$

$b \cos\theta = \frac{\underline{a}.\underline{b}}{|a|^2}\vec{a}$

$= \frac{-8}{(\sqrt{14})^2}\vec{a} = \frac{-8}{14}\vec{a}$

(ii) $a = 4\underline{i} - 2\underline{j} + 3\underline{k}$, $b = \underline{i} + \underline{j} + \underline{k}$

$\underline{a}.\underline{b} = |a||b|\cos\theta$

$\underline{a}.\underline{b} = (4\underline{i} - 2\underline{j} + 3\underline{k}).(\underline{i} + \underline{j} + \underline{k})$

$= 4\times1 - 2\times1 + 3\times1$

$= 4 - 2 + 3$

$a.b = 5$

$|a| = \sqrt{(4)^2 + (-2)^2 + (3)^2}$

$= \sqrt{16+4+9}$

$|a| = \sqrt{29}$

$|b| = \sqrt{(1)^2 + (1)^2 + (1)^2}$

$|b| = \sqrt{3}$

~~cos θ =~~

Projection of a along b:

$$a \cos\theta = \frac{a \cdot b}{|b|^2} \vec{b}$$

$$= \frac{5}{(\sqrt{3})^2} \vec{b}$$

$$= \frac{5}{3} \vec{b}$$

Projection of b along a:

$$b \cos\theta = \frac{a \cdot b}{|a|^2} \vec{a}$$

$$= \frac{5}{(\sqrt{29})^2} \vec{a}$$

$$= \frac{5}{29} \vec{a}$$

5. Find a real number α so that vector $\underline{u}$ and $\underline{v}$ are perpendicular:

(i) $\underline{u} = \alpha \underline{i} + 3\underline{j} + \underline{k}$, $\underline{v} = \underline{i} - 2\underline{j} + \alpha \underline{k}$

$\underline{u} \cdot \underline{v} = 0$

$(\alpha \underline{i} + 3\underline{j} + \underline{k}) \cdot (\underline{i} - 2\underline{j} + \alpha \underline{k}) = 0$

$\alpha \times 1 + 3 \times -2 + 1 \times \alpha = 0$

$a - 6 + a = 0$

$2a - 6 = 0$

$2a = 6$

$\boxed{a = 3}$

(ii) $\underline{u} = a\underline{i} + 2a\underline{j} - \underline{k}$, $\underline{v} = \underline{i} + a\underline{j} + 3\underline{k}$

$\underline{u} \cdot \underline{v} = 0$

$(a\underline{i} + 2a\underline{j} - \underline{k}) \cdot (\underline{i} + a\underline{j} + 3\underline{k}) = 0$

$a + 2a^2 - 3 = 0$

$2a^2 + a - 3 = 0$

$2a^2 + 3a - 2a - 3 = 0$

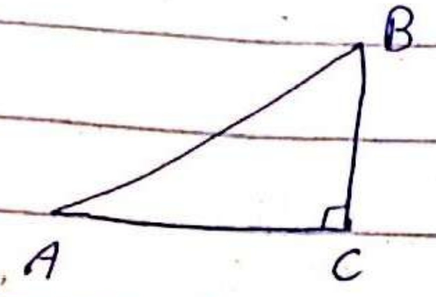
$a(2a + 3) - 1(2a + 3) = 0$

$(2a + 3)(a - 1) = 0$

$2a + 3 = 0$	$a - 1 = 0$
$\boxed{a = -3/2}$	$\boxed{a = 1}$

6. Find the number z so that the triangle with vertices A(3, 0, -2), B(0, 3, 1) and C(1, 1, z) is a triangle with right angle at C.

As



$\underline{CA} \perp \underline{CB} = 0$

$\underline{CA} \cdot \underline{CB} = 0$

$\underline{CA} = \underline{OA} - \underline{OC}$

$= 3\underline{i} + 0\underline{j} - 2\underline{k} - (\underline{i} + \underline{j} + z\underline{k})$

$= (3-1)\underline{i} + (0-1)\underline{j} - (2+z)\underline{k}$

$= 2\underline{i} - \underline{j} - (2+z)\underline{k}$

$\underline{CB} = \underline{OB} - \underline{OC}$

$= 0\underline{i} + 3\underline{j} + \underline{k} - (\underline{i} + \underline{j} + z\underline{k})$

$= -\underline{i} + 2\underline{j} + (1-z)\underline{k}$

$\underline{CA} \cdot \underline{CB} = 0$

$(2\underline{i} - \underline{j} - (2+z)\underline{k}) \cdot (-\underline{i} + 2\underline{j} + (1-z)\underline{k}) = 0$

$2 \times -1 + (-1) \times 2 + (1-z)(-(2+z)) = 0$

$-2 - 2 - (2 + z - 2z - z^2) = 0$

$-4 - 2 - z + 2z + z^2 = 0$

$z^2 + z - 6 = 0$

$z^2 + 3z - 2z - 6 = 0$

$z(z+3) - 2(z+3) = 0$

$(z+3)(z-2) = 0$

$\boxed{z = -3}$ $\boxed{z = +2}$

7. If $\hat{a}$ and $\hat{b}$ are unit vectors and 2θ is the angle between them, show that

$\sin\theta = \frac{1}{2}|\hat{a} - \hat{b}|.$

$|\hat{a}-\hat{b}|^2 = |\hat{a}|^2 + |\hat{b}|^2 - 2|\hat{a}||\hat{b}|\cos\theta$

$|\hat{a}-\hat{b}|^2 = 1^2 + 1^2 - 2(1)(1)\cos 2\theta$

$= 2 - 2\cos 2\theta$

$= 2(1-\cos 2\theta)$

$= 2[1-(1-2\sin^2\theta)]$

$= 2[1-1+2\sin^2\theta]$

$= 2(2\sin^2\theta)$

$|\hat{a}-\hat{b}|^2 = 4\sin^2\theta$

Taking square root on both sides

$\sqrt{|\hat{a}-\hat{b}|^2} = \sqrt{4\sin^2\theta}$

$|\hat{a}-\hat{b}| = 2\sin\theta$

$\frac{1}{2}|\hat{a}-\hat{b}| = \sin\theta$

8. If $|\underline{a}+\underline{b}| = |\underline{a}-\underline{b}|$, then show that $\underline{a}$ and $\underline{b}$ are perpendicular.

$|\underline{a}+\underline{b}|^2 = |\underline{a}-\underline{b}|^2$

$(\underline{a}+\underline{b}).(\underline{a}+\underline{b}) = (\underline{a}-\underline{b}).(\underline{a}-\underline{b})$

$\underline{a}.\underline{a} + \underline{a}.\underline{b} + \underline{b}.\underline{a} + \underline{b}.\underline{b} = \underline{a}.\underline{a} - \underline{a}.\underline{b} - \underline{b}.\underline{a} + \underline{b}.\underline{b}$

$|\underline{a}|^2 + 2\,\underline{a}.\underline{b} + |\underline{b}|^2 = |\underline{a}|^2 - 2\,\underline{a}.\underline{b} + |\underline{b}|^2$

$2\,\underline{a}.\underline{b} + 2\,\underline{a}.\underline{b} = 0$

$4\,\underline{a}.\underline{b} = 0$

$\underline{a} \cdot \underline{b} = 0$

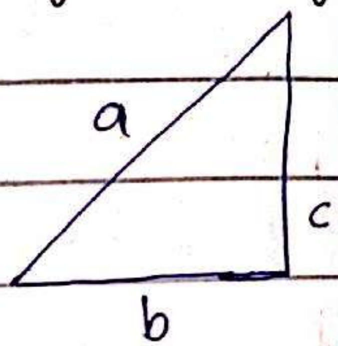
Hence, prove that are perpendicular.

9. (i) Show that the vectors $3\underline{i} - 2\underline{j} + \underline{k}$, $\underline{i} - 3\underline{j} + 5\underline{k}$ and $2\underline{i} + \underline{j} - 4\underline{k}$ form a right triangle.

$\underline{a} = 3\underline{i} - 2\underline{j} + \underline{k}$

$\underline{b} = \underline{i} - 3\underline{j} + 5\underline{k}$

$\underline{c} = 2\underline{i} + \underline{j} - 4\underline{k}$



From fig $\underline{b} + \underline{c} = \underline{a}$

$\underline{b} + \underline{c} = \underline{i} - 3\underline{j} + 5\underline{k} + 2\underline{i} + \underline{j} - 4\underline{k}$

$= 3\underline{i} - 2\underline{j} + \underline{k}$

$\underline{b + c} = \underline{a}$

For Right triangle :

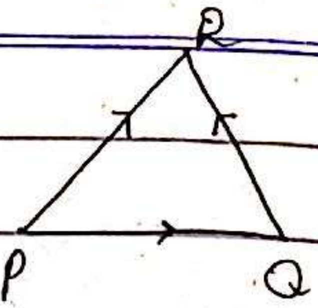
$\underline{a} \cdot \underline{c} = (3\underline{i} - 2\underline{j} + \underline{k}) \cdot (2\underline{i} + \underline{j} - 4\underline{k})$

$= 6 - 2 - 4 = 0$

$\underline{a} \cdot \underline{c} = 0$

So, $\underline{a} \perp \underline{c}$

(ii) Show that the set of points $P(4, -1, 2)$, $Q(1, 3, -1)$ and $R(-2, 4, 6)$ form a right triangle.



$\overline{PQ} = Q - \overline{P}$

$= (i + 3j - k) - (4i - j + 2k)$

$= i + 3j - k - 4i + j - 2k$

$\overline{PQ} = -3i + 4j - 3k$

$\overline{QR} = R - Q$

$= (-2i + 4j + 6k) - (i + 3j - k)$

$= -2i + 4j + 6k - i - 3j + k$

$\overline{QR} = -3i + j + 7k$

$\overline{PR} = R - P$

$= (-2i + 4j + 6k) - (4i - j + 2k)$

$= -2i + 4j + 6k - 4i + j - 2k$

$\overline{PR} = -6i + 5j + 4k$

For triangle:

$PR = PQ + QR$

$PQ + QR = (-3i + 4j - 3k) + (-3i + j + 7k)$

$= -6i + 5j + 4k$

$PQ + QR = PR$

For Right triangle:

$PQ \cdot QR = (-3i + 4j - 3k) \cdot (-3i + j + 7k)$

$= 9 + 4 - 21 \neq 0$

$PQ \cdot PR = (-3i + 4j - 3k) \cdot (-6i + 5j + 4k)$

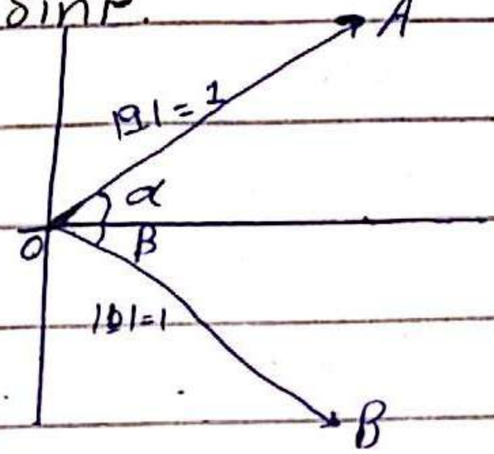
$= 18 + 20 - 12 \neq 0$

$\underline{QR} . \underline{PR} = (-3i + j + 7k) . (-6i + 5j + 4k)$

$= 18 + 5 + 28 \neq 0$

Not a Right angle triangle.

10\. Prove that $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.



$\vec{OA} = a = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$

$\vec{OB} = b = \cos\beta\,\hat{i} - \sin\alpha\,\hat{j}$

$\vec{OA} . \vec{OB} = (\cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}) . (\cos\beta\,\hat{i} - \sin\beta\,\hat{j})$

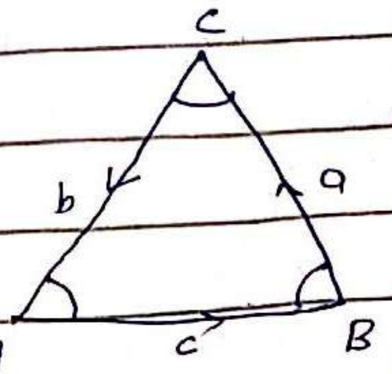
$|\vec{OA}||\vec{OB}| \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$1 . 1 \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

11\. Prove that in any triangle ABC.

(i) $b = c\cos A + a\cos C$



$\underline{a} + \underline{b} + \underline{c} = 0$

$\underline{b} = -(\underline{a} + \underline{c})$

$\underline{b} . \underline{b} = -(\underline{a} + \underline{c}) . \underline{b}$

$\underline{b} \cdot \underline{b} = -\underline{a} \cdot \underline{b} - \underline{c} \cdot \underline{b}$

$|\underline{b}||\underline{b}| \cos 0 = -|\underline{a}||\underline{b}| \cos(\pi - \angle C) - |\underline{c}||\underline{b}| \cos(\pi - \angle A)$

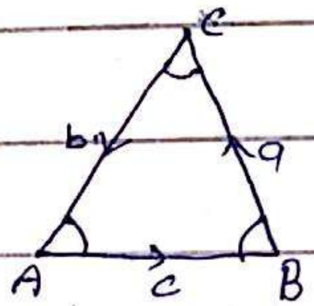
$|b|^2(1) = -ab(-\cos C) - cb(-\cos A)$

$b^2 = ab\cos C + cb \cos A$

$b^2 = b(a\cos C + c\cos A)$

$b = a\cos C + c\cos A = c\cos A + a\cos C$

(ii) $c = a\cos B + b\cos A$



$\underline{a} + \underline{b} + \underline{c} = 0$

$\underline{c} = -(\underline{a} + \underline{b})$

$\underline{c} \cdot \underline{c} = -(\underline{a} + \underline{b}) \cdot \underline{c}$

$\underline{c} \cdot \underline{c} = -(\underline{a} + \underline{b}) \cdot (-(\underline{a} + \underline{b}))$

$|\underline{c}||\underline{c}| \cos 0 = \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b} + \underline{b} \cdot \underline{a} + \underline{b} \cdot \underline{b}$

$|\underline{c}||\underline{c}| \cos 0 = |\underline{a}|^2 + 2\,\underline{a} \cdot \underline{b} + |\underline{b}|^2$

$c^2(1) = a^2 + b^2 + 2|a$

$\underline{c} \cdot \underline{c} = -\underline{a} \cdot \underline{c} - \underline{b} \cdot \underline{c}$

$|\underline{c}||\underline{c}| \cos 0 = -|\underline{a}||\underline{c}| \cos(\pi - \angle B) - |b||c| \cos(\pi - \angle A)$

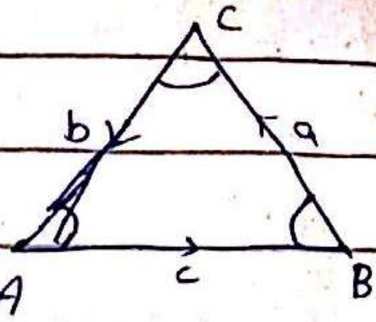
$c^2(1) = -ac(-\cos B) - bc(-\cos A)$

$c^2 = ac\cos B + bc\cos A$

$c^2 = c(a\cos B + b\cos A)$

$c = a\cos B + b\cos A$

(iii) $b^2 = c^2 + a^2 - 2ca\cos B$



$a + b + c = 0$

$b = -(a + c)$

$\underline{b} \cdot \underline{b} = -(\underline{a} + \underline{c}) \cdot \underline{b}$

$\underline{b} \cdot \underline{b} = -(\underline{a} + \underline{c}) \cdot (-(\underline{a} + \underline{c}))$

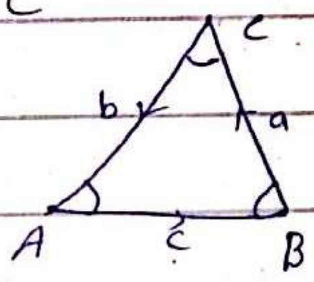
$|b||b|\cos 0 = \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{c} + \underline{c} \cdot \underline{a} + \underline{c} \cdot \underline{c}$

$b^2(1) = a^2 + c^2 + 2\, \underline{a} \cdot \underline{c} \cos(\pi - \angle B)$

$b^2 = c^2 + a^2 + 2ca\, \cos(-\cos B)$

$b^2 = c^2 + a^2 - 2ca\cos B$

(iv) $c^2 = a^2 + b^2 - 2ab\cos C$



$a + b + c = 0$

$c = -(a + b)$

$\underline{c} \cdot \underline{c} = -(\underline{a} + \underline{b}) \cdot \underline{c}$

$\underline{c} \cdot \underline{c} = -(\underline{a} + \underline{b}) \cdot (-(\underline{a} + \underline{b}))$

$\underline{c} \cdot \underline{c} = \underline{a} \cdot \underline{a} + \underline{a} \cdot b + \underline{b} \cdot \underline{a} + \underline{b} \cdot \underline{b}$

$|c||c|\cos 0 = |\underline{a}|^2 + 2\, a \cdot \underline{b} + |b|^2$

$c^2(1) = a^2 + b^2 + 2ab\cos(\pi - \angle C)$

$c^2 = a^2 + b^2 + 2ab(-\cos C)$

$c^2 = a^2 + b^2 - 2ab\cos C$

12. Show that for any vectors a and b, $||a| - |b|| \leq |a + b| \leq |a| + |b|$

$|\underline{a} + b|^2 = (a + b) \cdot (a + b)$

$|a+b|^2 = a.a + a.b + b.a + b.b$

$|a+b|^2 = |a|^2 + 2\,a.b + |b|^2$

$|a+b|^2 = |a|^2 + 2ab\cos\theta + |b|^2$

$|a+b|^2 \leq |a|^2 + 2\,a.b + |b|^2$

$\sqrt{|a+b|^2} \leq \sqrt{(|a|^2 + |b|^2)^2}$

$|a+b| \leq |a|^2 + |b|^2 \rightarrow (i)$

$|a| = |a+b-b|$

$|a| = |a+b| + |-b|$

$|a| \leq |a+b| + |b|$

$|a| - |b| \leq |a+b| \rightarrow (ii)$

$|b| = |a+b-a|$

$|b| = |a+b| + |-a|$

$|b| \leq |a+b| + |a|$

$|b| - |a| \leq |a+b|$

$-(|b| - |a|) \geq -|a+b|$

$|a| - |b| \geq -|a+b|$

$-|a+b| \leq |a| - |b|$

Combining (ii) and (iii)

$-|a+b| \leq |a| - |b| \leq |a+b|$

$||a| - |b|| \leq |a+b| \rightarrow (iv)$

Combining (i) and (iv)

$||a| - |b|| \leq |a+b| \leq |a| + |b|$

13. Find the work done, if the point at which the constant force $\underline{F} = 2\underline{i} + 5\underline{j} + 3\underline{k}$ is applied to an object, moves it from $P_1(2, -3, +1)$ to $P_2(7, 5, 3)$.

let

$$d = \overline{P_1P_2} = P_2 - P_1$$

$$\vec{d} = (7\underline{i} + 5\underline{j} + 3\underline{k}) - (2\underline{i} - 3\underline{j} + \underline{k})$$

$$\vec{d} = (7-2)\underline{i} + (5+3)\underline{j} + (3-1)\underline{k}$$

$$\vec{d} = 5\underline{i} + 8\underline{j} + 2\underline{k}$$

$$W = \vec{F}.\vec{d}$$

$$W = (2\underline{i} + 5\underline{j} + 3\underline{k}).(5\underline{i} + 8\underline{j} + 2\underline{k})$$

$$W = 2 \times 5 + 5 \times 8 + 3 \times 2$$

$$W = 10 + 40 + 6$$

$$W = 56 \text{ units}$$

14. A particle, acted by constant force $F_1 = 3\underline{i} + 4\underline{j} - 3\underline{k}$ and $F_2 = \underline{i} + 4\underline{j} - \underline{k}$, is displaced from $A(2, 1, 3)$ to $B(5, 4, 4)$. Find the work done.

$$F_1 = 3\underline{i} + 4\underline{j} - 3\underline{k}$$

$$F_2 = \underline{i} + 4\underline{j} - \underline{k}$$

$$F_1 + F_2 = 4\underline{i} + 8\underline{j} - 4\underline{k}$$

$F_1 + F_2 = \vec{F}$

$A = 2\underline{i} + \underline{j} + 3\underline{k}$, $B = 5\underline{i} + 4\underline{j} + 4\underline{k}$

$\vec{AB} = B - A$

$\vec{AB} = (5\underline{i} + 4\underline{j} + 4\underline{k}) - (2\underline{i} + \underline{j} + 3\underline{k})$

$\vec{AB} = (5-2)\underline{i} + (4-1)\underline{j} + (4-3)\underline{k}$

$\vec{AB} = 3\underline{i} + 3\underline{j} + \underline{k}$

$W = \vec{F} \cdot \vec{AB}$

$W = (4\underline{i} + 8\underline{j} - 4\underline{k}) \cdot (3\underline{i} + 3\underline{j} + \underline{k})$

$W = 12 + 24 - 4$

$W = 32$ units

15. A particle is displaced from the point $A(5, -5, -7)$ to the point $B(6, 2, -2)$ under the action of constant force defined by $10\underline{i} - \underline{j} + 11\underline{k}$, $4\underline{i} + 5\underline{j} + 9\underline{k}$ and $-2\underline{i} + \underline{j} - 9\underline{k}$. Show that the total work done by the force is 102 units.

Let

$F_1 = 10\underline{i} - \underline{j} + 11\underline{k}$, $F_2 = 4\underline{i} + 5\underline{j} + 9\underline{k}$

$F_3 = -2\underline{i} + \underline{j} - 9\underline{k}$

$F_1 + F_2 + F_3 = F$

$F = 12\underline{i} + 5\underline{j} + 11k$

$A = 5\underline{i} - 5\underline{j} - 7\underline{k}$, $B = (6\underline{i} + 2\underline{j} - 2\underline{k})$

$\vec{AB} = B - A$

$\vec{AB} = 6\underline{i} + 2\underline{j} - 2\underline{k} - 5\underline{i} + 5\underline{j} + 7\underline{k}$

$\vec{AB} = (6-5)\underline{i} + (2+5)\underline{j} + (-2+7)\underline{k}$

$\vec{AB} = \underline{i} + 7\underline{j} + 5k$

$W = \vec{F} \cdot \vec{AB}$

$W = (12\underline{i} + 5\underline{j} + 11\underline{k}) \cdot (\underline{i} + 7\underline{j} + 5\underline{k})$

$W = 12 \times 1 + 5 \times 7 + 11 \times 5$

$W = 102$ units

that's prove

16. A force of magnitude 6 units acting parallel to $4\underline{i} + 3\underline{j} - \underline{k}$ displace the point of application from $A(2, -1, 3)$ to $B(7, 3, 2)$. Find the work done.

Let $V = 4\underline{i} + 3\underline{j} - \underline{k}$

$\vec{F} = 6\hat{V}$

$|V| = \sqrt{(4)^2 + (3)^2 + (-1)^2}$

$|V| = \sqrt{16 + 9 + 1}$

$|V| = \sqrt{26}$

$\hat{V} = \frac{V}{|V|}$

$$\hat{v} = \frac{4}{\sqrt{26}}\underline{i} + \frac{3}{\sqrt{26}}\underline{j} - \frac{1}{\sqrt{26}}\underline{k}$$

$$\vec{F} = 6\left(\frac{4}{\sqrt{26}}\underline{i} + \frac{3}{\sqrt{26}}\underline{j} - \frac{1}{\sqrt{26}}\underline{k}\right)$$

$$A = (2\underline{i} - \underline{j} + 3\underline{k}) \quad , \quad B = (7\underline{i} + 3\underline{j} + 2\underline{k})$$

$$\vec{AB} = B - A$$

$$= (7i + 3\underline{j} + 2\underline{k}) - (2\underline{i} - \underline{j} + 3\underline{k})$$

$$= 7i + 3\underline{j} + 2\underline{k} - 2\underline{i} + \underline{j} - 3\underline{k}$$

$$\vec{AB} = 5\underline{i} + 4\underline{j} - \underline{k}$$

$$W = \vec{F} \cdot \vec{AB}$$

$$= \frac{6}{\sqrt{26}}(4\underline{i} + 3\underline{j} - \underline{k}) \cdot (5\underline{i} + 4\underline{j} - \underline{k})$$

$$= \frac{6}{\sqrt{26}}(20 + 12 + 1)$$

$$= \frac{6}{\sqrt{26}} \times 33$$

$$W = \frac{198}{\sqrt{26}} \text{ units}$$

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