10.4

1. Express the following products as sums or differences:

(i) $2\sin 3\theta \cos\theta$

$= \sin(3\theta+\theta)+\sin(3\theta-\theta)$

$= \sin 4\theta + \sin 2\theta$

(ii) $2\cos 5\theta \sin 3\theta$

$= \sin(5\theta+3\theta)-\sin(5\theta-3\theta)$

$= \sin 8\theta - \sin 2\theta$

(iii) $\sin 5\theta \cos 2\theta$

$= \dfrac{(\sin(5\theta+2\theta)+\sin(5\theta-2\theta))}{2}$

$= \dfrac{(\sin 7\theta + \sin 3\theta)}{2}$

$= \dfrac{1}{2}(\sin 7\theta + \sin 3\theta)$

(iv) $2\sin 7\theta \sin 2\theta$

$= -[\cos(7\theta+2\theta)-\cos(7\theta-2\theta)]$

$= -\cos 9\theta + \cos 5\theta$

$= \cos 5\theta - \cos 9\theta$

تاریخ: ______

(v) $\cos(x+y)\sin(x-y)$

$= \frac{1}{2}[\sin(x+\cancel{y}+x-\cancel{y}) - \sin(x+y-(x-y))]$

$= \frac{1}{2}[\sin 2x - \sin(\cancel{x}+y-\cancel{x}+y]$

$= \frac{1}{2}[\sin 2x - \sin 2y]$

(vi) $\cos(2x+30^\circ)\cos(2x-30^\circ)$

$= \frac{1}{2}[\cos(2x+\cancel{30}+2x-\cancel{30}) + \cos(2x+30-(2x-30))]$

$= \frac{1}{2}[\cos 4x + \cos(\cancel{2x}+30-\cancel{2x}+30)]$

$= \frac{1}{2}[\cos 4x + \cos(\cancel{2x+30}\,60^\circ]$

(vii) $\sin 12^\circ \sin 46^\circ$

$= -\frac{1}{2}[\cos(12^\circ+46^\circ) - \cos(\overset{12}{\cancel{12}} - 46^\circ)]$

$= -\frac{1}{2}[\cos 58^\circ - \cos 34^\circ]$

$= \frac{1}{2}[\cos 34 - \cos 58^\circ]$

(viii) $\sin(x+45^\circ)\sin(x-45^\circ)$

$= -\frac{1}{2}[\cos(x+45^\circ+(x-45^\circ)) - \cos(x+45^\circ-(x-45^\circ))]$

$= -\frac{1}{2}[\cos(x+\cancel{45^\circ}+x-\cancel{45^\circ}) - \cos(\cancel{x}+45^\circ-\cancel{x}+45^\circ)]$

$= -\frac{1}{2}[\cos 2x - \cos 90^\circ]$

$= \frac{1}{2}(\cos 90^\circ - \cos 2x)$

2. Express the following sums or differences as products:

(i) $\sin 5\theta + \sin 3\theta$

$= 2 \sin \frac{5\theta+3\theta}{2} \cos \frac{5\theta-3\theta}{2}$

$= 2 \sin \frac{8\theta}{2} \cos \frac{2\theta}{2}$

$= 2 \sin 4\theta \cos\theta$

(ii) $\sin 8\theta - \sin 4\theta$

$= 2 \cos \frac{8\theta+4\theta}{2} \sin \frac{8\theta-4\theta}{2}$

$= 2 \cos \frac{12\theta}{2} \sin \frac{4\theta}{2}$

$= 2 \cos 6\theta \sin 2\theta$

(iii) $\cos 6\theta + \cos 3\theta$

$= 2 \cos \frac{6\theta + 3\theta}{2} \cos \frac{6\theta - 3\theta}{2}$

$= 2 \cos \frac{9\theta}{2} \cos \frac{3\theta}{2}$

(iv) $\cos 7\theta - \cos \theta$

$= -2 \sin \frac{7\theta + \theta}{2} \sin \frac{7\theta - \theta}{2}$

$= -2 \sin \frac{8\theta}{2} \sin \frac{6\theta}{2}$

$= -2 \sin 4\theta \sin 3\theta$

(v) $\cos 12^\circ + \cos 48^\circ$

$= 2 \cos \frac{12^\circ + 48^\circ}{2} \cos \frac{12^\circ - 48^\circ}{2}$

$= 2 \cos \frac{60^\circ}{2} \cos \frac{-36^\circ}{2}$

$= 2 \cos 30^\circ \cos -18^\circ$

$= 2 \cos 30^\circ \cos 18^\circ$

(vi) $\sin(x + 30^\circ) + \sin(x - 30^\circ)$

$= 2 \sin \frac{(x + \cancel{30^\circ} + x - \cancel{30^\circ})}{2} \cos(\cancel{x} + 30^\circ - \cancel{x} + 30^\circ)$

$= 2 \sin \frac{2x}{2} \cos \frac{60^\circ}{2}$

$= 2 \sin x \cos 30^\circ$

3\. Prove following identities:

(i) $\dfrac{\sin 3x - \sin x}{\cos x - \cos 3x} = \cot 2x$

L.H.S

$$= \frac{2 \cos \frac{3x+x}{2} \sin \frac{3x-x}{2}}{-2 \sin \frac{x+3x}{2} \sin \frac{x-3x}{2}}$$

$$= \frac{2 \cos \frac{4x}{2} \sin \frac{2x}{2}}{-2 \sin \frac{4x}{2} \sin -\frac{2x}{2}}$$

$$= \frac{\cancel{2} \cos 2x \cancel{\sin x}}{\cancel{2} \sin 2x \cancel{\sin x}}$$

$$= \frac{\cos 2x}{\sin 2x}$$

$= \cot 2x \Rightarrow$ R.H.S

Proved L.H.S = R.H.S

(ii) $\dfrac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x} = \tan 5x$

L.H.S

$$= \frac{2 \sin \frac{8x+2x}{2} \cos \frac{8x-2x}{2}}{2 \cos \frac{8x+2x}{2} \cos \frac{8x-2x}{2}}$$

$$= \frac{\cancel{2} \sin \frac{10x}{2} \cos \frac{6x}{2}}{\cancel{2} \cos \frac{10x}{2} \cos \frac{6x}{2}}$$

$$= \frac{\sin 5x \, \cancel{\cos 3x}}{\cos 5x \, \cancel{\cos 3x}}$$

$$= \tan 5x \Rightarrow R.H.S$$

Proved L.H.S = R.H.S

(iii) $\frac{\sin A - \sin B}{\sin A + \sin B} = \tan \frac{A-B}{2} \cot \frac{A+B}{2}$.

L.H.S

$$= \frac{\cancel{2} \cos \frac{A+B}{2} \sin \frac{A-B}{2}}{\cancel{2} \sin \frac{A+B}{2} \cos \frac{A-B}{2}}$$

$$= \cot \frac{A+B}{2} \tan \frac{A-B}{2}$$

$$= \tan \frac{A-B}{2} \cot \frac{A+B}{2} \Rightarrow R.H.S$$

Proved L.H.S = R.H.S

(iv) $\frac{\sin 80^\circ + \sin 40^\circ}{\cos 80^\circ + \cos 40^\circ} = \sqrt{3}$

L.H.S

$$= \frac{\cancel{2}\sin\frac{80+40}{2}\cos\frac{80-40}{2}}{\cancel{2}\cos\frac{80+40}{2}\cos\frac{80-40}{2}}$$

$$= \frac{\sin\frac{120^\circ}{2}\cos\frac{40^\circ}{2}}{\cos\frac{120}{2}\cos\frac{40}{2}}$$

$$= \frac{\sin 60^\circ\ \cancel{\cos 20^\circ}}{\cos 60^\circ\ \cancel{\cos 20^\circ}}$$

$$= \frac{\sqrt{3}/\cancel{2}}{1/\cancel{2}}$$

$= \sqrt{3} \Rightarrow$ R.H.S

Proved L.H.S = R.H.S

4. Prove that:

(i) $\cos 15^\circ + \cos 105^\circ + \cos 195^\circ + \cos 160^\circ + \cos 285^\circ = 0$

$= \cos 15^\circ + \cos 105^\circ + \cos 195^\circ + \cos 225^\circ$

$= \cos 15^\circ + \cos 195^\circ + \cos 105^\circ + \cos 225^\circ$

$= \left[2\cos\frac{15+195}{2}\cos\frac{15-195}{2}\right] + \left[2\cos\frac{105+285}{2}\cos\frac{105-285}{2}\right]$

$= (2\cos 105 \cos 90) + (2\cos 195^\circ \cos 90^\circ)$

$= (2\cos 105^\circ(0) + 2\cos 195(0))$

$= 0 + 0$

$= 0 \Rightarrow$ R.H.S

Proved L.H.S = R.H.S

تاریخ ____

(ii) $\dfrac{\sin 2\theta + \sin 4\theta + \sin 6\theta + \sin 8\theta}{\cos 2\theta + \cos 4\theta + \cos 6\theta + \cos 8\theta} = \tan 5\theta$

L.H.S

$= \dfrac{\sin 2\theta + \sin 8\theta + \sin 4\theta + \sin 6\theta}{\cos 2\theta + \cos 8\theta + \cos 4\theta + \cos 6\theta}$

$= \dfrac{2\sin\frac{2\theta+8\theta}{2}\cos\frac{2\theta-8\theta}{2} + 2\sin\frac{4\theta+6\theta}{2}\cos\frac{4\theta-6\theta}{2}}{2\cos\frac{2\theta+8\theta}{2}\cos\frac{2\theta-8\theta}{2} + 2\cos\frac{4\theta+6\theta}{2}\cos\frac{4\theta-6\theta}{2}}$

$= \dfrac{\cancel{2}(\sin 5\theta\cos(-3\theta)) + \sin 5\theta\cos(-\theta)]}{\cancel{2}[\cos 5\theta\cos(-3\theta) + \cos 5\theta\cos(-\theta)]}$

$= \dfrac{\sin 5\theta(\cancel{\cos 3\theta + \cos\theta})}{\cos 5\theta(\cancel{\cos 3\theta + \cos\theta})}$

$= \dfrac{\sin 5\theta}{\cos 5\theta}$

$= \tan 5\theta \Rightarrow$ R.H.S

Proved L.H.S = R.H.S

(iii) $\cos^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) - \cos^2\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) = \sin\alpha$

$= \cancel{\left[\cos\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) + \cos\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right]\left[\cos\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) - \cos\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right]}$

$\cancel{\frac{1}{2}\left[\cos\left(\frac{\frac{\pi}{4} - \frac{\alpha}{2} + \frac{\pi}{4} + \frac{\alpha}{2}}{2}\right)\cos\left(\frac{\frac{\pi}{4} - \frac{\alpha}{2} - \frac{\pi}{4} - \frac{\alpha}{2}}{2}\right)\right] - \frac{1}{2}\left(\sin\left(\frac{\frac{\pi}{4} - \frac{\alpha}{2} + \frac{\pi}{4} + \frac{\alpha}{2}}{2}\right)\sin\left(\frac{\frac{\pi}{4} - \frac{\alpha}{2} - \frac{\pi}{4} - \frac{\alpha}{2}}{2}\right)\right.}$

$$-\frac{1}{4}\left[\cos\frac{\pi}{4}\cos\frac{(-\alpha)}{2}\right]\left[\sin\left(\frac{\pi}{4}\right)\sin\frac{(-\alpha)}{2}\right]$$

L.H.S

$$\left[\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)+\cos\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right]\left[\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)-\cos\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right]$$

$$=\left[2\cos\left(\frac{\frac{\pi}{4}-\cancel{\frac{\alpha}{2}}+\frac{\pi}{4}+\cancel{\frac{\alpha}{2}}}{2}\right)\cos\left(\frac{\cancel{\frac{\pi}{4}}-\frac{\alpha}{2}-\cancel{\frac{\pi}{4}}-\frac{\alpha}{2}}{2}\right)\right]$$

$$\left[-2\sin\left(\frac{\pi}{4}-\cancel{\frac{\alpha}{2}}+\frac{\pi}{4}+\cancel{\frac{\alpha}{2}}\right)\sin\left(\cancel{\frac{\pi}{4}}-\frac{\alpha}{2}-\cancel{\frac{\pi}{4}}-\frac{\alpha}{2}\right)\right]$$

$$=-4\left[\cos\frac{\cancel{2}\left(\frac{\pi}{4}\right)}{\cancel{2}}\cos\frac{-\cancel{2}\left(\frac{\alpha}{2}\right)}{\cancel{2}}\right]$$

$$\left[\sin\frac{\cancel{2}\left(\frac{\pi}{4}\right)}{\cancel{2}}\sin\frac{-\cancel{2}\left(\frac{\alpha}{2}\right)}{\cancel{2}}\right]$$

$$=-4\left[\cos\frac{\pi}{4}\cos\left(-\frac{\alpha}{2}\right)\right]\left[\sin\frac{\pi}{4}\sin\left(-\frac{\alpha}{2}\right)\right]$$

$$=4\times\frac{1}{\sqrt{2}}\cos\frac{\alpha}{2}\cdot\frac{1}{\sqrt{2}}\cdot\sin\frac{\alpha}{2}$$

$$=\overset{2}{\cancel{4}}\times\frac{1}{\cancel{2}}\cdot\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}$$

$$=2\sin\alpha/2\cos\alpha/2$$

$$=\sin\alpha \Rightarrow R.H.S$$

Proved

L.H.S = R.H.S

تاریخ ______

(iv) $\sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}+\theta\right)=\frac{1}{2}\cos 2\theta$

L.H.S

$=\sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}+\theta\right)$

$=-\frac{1}{2}\left[\cos\left(\frac{\pi}{4}-\cancel{\theta}+\frac{\pi}{4}+\cancel{\theta}\right)-\cos\left(\cancel{\frac{\pi}{4}}-\theta-\cancel{\frac{\pi}{4}}-\theta\right)\right]$

$=-\frac{1}{2}\left[\cos 2\left(\frac{\pi}{4}\right)-\cos(-2\theta)\right]$

$=-\frac{1}{2}\left(\cos\frac{\pi}{2}-\cos 2\theta\right)$

$=-\frac{1}{2}\times(0-\cos 2\theta)$

$=\frac{1}{2}\cos 2\theta \Rightarrow R.H.S$

Proved L.H.S = R.H.S

(v) $\frac{\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta}{\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta}=\tan 4\theta$

L.H.S

$=\frac{\sin 7\theta+\sin\theta+\sin 5\theta+\sin 3\theta}{\cos 7\theta+\cos\theta+\cos 5\theta+\cos 3\theta}$

$=\frac{\left(2\sin\frac{7\theta+\theta}{2}\cos\frac{7\theta-\theta}{2}\right)+\left(2\sin\frac{5\theta+3\theta}{2}\cos\frac{5\theta-3\theta}{2}\right)}{\left(2\cos\frac{7\theta+\theta}{2}\cos\frac{7\theta-\theta}{2}\right)+\left(2\cos\frac{5\theta+3\theta}{2}\cos\frac{5\theta-3\theta}{2}\right)}$

$=\frac{\cancel{2}\left[\sin 4\theta\cos 3\theta+\sin 4\theta\cos\theta\right]}{\cancel{2}\left[\cos 4\theta\cos 3\theta+\cos 4\theta\cos\theta\right]}$

$=\frac{\sin 4\theta(\cancel{\cos 3\theta+\cos\theta})}{\cos 4\theta(\cancel{\cos 3\theta+\cos\theta})}$

$= \tan 4\theta \Rightarrow R.H.S$

Proved $L.H.S = R.H.S$

5. Prove that:

(i) $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

L.H.S

$= \frac{1}{2} \cos 20 (\cos 40^\circ \cos 80^\circ)$

$= \frac{1}{2} \cos 20 \cdot \frac{1}{2} (\cos(40+80) + \underbrace{\cos(40-80)}_{(-40)})$

$= \frac{1}{4} \cos 20 (\cos 120 + \cos(\;))$

$= \frac{1}{4} \cos 20 (-\frac{1}{2} + \cos 40)$

$= -\frac{1}{8} \cos 20 + \frac{1}{4} \cos 20^\circ \cos 40^\circ$

$= -\frac{1}{8} \cos 20 + \frac{1}{8} (\cos(20+40^\circ) + \cos(20-40^\circ))$

$= -\frac{1}{8} \cos 20 + \frac{1}{8} (\cos 60 + \cos(-20))$

$= -\frac{1}{8} \cos 20 + \frac{1}{8} (\frac{1}{2} + \cos 20)$

$= \cancel{-\frac{1}{8} \cos 20} + \frac{1}{16} + \cancel{\frac{1}{8} \cos 20}$

تاریخ ______

$= 1/16 \Rightarrow$ R.H.S

Proved L.H.S = R.H.S

(ii) $\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{\pi}{3}\sin\frac{4\pi}{9} = \frac{3}{16}$

L.H.S

$= \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$

$= \frac{\sqrt{3}}{2}\sin 20^\circ(\sin 80^\circ \sin 40^\circ)$

$= \frac{\sqrt{3}}{2}\sin 20^\circ\left[-\frac{1}{2}(\cos(80+40) - \cos(80-40))\right]$

$= -\frac{\sqrt{3}}{4}\sin 20\left[\cos 120^\circ - \cos 40^\circ\right]$

$= -\frac{\sqrt{3}}{4}\sin 20\left(-\frac{1}{2} - \cos 40^\circ\right)$

$= \frac{\sqrt{3}}{8}\sin 20 + \frac{\sqrt{3}}{4}\sin 20\cos 40$

$= \frac{\sqrt{3}}{8}\sin 20 + \frac{\sqrt{3}}{4}\left(\frac{1}{2}\sin(20+40) + \sin(20-40)\right)$

$= \frac{\sqrt{3}}{8}\sin 20 + \frac{\sqrt{3}}{8}(\sin 60^\circ - \sin 20^\circ)$

$= \frac{\sqrt{3}}{8}\sin 20 + \frac{\sqrt{3}}{8}\left(\frac{\sqrt{3}}{2} - \sin 20^\circ\right)$

$= \cancel{\frac{\sqrt{3}}{8}\sin 20} + \frac{(\sqrt{3})^2}{16} - \cancel{\frac{\sqrt{3}}{8}\sin 20}$

$= \frac{3}{16} \Rightarrow R.H.S$

Proved L.H.S = R.H.S

(iii) $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$

L.H.S

$= \frac{1}{2}\sin 10^\circ (\sin 70^\circ \sin 50^\circ)$

$= \frac{1}{2}\sin 10^\circ [-\frac{1}{2} (\cos(70^\circ+50^\circ) - \cos(70^\circ-50^\circ))]$

$= -\frac{1}{4}\sin 10^\circ [\cos 120^\circ - \cos 20^\circ]$

$= -\frac{1}{4}\sin 10^\circ (-\frac{1}{2} - \cos 20^\circ)$

$= \frac{1}{8}\sin 10^\circ + \frac{1}{4}\sin 10^\circ \cos 20^\circ$

$= \frac{1}{8}\sin 10 + \frac{1}{4}[\frac{1}{2}(\sin(10+20) + \sin(10-20))]$

$= \frac{1}{8}\sin 10 + \frac{1}{8}(\sin 30 - \sin 10)$

$= \frac{1}{8}\sin 10 + \frac{1}{8}(\frac{1}{2} - \sin 10^\circ)$

تاریخ:

$$= \frac{1}{8}\cancel{\sin 10^\circ} + \frac{1}{16} - \frac{1}{8}\cancel{\sin 10^\circ}$$

$$= \frac{1}{16} \Rightarrow R.H.S$$

Proved $L.H.S = R.H.S$

M T W T F S Date:

6. Prove that: $\frac{\sin 3\theta}{1+2\cos 2\theta} = \sin\theta$; deduce the value of $\sin 15°$.

L.H.S

$= \frac{\sin 3\theta}{1+2\cos 2\theta}$

$= \frac{3\sin\theta - 4\sin^3\theta}{1+2(1-2\sin^2\theta)}$

$= \frac{\sin\theta(3-4\sin^2\theta)}{1+2-4\sin^2\theta}$

$= \frac{\sin\theta\,\cancel{(3-4\sin^2\theta)}}{\cancel{3-4\sin^2\theta}}$

$= \sin\theta \Rightarrow$ R.H.S

Put $\theta = 15°$

$= \frac{\sin 3(15°)}{1+2\cos 2(15°)}$

$= \frac{\sin 45°}{1+2\cos 30}$

$= \frac{\frac{1}{\sqrt{2}}}{1+\cancel{2}\frac{\sqrt{3}}{\cancel{2}}}$

$= \frac{1}{\sqrt{2}(1+\sqrt{3})}$

7. Prove that: $\tan 75^\circ - \tan 15^\circ = 2\sqrt{3}$

L.H.S

$= \tan 75^\circ - \tan 15^\circ$

$= \tan(45^\circ + 30^\circ) - \tan(45^\circ - 30^\circ)$

$= \dfrac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} - \dfrac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$

$= \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}} - \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}$

$= \dfrac{\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}} - \dfrac{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}}}$

$= \dfrac{\sqrt{3}+1}{\sqrt{3}-1} - \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \dfrac{(\sqrt{3}+1)^2 - (\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \dfrac{3+1+2\sqrt{3} - (3+1-2\sqrt{3})}{(\sqrt{3})^2 - (1)^2}$

$= \dfrac{4+2\sqrt{3} - (4-2\sqrt{3})}{3-1}$

$= \dfrac{\cancel{4} + 2\sqrt{3} - \cancel{4} + 2\sqrt{3}}{2}$

$= \dfrac{\overset{2}{\cancel{4}}\sqrt{3}}{\cancel{2}}$

$= 2\sqrt{3} \Rightarrow$ R.H.S

8. Prove that: $\cos 15^\circ - \sin 15^\circ = \frac{1}{\sqrt{2}}$

L.H.S

$= \cos 15^\circ - \sin 15^\circ$

$= \cos(45^\circ - 30^\circ) - \sin(45^\circ - 30^\circ)$

$= \cos 45 \cos 30 + \sin 45 \sin 30 - \sin 45 \cos 30 + \cos 45 \sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} - \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\cancel{\sqrt{3}} + 1 - \cancel{\sqrt{3}} + 1}{2\sqrt{2}}$

$= \frac{\cancel{2}}{\cancel{2}\sqrt{2}}$

$= \frac{1}{\sqrt{2}} \Rightarrow$ R.H.S

9. Prove that: $\frac{\sin^2\alpha - \sin^2\beta}{\sin\alpha\cos\alpha - \sin\beta\cos\beta} = \tan(\alpha+\beta)$

L.H.S

$= \frac{(\sin\alpha + \sin\beta)(\sin\alpha - \sin\beta)}{\sin\alpha\cos\alpha - \sin\beta\cos\beta}$

$= \frac{2\left(2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}\right)\left(2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right)}{[2\sin\alpha\cos\alpha - 2\sin\beta\cos\beta]}$

M T W T F S Date:

$$= \frac{8\left(\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha-\beta}{2}\right)}{\sin 2\alpha - \sin 2\beta}$$

$$= \frac{\overset{4}{\cancel{8}}\left(\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha-\beta}{2}\right)}{\cancel{2}\cos\frac{2\alpha+2\beta}{2}\sin\frac{2\alpha-2\beta}{2}}$$

$$= \frac{4\left(\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha-\beta}{2}\right)}{\cos\frac{\cancel{2}(\alpha+\beta)}{\cancel{2}}\sin\frac{\cancel{2}(\alpha-\beta)}{\cancel{2}}}$$

$$= \frac{\left(2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}\right)\left(2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha-\beta}{2}\right)}{\cos(\alpha+\beta)\sin(\alpha-\beta)}$$

$$= \frac{\sin\cancel{2}\left(\frac{\alpha+\beta}{\cancel{2}}\right)\sin\cancel{2}\left(\frac{\alpha-\beta}{\cancel{2}}\right)}{\cos(\alpha+\beta)\sin(\alpha-\beta)}$$

$$= \frac{\sin(\alpha+\beta).\cancel{\sin(\alpha-\beta)}}{\cos(\alpha+\beta).\cancel{\sin(\alpha-\beta)}}$$

$$= \tan(\alpha+\beta) \Rightarrow R.H.S$$

10. Prove that:

$$\sin\alpha+\sin\beta+\sin\gamma-\sin(\alpha+\beta+\gamma) = 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\gamma+\alpha}{2}\right)$$

L.H.S

$$= \sin\alpha+\sin\beta+\sin\gamma-\sin(\alpha+\beta+\gamma)$$

$$= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + 2\cos\frac{\gamma+\alpha+\beta+\gamma}{2}\sin\frac{\gamma-\alpha-\beta-\gamma}{2}$$

$$= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + 2\cos\frac{\alpha+\beta+2\gamma}{2}\sin\left(\frac{-\alpha-\beta}{2}\right)$$

$$= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} - 2\cos\frac{\alpha+\beta+2\gamma}{2}\sin\left(\frac{\alpha+\beta}{2}\right)$$

M T W T F S Date:

$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[\cos\frac{\alpha-\beta}{2} - \cos\frac{\alpha+\beta+2\gamma}{2}\right]$

$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[-2\sin\frac{\frac{\alpha-\beta}{2}+\frac{\alpha+\beta+2\gamma}{2}}{2}\sin\frac{\frac{\alpha-\beta}{2}-\frac{\alpha+\beta+2\gamma}{2}}{2}\right]$

$= -4\sin\left(\frac{\alpha+\beta}{2}\right)\left[\sin\frac{\frac{\alpha-\cancel{\beta}+\alpha+\cancel{\beta}+2\gamma}{2}}{2} \cdot \sin\frac{\frac{\cancel{\alpha}-\beta-\cancel{\alpha}-\beta-2\gamma}{2}}{2}\right]$

$= -4\sin\frac{\alpha+\beta}{2} \cdot \sin\frac{2\alpha+2\gamma}{4} \cdot \sin\frac{(-2\beta-2\gamma)}{4}$

$= 4\sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\cancel{2}(\alpha+\gamma)}{\cancel{4}\,2} \cdot \sin\frac{\cancel{2}(\beta+\gamma)}{\cancel{4}\,2}$

$= 4\sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha+\gamma}{2} \cdot \sin\frac{\beta+\gamma}{2}$

Proved that

L.H.S = R.H.S

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S.S.S (Math)